

## A new method for calculating single-point productivity equation coefficient $\alpha$ in heterogeneous gas reservoir

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**Abstract.** Single-point test is mainly based on quantitative statistics of productivity well testing data to obtain average  $\alpha$  value of gas fields or blocks, then corresponding single-point productivity equation can be obtained. However, for heterogeneous gas reservoir in Ordos Basin, because of the influences of reservoir physical property and test data, there are relatively big error between calculated average  $\alpha$  value of gas field and actual  $\alpha$  value of single gas well, which results in application of single-point empirical formula is limited. Through derivation of binomial productivity equation in this paper, it is found that there are good linear relationship between  $\alpha$  and reciprocal of the square of pressure  $1/p_R^2$ . Combined with actual test data of Jingbian gas field, the empirical formula between  $\alpha$  and  $1/p_R^2$  for 20 gas wells is got through regression analysis. The results indicate that new method greatly improves computational accuracy for absolute open flow of gas wells, which has good application prospect in similar gas field.

### 1. Introduction

Regular productivity well testing include systematic well test, isochronal test, modified isochronal test and single-point test[1-4]. Due to low permeability of lithologic gas reservoir, strong heterogeneity, difference of single well production capacity and large amount of gas wells, regular multipoint production well test are restricted in Ordos Basin. Thus, single-point test is applied in most of gas wells[5-7].

Empirical formula of single-point test is based on large quantity of steady well test data. In another word, single-point productivity equation is derived from average  $\alpha$  which is based on reliable productivity equation and corresponding absolute open flow. Generally, the more stable well testing data of gas field, the more representative single-point productivity equation will be[8-10]. However, because of strong heterogeneity, there are big errors between average  $\alpha$  which is obtained from a gas block by statistics data and single wells. Considering above problems, in order to accurately forecast gas well productivity, it is essential to find a new method to improve computational accuracy.



## 2. Derivation of basic theory

### 2.1 Single-point productivity equation

Making production of gas wells reach steady state under single working system, single-point test can obtain absolute open flow through substituting reservoir pressure, gas production and corresponding steady flowing bottom pressure into empirical productivity formula.

Binomial productivity equation is defined as follow:

$$P_R^2 - P_{wf}^2 = Aq_g + Bq_g^2 \quad (1)$$

where, the coefficient A expresses viscous resistance and coefficient B expresses inertia resistance along flow path, which can all be got through high pressure physical parameters of reservoir and fluid[11-12]. When  $p_{wf}=0.101\text{MPa}(1\text{atm})$ , the maximum potential production capacity is absolute open flow :

$$P_R^2 - (0.101)^2 = Aq_{AOF} + Bq_{AOF}^2 \quad (2)$$

Combination Eq. (1) and Eq. (2), this correlation is given in Eq. (3)

$$\frac{P_R^2 - P_{wf}^2}{P_R^2} = \frac{Aq_g + Bq_g^2}{Aq_{AOF} + Bq_{AOF}^2} \quad (3)$$

where

$$q_D = \frac{q_g}{q_{AOF}} \quad (4)$$

$$p_D = \frac{P_R^2 - P_{wf}^2}{P_R^2} \quad (5)$$

$$\alpha = \frac{A}{A + Bq_{AOF}} \quad (6)$$

Through derivation, single-point productivity equation can be written as:

$$q_{AOF} = \frac{2(1-\alpha)q_g}{\alpha \left[ \sqrt{1 + 4 \left( \frac{1-\alpha}{\alpha^2} \right) \left( \frac{P_R^2 - P_{wf}^2}{P_R^2} \right)} - 1 \right]} \quad (7)$$

### 2.2 Analysis of influence factor

Binomial productivity equation also can be given as[13-15]:

$$P_R^2 - P_{wf}^2 = \frac{1.291 \times 10^{-3} q_{sc} T \bar{\mu} \bar{z}}{kh} \left( \ln \frac{0.472 r_e}{r_w} + S' \right) \quad (8)$$

where

$$S' = S + Dq_{sc} \quad (9)$$

$$D = 2.191 \times 10^{-18} \frac{\beta r_g K}{\bar{\mu} h r_w} \quad (10)$$

$$\beta = 7.644 \times 10^{10} / k^{1.5} \quad (11)$$

Combination Eq. (8) , Eq. (9) , Eq. (10) and Eq. (11), Eq. (12) can be written as:

$$P_R^2 - P_{wf}^2 = \frac{1.291 \times 10^{-3} q_{sc} T \bar{\mu} \bar{z}}{kh} \left( \ln \frac{0.472 r_e}{r_w} + S \right) + \frac{2.828 \times 10^{-21} \beta \gamma_g \bar{z} T q_{sc}^2}{r_w h^2} \quad (12)$$

where

$$A = \frac{1.291 \times 10^{-3} T \bar{\mu} \bar{z}}{kh} \left( \ln \frac{0.472 r_e}{r_w} + S \right) \quad (13)$$

$$B = \frac{2.828 \times 10^{-21} \beta \gamma_g \bar{Z} T}{r_w h^2} \quad (14)$$

For dry gas reservoir, following equation can be used to calculate production rate

$$q_{sc} = \frac{774.6 kh (p_R^2 - p_{wf}^2)}{T \bar{\mu} \bar{z} \left( \ln \frac{0.472 r_e}{r_w} + s' \right)} \quad (15)$$

In fact, wellbore radius, gas viscosity, reservoir temperature, Z-factor and relative gas density is almost the same in the same gas reservoir if ignoring skin effect caused by turbulence flow. Combination Eq.(6), Eq.(13), Eq.(14) and Eq.(15), following simplified formula can be obtained:

$$\alpha = \frac{1}{1 + C_1 k^{0.5} P_R^2} \quad (16)$$

In low-permeability gas reservoir, permeability is related to formation pressure, especially under the condition of stress sensitive effect[16-17]. According to Eq. (16), there is some correlation between  $\alpha$  and permeability as well as reservoir pressure. Actually, coefficient  $\alpha$  is an empirical parameter, which is various in different gas wells or blocks along changes of reservoir characteristics[18-19]. For gas wells in Ordos Basin, it is found that relatively to reservoir pressure, the error of permeability  $k$  is big and the influence on result is small. Thus, Eq. (17) can be written as

$$\alpha = \frac{1}{1 + C_2 P_R^2} \quad (17)$$

$P_R$  and  $\alpha$  are determined by actual test data of gas field. Empirical formula about reciprocal of pressure square  $\frac{1}{P_R^2}$  and  $\alpha$  can be obtained through regression analysis of series of  $P_R$  and  $\alpha$  value for every well.

### 3. Field examples

Based on test data of 20 gas wells in Jingbian gas field, the value of  $\alpha$  can be calculated from static pressure( $p_R$ ),  $p_{wf}$ ,  $q_{sc}$ , an  $q_{AOF}$ . Range of  $\alpha$  value in low-permeability and high-permeability area are 0.548~0.853 and 0.218~0.781, with average value of 0.684 and 0.495 respectively (**Table 1**).

Calculation results show that  $\alpha$  is variable for each well. Average error in low-permeability and high-permeability area is 13.3% and 41.9%, with range of 3.4%~23.7 and 15.5%~127.1% respectively.

From Eq.(7), it is found that it is relevant between  $\alpha$  and reservoir pressure. Through regression analysis of  $\alpha$  value and  $p_R$  in 20 gas wells, it is shown that as decrease of  $1/P_R^2$ , value of  $\alpha$  will increase, with good linear relation.

Thus,  $\alpha$  can be calculated through  $1/p_R^2$  which is proposed as follow.

For  $\alpha$  in low-permeability area (**Figure 1**):

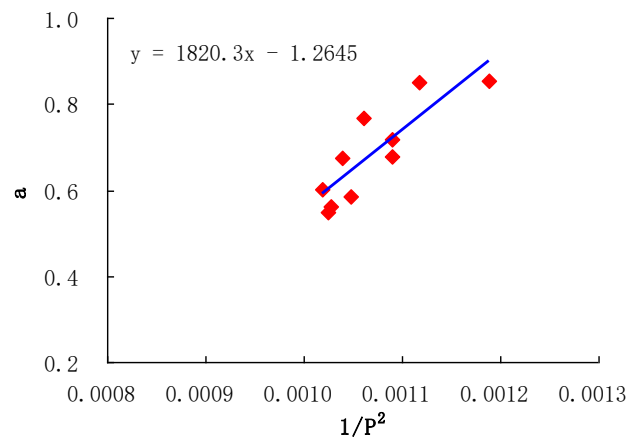
$$\alpha = 1820.3/P^2 - 1.2645 \quad (18)$$

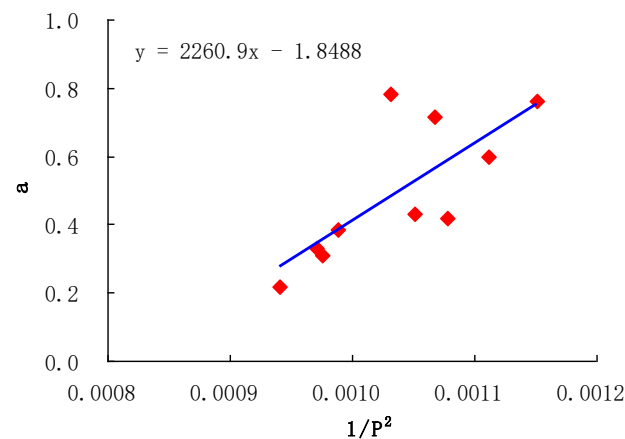
For  $\alpha$  in high-permeability area (**Figure 2**):

$$\alpha = 2260.9/P^2 - 1.8488 \quad (19)$$

**Table 1.** Statistic results of productivity well testing

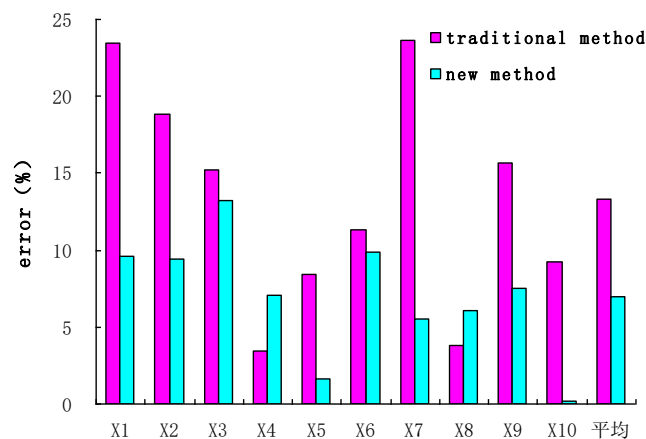
Reservoir types	Wells	$q_g$	$p_{wf}$	$p_R$	$q_{AOF}$	$\alpha$	
		( $10^4 m^3/d$ )	(MPa)	(MPa)	( $10^4 m^3/d$ )	Result	Absolute error(%)
Low permeability	X1	1.49	22.8	29.9	3.26	0.851	23.5
	X2	2.00	28.7	31.3	8.40	0.548	18.8
	X3	2.00	28.3	30.7	10.77	0.768	15.2
	X4	4.00	27.1	31.0	12.99	0.674	3.4
	X5	4.00	28.4	31.3	15.87	0.601	8.4
	X6	2.50	29.5	30.9	18.58	0.585	11.3
	X7	3.00	27.4	29.0	24.36	0.853	23.7
	X8	2.80	28.7	30.3	19.69	0.677	3.8
	X9	3.50	28.2	31.2	13.01	0.563	15.6
	X10	4.50	29.0	30.3	40.13	0.717	9.2
	Average	2.98	27.8	30.6	16.71	0.684	13.3
High permeability	X11	3.01	29.5	31.1	23.15	0.781	36.5
	X12	4.00	30.0	30.8	34.98	0.430	15.5
	X13	8.01	26.7	29.5	36.46	0.761	34.8
	X14	5.97	31.0	32.1	37.50	0.332	49.5
	X15	8.00	28.3	30.6	43.00	0.715	30.7
	X16	15.03	30.8	32.6	58.19	0.218	127.1
	X17	9.85	29.3	30.5	65.17	0.419	18.3
	X18	9.99	31.2	32.0	78.32	0.309	60.6
	X19	4.60	30.5	31.8	27.93	0.385	28.8
	X20	4.10	29.3	30.0	55.71	0.597	16.9
	Average	7.26	29.6	31.1	46.04	0.495	41.9

**Figure 1.**  $\alpha$  vs.  $1/p_R^2$  in low-permeability area

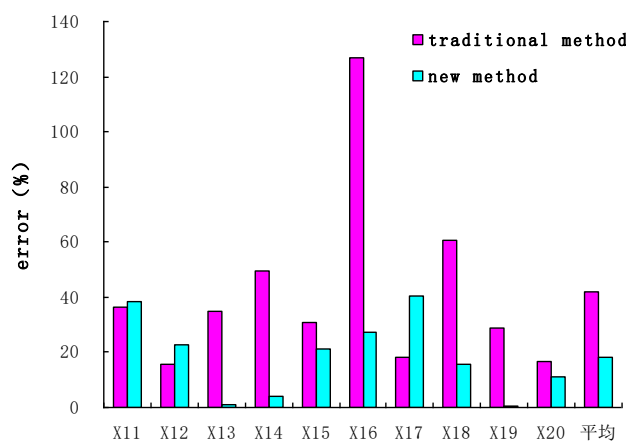


**Figure 2.**  $\alpha$  vs.  $1/p_R^2$  in high-permeability area

The error of new method is obviously smaller than traditional method, where error decline from 13.3% to 7.0% and 41.9% to 18.2 % in low and high permeability area respectively (Table 2, Figure 3 and Figure 4). Contrasting with traditional method, there are high accuracy for calculating absolute open flow of gas well with new method, which has good application prospects in similar gas field.



**Figure 3.** Bar graph for error of  $\alpha$  in low-permeability area



**Figure 4.** Bar graph for error of  $\alpha$  in high-permeability area

**Table 2.** Contrast results of traditional and new methods

Reservoir types	Wells	Traditional method ( $\alpha$ )		New method ( $\alpha$ )	
		Results	Absolute error(%)	Results	Absolute error(%)
Low permeability	X1	0.851	23.5	0.769	9.6
	X2	0.548	18.8	0.599	9.4
	X3	0.768	15.2	0.666	13.2
	X4	0.674	3.4	0.626	7.1
	X5	0.601	8.4	0.591	1.6
	X6	0.585	11.3	0.643	9.9
	X7	0.853	23.7	0.900	5.5
	X8	0.677	3.8	0.718	6.1
	X9	0.563	15.6	0.605	7.5
	X10	0.717	9.2	0.718	0.2
	Average	0.684	13.3	0.684	7.0
High permeability	X11	0.781	36.5	0.482	38.3
	X12	0.430	15.5	0.527	22.7
	X13	0.761	34.8	0.755	0.8
	X14	0.332	49.5	0.345	4.1
	X15	0.715	30.7	0.564	21.2
	X16	0.218	127.1	0.278	27.1
	X17	0.419	18.3	0.589	40.4
	X18	0.309	60.6	0.358	15.8
	X19	0.385	28.8	0.387	0.5
	X20	0.597	16.9	0.663	11.1
	Average	0.495	41.9	0.495	18.2

#### 4. Conclusions

(1) Because of influences of reservoir property and systematic well testing data, value of  $\alpha$  in traditional single-point productivity equation is different from actual  $\alpha$  value of every gas well, which results in error generation of absolute open flow.

(2) There are some correlation between  $\alpha$  value and formation pressure. Empirical formula calculating  $\alpha$  value in high and low permeability area in Jingbian gas field are established.

(3) Contrasting with traditional method, calculation error of new method is obviously smaller, which declines from 13.3% to 7.0% in low permeability area, from 41.9% to 18.2 % in high permeability area respectively. New method can effectively improve calculation accuracy of  $\alpha$  value and absolute open flow.

#### Nomenclature

$q_g$ —wellhead production rate of gas well,  $10^4\text{m}^3/\text{d}$ ;

$q_{\text{AOF}}$ —absolute open flow,  $10^4\text{m}^3/\text{d}$ ;

$k$ —permeability, mD;

$h$ —thickness of gas reservoir, m;

$p_R$ —reservoir pressure, MPa;

$p_{wf}$ —flow bottom hole pressure, MPa;

$T$ —reservoir temperature, K;

$\mu$ —gas viscosity,  $\text{mPa}\cdot\text{s}$ ;

—  
 $Z$  — gas deviation factor;  
 $r_e$  — gas drainage radius, m;  
 $r_w$  — well radius, m;  
 $s'$  — apparent skin factor;  
 $s$  — skin factor;  
 $D$  — non-darcy factor;  
 $\beta$  — velocity factor describing influence of turbulent flow in porous media,  $m^{-1}$ ;  
 $\gamma_g$  — gas relative density;  
 $C_1$  — constant;  
 $C_2$  — constant.

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