

## Energy flows in rock mass under tidal deformation

SV Klishin\* and AF Revuzhenko

Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences,  
Novosibirsk, Russia

E-mail: \*sv.klishin@gmail.com

**Abstract.** Under analysis is the stress state of an elliptical domain under varying loading conditions. The energy flow lines are plotted. The paper demonstrates the effect of the boundary conditions on the shape of the flow lines.

Tidal deformation of the Earth is a global process. It exerts influence, in a varying degree, on all geological processes on the planet: evolution of stresses in the interior of the Earth, including mineral mining depths, mass transfer, deformation of blocks on different scales, behavior of interface areas between the blocks, etc. [1]. Investigation and understanding of the tidal deformation of the Earth is an interesting challenge at this time.

Tidal deformation is connected with the energy physics. On the whole, tidal deformation is induced by kinetic energy of rotation of the Earth. Transformation of the energy of rotation into the energy of deformation of the solid and viscous Earth results in deceleration of Earth's rotation and in elongation of the day length. In kinematic models of tides, energy flows through the boundary of a body. This energy, in a specified approximation, ensures the same kinematics of mass movement as in deformation under body forces.

It seems interesting to address energy flows in these situations. Let the Cartesian coordinate system  $Oxy$  contain an elliptical domain with a center  $(0, 0)$  and axes oriented along the axes  $Ox$  and  $Oy$

$$\frac{x^2}{(1+m)^2} + \frac{y^2}{(1-m)^2} \leq 1, \quad (1)$$

where  $0 \leq m \ll 1$ .

Let us compare deformation of a perfect granular medium and a uniform viscous fluid. According to [2], when Keplerian velocities are assigned at the boundary of an elliptical domain, velocity distribution inside the domains is always linear. And this fact is independent of rheology of a medium. Deviation of boundary conditions from a Keplerian kind results in nonlinearity of velocity field and, consequently, to the velocity dependence on rheology of the medium. The basic characteristics of a nonlinear field are observable in the models of a perfect granular medium and Newtonian viscous fluid. The both models in a first approximation with respect to  $m \ll 1$  yield the same kinematics. On this basis, we analyze the stationary solutions of the Navier–Stokes equations

$$\begin{aligned} \nu \Delta u - \frac{1}{\rho} \frac{\partial p}{\partial x} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}, \quad \nu \Delta v - \frac{1}{\rho} \frac{\partial p}{\partial y} = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \end{aligned} \quad (2)$$



where  $\Delta$  is the Laplace operator;  $u(x, y)$ ,  $v(x, y)$  are the components of the velocity vector  $\mathbf{v}$ ;  $p(x, y)$  is the pressure;  $\rho$  is the density;  $\nu$  is the viscosity coefficient. The boundary conditions for the elliptical domain (1) are set as

$$|\mathbf{v}| = v^0 = \text{const}; \mathbf{v} \cdot \mathbf{n} = 0. \quad (3)$$

Here,  $\mathbf{n}$  is the vector of the outer normal drawn to the boundary. With the scheme from [3], the solution to the problem (2) with the boundary conditions (3) is given by:

$$\begin{aligned} u(x, y) &= -y + m(-3y + 2y^3) + m^2 \left( \frac{7}{4}y + \frac{7}{2}y^3 - \frac{9}{4}y^5 - \frac{21}{2}x^2y + \frac{15}{2}x^2y^3 + \frac{15}{4}x^4y \right), \\ v(x, y) &= x + m(-3x + 2x^3) + m^2 \left( -\frac{7}{4}x - \frac{7}{2}x^3 + \frac{9}{4}x^5 + \frac{21}{2}xy^2 - \frac{15}{2}x^3y^2 - \frac{15}{4}xy^4 \right). \end{aligned} \quad (4)$$

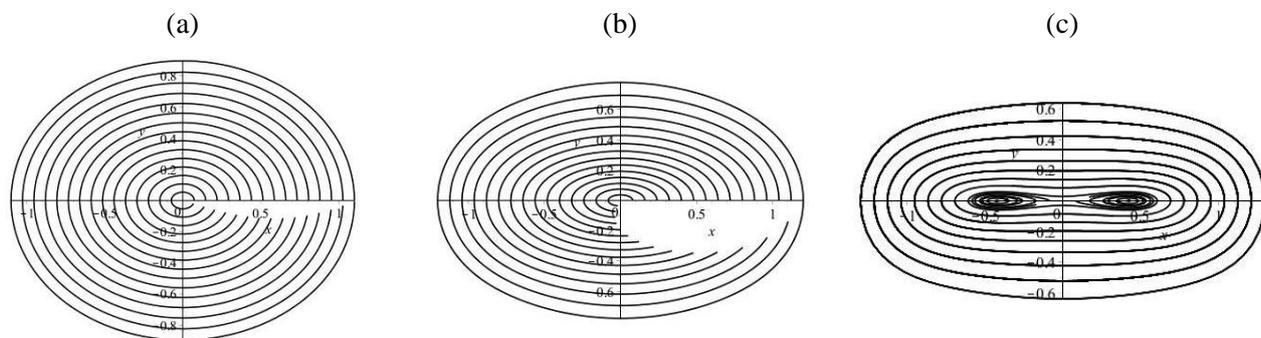
Particle transport in a medium and the mechanism of deformation are determined from the numerical integrating of the system of ordinary differential equations

$$\frac{dx(t)}{dt} = u(x, y), \quad \frac{dy(t)}{dt} = v(x, y)$$

with the initial conditions

$$x(0) = x^0, \quad y(0) = y^0.$$

Figure 1 depicts the trajectories of particles that are in the major semiaxis of the ellipse at a reference time for  $m = 0.1, 0.2$  and  $0.3$ . It is seen that all particles move along closed trajectories, and the periods of revolution around the center are different along different trajectories. Because of this difference, internal deformation of the body grows infinitely with time. According to [2], this effect can be interpreted as the directional mass transfer. In this way, the proposed method of loading can be considered as specification of a definite wave flow on a body surface.



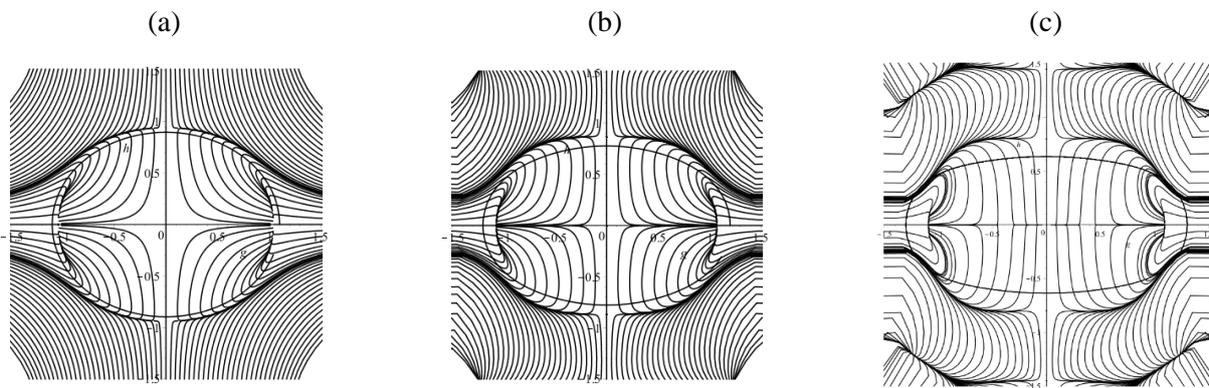
**Figure 1.** Trajectories of particles at (a)  $m = 0.1$ ; (b)  $m = 0.2$ ; (c)  $m = 0.3$ .

Plotting of lines of energy flow reduces to finding a vector field [4–6]

$$\mathbf{E} = -\{\sigma_{xx}u + \sigma_{xy}v, \sigma_{xy}u + \sigma_{yy}v\} \quad (5)$$

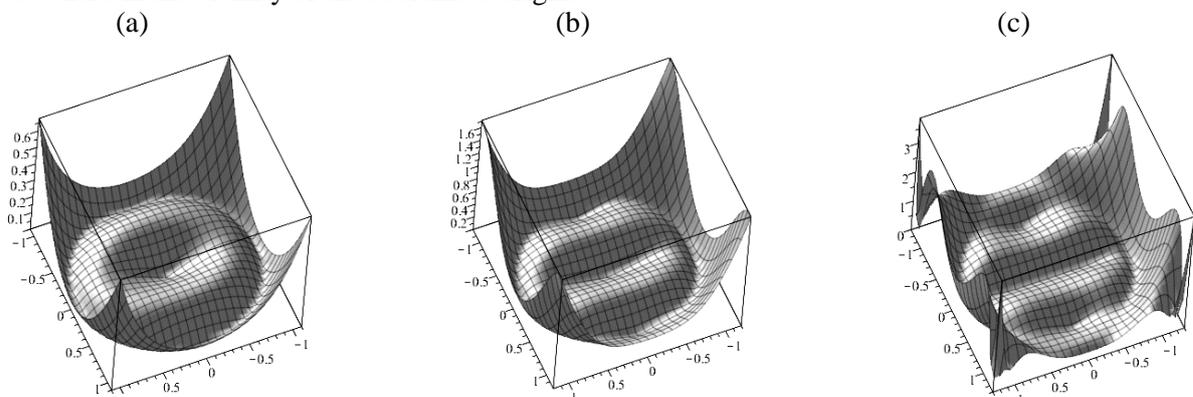
conditioned by stresses  $\sigma_{xx}$ ,  $\sigma_{yy}$ ,  $\sigma_{xy}$  and velocities  $u$  and  $v$ .

It follows from the analysis of deformation processes in different media that stresses in an arbitrary element of a body depend on forces and displacements pre-set at the entire boundary of the body. On the other hand, energy flows into the body not from the entire boundary but from its specific sectors. For the problem under consideration, energy flow lines at different  $m$  are shown in Figure 2. It is seen that at small  $m$ , the energy flow lines envelope the origin of the coordinate system and shape a narrow region of confluence at the intersections of the boundary and major semiaxis of the ellipse. As  $m$  is increased, the flow gets closer to the  $Ox$  axis (Figure 2b), and the confluence boundary drifts toward the  $Oy$  axis. The further increase in  $m$  results in generation of an interface at which some lines of energy flow go back to the domain boundary while the other energy flow lines tend toward the coordinate origin.



**Figure 2.** Energy flow lines for (a)  $m = 0.1$ ; (b)  $m = 0.2$ ; (c)  $m = 0.3$ .

Figure 3 shows the absolute vectors  $E$  for different  $m$ . In all the cases, the absolute vector (5) is close to zero in the vicinity of the coordinate origin.



**Figure 3.** Absolute vectors of energy flow for (a)  $m = 0.1$ ; (b)  $m = 0.2$ ; (c)  $m = 0.3$ .

## Conclusion

During deformation of a solid, energy flow in an elastic material is similar to the compressible fluid flow, and it is always possible to identify segment of the body boundary where energy flows in the body. The kinematic model of tidal deformation discussed in this paper illustrates that energy inflow in the domain induces transport of internal mass.

## References

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