

Evolution of stress state of granular medium under multiple dynamic impacts

AP Bobryakov, VP Kosykh* and AF Revuzhenko**

Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences,
Novosibirsk, Russia

E-mail: *v-kosykh@yandex.ru, **revuzhenko@yandex.ru

Abstract. The paper describes experimental measurement of pressure in marble chips placed in cylindrical reservoir subjected to weak impacting within a long time. It is found that pressure in the granular material changes non-monotonically but fluctuates relative to hydrostatic pressure with the amplitude equaling order of magnitude of the latter. The fractal analysis of the experimental curves of pressure and number of impacts is performed. The Hurst exponent ranges as 0.67–0.73, and the process of pressure variation is fractal and possesses long-term “memory.”

It is common knowledge that geomaterials—rocks, soils, granular media—have various kinds of equilibria. This means that having barely the information on body forces and boundary conditions, it is impossible to assess stresses inside a domain. On the other hand, stress assessment is the basic objective of geomechanics. The question is that there can be self-balancing stresses in a medium [1–3]. When solving a linear problem on secondary stresses, it is allowed to neglect initial stresses. However in case that local damage or local transition to limiting state is probable, the role of the initial stress field may be critical, for instance, in a problem on weak impacts exerted on a medium. Experiments show that such impacts induce irreversible deformations in the medium. So, a medium evolves from one “intrinsic” state to another.

This study focuses on the effect of weak but long-term impacts on a medium.

Which impacts can be assumed weak? Initially, a granular medium has only one characteristic of the dimension of stress. This is the cohesion k . And, when a medium is in the limiting state, it has one more characteristic—internal friction angle φ . The latter is dimensionless and is included in the limiting state conditions together with the uniform compression:

$$\tau = -\sin \varphi \cdot \sigma + k \cos \varphi.$$

Impacts may be characterized by a pressure p^* . This allows a natural dimensionless combination of stresses

$$\lambda = \frac{p^*}{-\sin \varphi \cdot \sigma + k \cos \varphi}.$$

Accordingly, “weakness” of an impact is conditioned by a specific element of a medium, and by strength characteristic and stress state of this element. Consequently, an explosion load is high in the near and medium-range zones and is weak in the rest (major part) of rock mass.



A granular medium is composed of many discrete solid particles that can carry out translational or rotational movements under certain energy input. This motion can include both differential movements of individual particles and coordinated movement of particle clusters temporarily composed and decomposed. The detailed research has shown that particles in a granular medium under deformation make clusters (actually, blocks) [4]. Deformations mainly occur at the boundaries of the clusters and are comparatively low inside the clusters. Clusters of particles with flat faces are better observed. By way of illustration, the squashing test of a bunch of six-sided pencils tied up by an elastic rubber from outside can be presented [5].

The evolution of stress state of a specimen composed of marble chips was studied in the experimental treatment of the specimen by weak impacts within a long time (see Figure 1). A steel cylindrical glass 1 with a diameter of 100 mm and 220 mm high, with the wall thickness of 0.3 mm was filled with marble chips up to the top using a funnel. Two sensors 3 were embedded in marble chips specimen to measure vertical pressure. The sensors were $12 \times 25 \times 3$ mm in dimension. The sensors were buried at a depth of 150 and 110 mm from the specimen top surface. The glass was fixed on a stiff basis. An impacting tool was arranged nearby. The nose-piece of the impacting tool was made of rubber. The distance between the glass bottom and the impact point was 60 mm.

To avoid temperature effect on the sensors, the test system was placed in a thermostat. As soon as the system was heated and the sensors gave steady-state reaching, the impacting tool was actuated. The impact energy E was constantly $3.85 \cdot 10^{-3}$ J during the test and the blow frequency was 80 impacts per minute. Sampling period was 10 s. Particles of marble chips were 1 mm in size on average.

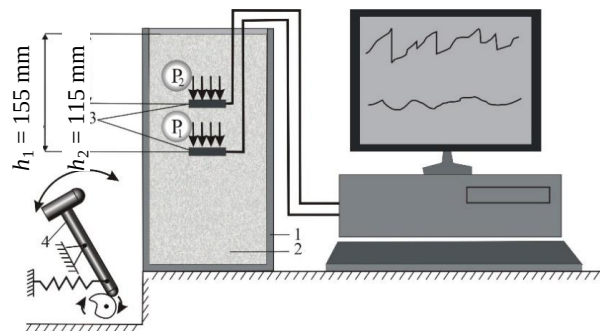


Figure 1. Schematic of the test: 1—cylindrical glass; 2—marble chips; 3—normal stress sensors; 4—impacting device.

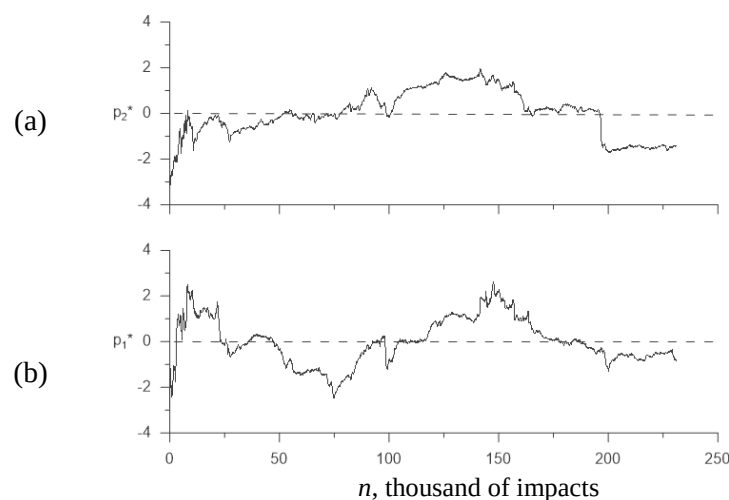


Figure 2. Normalized plots of vertical pressure in marble chips package versus multiple weak impacts: (a) upper sensor; (b) lower sensor.

The pressure–impact curves plotted based on the test data were processed. At the first stage, linear trends adjusted to the experimental curves using the least square method were withdrawn from the plots. The second stage of the processing was normalization of the plots through dividing of pressure readings by standard deviation. The resultant normalized curves are presented in Figure 2.

Evidently, the pressure curves in Figure 2 are non-monotonic. There are peaks and troughs. The same pressure pattern was experimentally obtained in quartz sand tests [6], and the tests showed that stress deviation from uniform compression could reach 20–150% in the course of the experiment.

By this time, researchers have found that many time series, although looking random, are not random actually. Using many model and natural series, it has been shown that they often contain information on begetting mechanisms. For example, the R/S analysis reveals the fractal nature of a process and the process “memory” [7–9]. The R/S analysis was introduced in the practice of the empirical analysis of time series by hydrologist Harold Edwin Hurst in 1960s. He discovered that for many experimental time series, the ratio R/S grows exponentially with an increase in the series length L

$$(R_L / S_L)^H / L \sim \text{const}(L),$$

where R_L and S_L are, respectively, the series amplitude and the standard deviation dependent on the series length L ; H is the parameter of the self-similarity of a time series (Hurst exponent).

For a time series X_t at $t = 1, 2, 3, \dots, L$, the amplitude R_L is determined as the difference between the maximum and minimum deviations of X_t from the average X_0

$$R_L = \max[Y_t(k) |_{k=1..L}] - \min[Y_t(k) |_{k=1..L}],$$

where Y_t is the sum of the deviations from the current average X_0 over the interval from 1 to L

$$Y_t = (X_1 - X_0) + (X_2 - X_0) + (X_3 - X_0) + \dots + (X_k - X_0).$$

Hurst analysis is intended to find slope of the plot $R/S \sim \tau^H$ in a log–log frame, where R is the amplitude of the time series in the given time interval τ ; S is the standard deviation in the same interval.

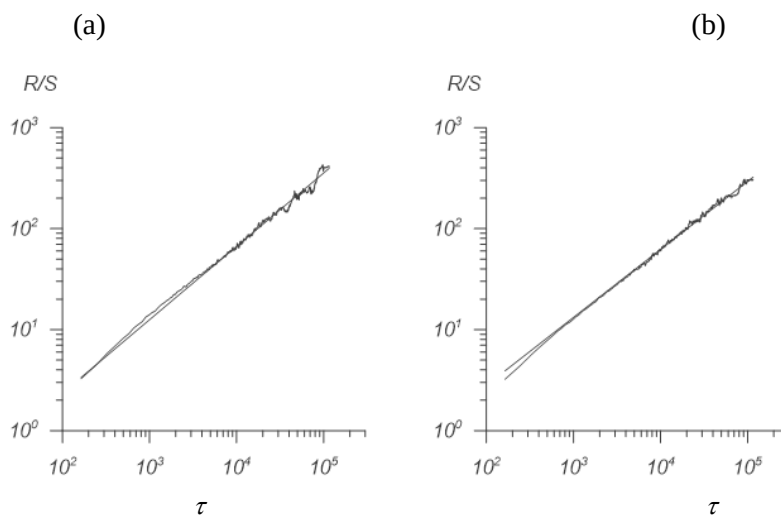


Figure 3. Normalized pressure amplitude versus the width of interval τ (number of impacts): (a) lower sensor; (b) upper sensor.

For a random (stochastic) process, with independent increments and finite dispersion, $H = 0.5$. Deviation of H from 0.5 takes place when the analyzed processes possess “memory” and is the

evidence of fractal properties of these processes. When $H > 0.5$ time series have a sustainable tendency (persistence). When $H < 0.5$ the increase in the observed value changes to a decrease and vice versa (anti-persistence). The Hurst method is very stable and needs no a priori assumption on the character of distribution of values. For many natural processes, e.g. river flows, river and lake stages, amount of precipitation, thickness of bedded precipitation, thickness of growth rings, number sunspots etc., H range is 0.6–0.8 [7, 8].

The outcome of the R/S processing of the experimental pressure curves from Figure 2 is depicted in Figure 3.

It is seen that the curves of R/S and the interval width (number of impacts) τ are well approximated by exponential functions. This implies the fractal nature of the studied process, the Hurst exponents for the lower and upper sensors are 0.73 and 0.67, respectively.

Thus, the time series under analysis have a stable tendency (persistence) while the process that induces them is fractal and possesses long-term “memory.”

Apparently, this is connected with that the impulse given to particles under percussion depends on arrangement of the particles at any specific time. Mutual arrangement of particles is changed from blow to blow and, consequently, conditions of contact interaction between particles (orientation of interfaces, contact stresses) are altered, too. For this reason, it is possible to state that displacement of a particle under each impact is more or less random. The result of this process is continuous re-packing of material, with formation of stable clusters of hierarchical (self-similar) structure here and there. Under long-term impact, clusters are decomposed and originated again.

These research findings conform with the results obtained in [10–12], where it is shown that movement and deformation in granular media are characterized by fluctuations and jumps of pressure. Deformations localize and block structures similar to fault-and-block systems are formed. The fractal distribution of sizes of blocks formed in a granular medium under external impact was observed in [13].

References

- [1] Moroz AI 2004 *Self-Stress State of Rocks* Moscow: MGGU (in Russian)
- [2] Birger IA 1963 *Residual Stresses* Moscow: Mashgiz (in Russian)
- [3] *Organizational Standard STO 36554501-019-2009 Identification of Self-Stress State in Rocks. NITs Stroitelstvo* Moscow 2010 (in Russian)
- [4] Kosykh VP 2012 Experimental investigation in variable density of granular material under deformation cycling *Journal of Mining Science* Vol 48 No 6 pp 990–995
- [5] Lavrikov SV and Revuzhenko AF 1991 One experimental rock model *Journal of Mining Science* Vol 27 No 4 pp 288–293
- [6] Kazantsev AA Kosykh VP and Revuzhenko AF 2015 Spectral analysis of granular material reaction to long-term weak dynamic effect, *IOP Conf. Series: Materials Science and Engineering* 91 012089 doi:10.1088/1757-899X/91/1/012089.
- [7] Feder J 1988 *Fractals* Springer US
- [8] Schroeder M 2009 *Fractals, Chaos, Power Laws* Dover Publications
- [9] Lukk AA, Deshcherevsky AV, Sidorin AYa and Sidorin IA 1996 *Variation of Geophysical Fields as the Deterministic Chaos in Fractal Media* Moscow OIFZ RAN (in Russian)
- [10] Leonov MG 2002 Tectonics of complex-dislocated areas in the Earth's crust (Problems and solutions) *Litosfera* No 1
- [11] Behringer RP, Howell D, Kondic L, Tennakoon S and Veje C 1999 Predictability and granular materials *Physica D* Vol 133 Nos 1–4
- [12] Revuzhenko AF 2006 *Mechanics of Granular Media* Springer-Verlag Berlin Heidelberg
- [13] McDowell GR, Bolton MD and Robertson D 1996 The fractal crushing of granular materials *J. Mech. Phys. Solids* Vol 44 No 2