

Optimal power allocation based on sum-throughput maximization for energy harvesting cognitive radio networks

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Abstract. In this study, an optimal power allocation algorithm by maximizing the sum-throughput in energy harvesting cognitive radio networks is proposed. Under the causality constraints of the harvested energy by solar radiation, electromagnetic waves and so on in the two secondary users (SUs), and the interference constraint in the primary user (PU), the sum-throughput maximization problem is formulated. The algorithm decomposes the interference threshold constraint to the power upper bounds of the two SUs. Then, the power allocation problems of the two SUs can be solved by a directional water-filling algorithm (DWA) with the power upper bounds, respectively. The paper gives the algorithm steps and simulation results, and the simulation results verify that the proposed algorithm has obvious advantages over the other two algorithms.

1. Introduction

Energy harvesting technology can continuously gather energy from the surrounding environment, such as solar radiation, electromagnetic waves and so on, which greatly extends the lifecycle of equipment, reduces the use of traditional energy sources, protects environment and improves the performance of wireless networks [1], [2]. In addition, the cognitive radio technology can find and rationally use spectrum holes [3], which improves the spectral resource utilization rate of the system. So it is very significant to study the resource allocation problems in energy harvesting cognitive radio networks.

The power allocation problems in energy harvesting networks have been studied extensively. In [4], the authors assume that the source can utilize conventional energy and harvested energy, and harvested energy spends less than conventional energy. The problem of the total energy cost minimization is solved by the theory of linear programming limited by a minimum capacity and harvested energy. In [5], the problem to maximize throughput is studied by Lagrangian function and KKT conditions in a two-node energy harvesting network, and a DWA scheme is proposed. In [6], the authors assume that data packets and harvested energy gradually arrive in the process of the data transmission, and then the throughput maximization problem is decoupled to a power allocation problem and a data transmission problem, which can be solved by alternating maximization method.

In [7], [8], both cognitive radio technology and energy harvesting technology are considered. In [7], the authors deduce the upper bound of the throughput that the SU can achieve under a collision constraint. In [8], the authors propose a geometric water-filling algorithm, in which the interference constraint is converted into the power upper bound for the SU.



The main contributions of this paper are as follows:

- This paper considers a new energy harvesting wireless network scenario in which two SUs can share a common spectrum of the PU [9], so we must jointly optimize the power allocations of the two SUs. But both [7], [8] consider that a single SU occupies the spectrum of the PU, so the interference constraint is easily converted into the power upper bound of the SU.
- The harvested energy can come from solar radiation, the wind, electromagnetic waves and so on, so the network can utilize a variety of new energy, reduce the use of traditional energy, and protect environment.
- In [6], the constraints of harvested energy and data are separable, and the original problem can be decoupled directly. But this paper need to decompose the interference threshold constraint to the power upper bounds of the two SUs. Then an iteration water-filling algorithm is proposed for the power upper bounds of the two SUs. In addition, we prove that the equivalence problem is a convex optimization problem.

2. Energy harvesting cognitive radio networks model and problem formulation

This paper considers an energy harvesting cognitive radio network, which consists of a PU, two SUs and a base station (BS), and the two SUs are both energy harvesting nodes, as shown in figure 1. Assuming that the timeslot is the minimum transmission unit, and there are N equal length timeslots. For convenience, we set the timeslot length is 1s, which can be extended to arbitrary timeslot length. Assuming that harvested energy arrives before the data transmission of each timeslot starts. We denote the channel fading coefficients from the two SUs to the BS as $h_{1,i}$ and $h_{2,i}$, and the channel fading coefficients from the two SUs to the PU as g_i and f_i in the i -th timeslot. All channels are additive white Gaussian noise channels with the noise power spectral density N_0 . The bandwidth of the PU is W . Let $E_{1,i}$ and $E_{2,i}$ denote the harvested energy amounts of the two SUs in the i -th timeslot.

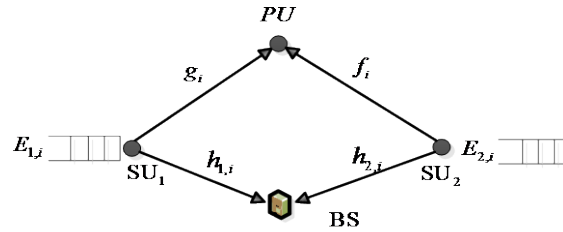


Figure 1. Energy harvesting cognitive radio networks model.

Assuming that the two SUs work simultaneously in each timeslot with half the bandwidth of the PU, that is, FDMA (frequency division multiple access) mode. The two SUs work with power $P_{1,i}$ and $P_{2,i}$ in the i -th timeslot. Meanwhile, the sum of the interference in the PU is required to be less than the interference threshold I [9], so the interference threshold constraint is

$$P_{1,i}g_i + P_{2,i}f_i \leq I \quad (1)$$

At the same time the causality constraints of the harvested energy in the two SUs are

$$\sum_{i=1}^n P_{1,i} \leq \sum_{i=1}^n E_{1,i} \quad (2)$$

$$\sum_{i=1}^n P_{2,i} \leq \sum_{i=1}^n E_{2,i} \quad (3)$$

The sum-throughput within N timeslots can be expressed as

$$C = \sum_{i=1}^N \left[\frac{1}{2} W \log_2 \left(1 + \frac{h_{1,i} P_{1,i}}{WN_0/2} \right) + \frac{1}{2} W \log_2 \left(1 + \frac{h_{2,i} P_{2,i}}{WN_0/2} \right) \right] \quad (4)$$

where we denote $\bar{h}_{1,i} = \frac{h_{1,i}}{WN_0/2}$ and $\bar{h}_{2,i} = \frac{h_{2,i}}{WN_0/2}$ to simplify the derivation process since $WN_0/2$ is a constant.

Hence, under the criterion of the sum-throughput maximization, we can formulate the problem as

$$\begin{aligned} \max_{\{P_{1,i}, P_{2,i}\}} \quad & \sum_{i=1}^N \left[\log_2(1 + \bar{h}_{1,i} P_{1,i}) + \log_2(1 + \bar{h}_{2,i} P_{2,i}) \right] \\ \text{s.t.} \quad & P_{1,i} g_i + P_{2,i} f_i \leq I, \quad \forall i \\ & \sum_{i=1}^n P_{1,i} \leq \sum_{i=1}^n E_{1,i}, \quad \forall n \\ & \sum_{i=1}^n P_{2,i} \leq \sum_{i=1}^n E_{2,i}, \quad \forall n \\ & 0 \leq P_{1,i}, \quad \forall i \\ & 0 \leq P_{2,i}, \quad \forall i \end{aligned} \quad (5)$$

where we omit the $\frac{1}{2}W$ in the objective function, which does not affect the optimal power allocation result.

Since the objective function in equation (5) is concave and the constraint conditions are linear, equation (5) is a convex optimization problem with a unique optimal value.

3. Proposed power allocation algorithm

This paper proposes a new iteration water-filling algorithm for the power upper bounds of the two SUs through converting an inequality constraint $P_{1,i} g_i + P_{2,i} f_i \leq I$ to an equality constraint $P_{1,i} g_i + P_{2,i} f_i = I$.

As shown in figure 2, we denote the power upper bounds of the two SUs as $\{P_{1,i}^I\}$ and $\left\{ P_{2,i}^I = \frac{I - P_{1,i}^I g_i}{f_i} \right\}$, and then we must have $P_{1,i} \leq P_{1,i}^I, P_{2,i} \leq P_{2,i}^I, \forall i$ for figure 2(a) and (b).

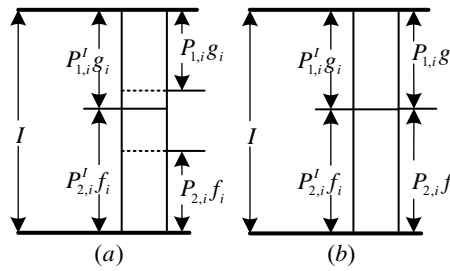


Figure 2. Schematic diagram of converting an inequality constraint to an equality constraint.

Assuming that the power upper bounds of the two SUs are known, and $\{P_{1,i}^I\}$ is denoted as vector \mathbf{P}_1^I . The equation (5) can be decoupled into the following two sub-problems:

$$\begin{aligned}
 g(\mathbf{P}_1^I) = \max_{\{P_{1,i}\}} & \sum_{i=1}^N \log_2(1 + \bar{h}_{1,i} P_{1,i}) \\
 s.t. & P_{1,i} \leq P_{1,i}^I, \quad \forall i \\
 & \sum_{i=1}^n P_{1,i} \leq \sum_{i=1}^n E_{1,i}, \quad \forall n \\
 & 0 \leq P_{1,i}, \quad \forall i
 \end{aligned} \tag{6}$$

where $g(\mathbf{P}_1^I)$ denotes the maximum value of equation (6) in the case of \mathbf{P}_1^I is determined.

$$\begin{aligned}
 f(\mathbf{P}_1^I) = \max_{\{P_{2,i}\}} & \sum_{i=1}^N \log_2(1 + \bar{h}_{2,i} P_{2,i}) \\
 s.t. & P_{2,i} \leq \frac{I - P_{1,i}^I g_i}{f_i}, \quad \forall i \\
 & \sum_{i=1}^n P_{2,i} \leq \sum_{i=1}^n E_{2,i}, \quad \forall n \\
 & 0 \leq P_{2,i}, \quad \forall i
 \end{aligned} \tag{7}$$

where $f(\mathbf{P}_1^I)$ denotes the maximum value of equation (7) in the case of \mathbf{P}_1^I is determined. Since the objective function in equation (6) is concave for $\{P_{1,i}\}$, and all the constraint conditions are linear, equation (6) is a convex optimization problem. Similarly, equation (7) is also a convex optimization problem for $\{P_{2,i}\}$. Then, equations (6) and (7) can be solved by a DWA with the power upper bounds [6], respectively.

Now, the causality constraints of the harvested energy in the two SUs and the interference constraint in the PU both are satisfied, and equation (5) is converted into optimizing $\{P_{1,i}^I\}$ to maximize the sum-throughput of equations (6) and (7). The equation (5) is equivalent to

$$\begin{aligned}
 \max_{\{P_{1,i}^I\}} & g(\mathbf{P}_1^I) + f(\mathbf{P}_1^I) \\
 s.t. & P_{1,i}^I g_i \leq I, \quad \forall i
 \end{aligned} \tag{8}$$

Then we will prove that equation (8) is a convex optimization problem for \mathbf{P}_1^I .

Proof: First, we prove equations (6) and (7) both are concave for \mathbf{P}_1^I .

For equation (7), let $0 \leq \xi \leq 1$, and $\mathbf{Q}_1^I, \mathbf{Q}_2^I$ denote two power upper bound vectors to substitute \mathbf{P}_1^I . Then, \mathbf{Q}_1 denotes the solution of $f(\mathbf{Q}_1^I)$, and \mathbf{Q}_2 denotes the solution of $f(\mathbf{Q}_2^I)$, respectively. Since equation (7) is a convex optimization problem for $\{P_{2,i}\}$, \mathbf{Q}_1 and \mathbf{Q}_2 are existing and unique because of convexity. Since the constraint conditions are linear, the vector $\xi \mathbf{Q}_1 + (1-\xi) \mathbf{Q}_2$ is feasible for $f(\xi \mathbf{Q}_1^I + (1-\xi) \mathbf{Q}_2^I)$. So

$$\begin{aligned}
 f(\xi \mathbf{Q}_1^I + (1-\xi) \mathbf{Q}_2^I) & \stackrel{(a)}{\geq} \sum_{i=1}^N \log_2(1 + \xi Q_{1,i} + (1-\xi) Q_{2,i}) \\
 & \stackrel{(b)}{\geq} \sum_{i=1}^N \xi \log_2(1 + Q_{1,i}) + \sum_{i=1}^N (1-\xi) \log_2(1 + Q_{2,i}) \\
 & \stackrel{(c)}{=} \xi f(\mathbf{Q}_1^I) + (1-\xi) f(\mathbf{Q}_2^I)
 \end{aligned} \tag{9}$$

where the reason of (a) is that $f(\xi \mathbf{Q}_1^I + (1-\xi) \mathbf{Q}_2^I)$ denotes the maximum value of equation (7) in the case of \mathbf{P}_1^I is substituted by $\xi \mathbf{Q}_1^I + (1-\xi) \mathbf{Q}_2^I$, but $\sum_{i=1}^N \log_2(1 + \xi Q_{1,i} + (1-\xi) Q_{2,i})$ denotes the sum-throughput which is calculated by the feasible point $\xi \mathbf{Q}_1 + (1-\xi) \mathbf{Q}_2$, and then we must have that the former is greater than the latter. The reason of (b) is that \log is a concave function. The reason of (c) is that \mathbf{Q}_1 denotes the solution of $f(\mathbf{Q}_1^I)$, and \mathbf{Q}_2 denotes the solution of $f(\mathbf{Q}_2^I)$, respectively.

Similarly, we can prove that equation (6) is concave for \mathbf{P}_1^I .

Then, we can obtain that $g(\mathbf{P}_1^I) + f(\mathbf{P}_1^I)$ is concave for \mathbf{P}_1^I because all the independent variables in log are linear. Meanwhile, the constraint condition $P_{1,i}^I g_i \leq I$ is linear, so equation (8) is a convex optimization problem and it can be solved by the iteration water-filling algorithm in table 1.

In table 1, note that the prerequisite for the next iteration is that the sum-throughput increases, if we repeat iteration many times, the objective function will increase monotonically until it converges to the optimal value. Since each iteration increases the sum-throughput, and equation (8) is a convex optimization problem with a unique optimal value, the convergence of the problem is guaranteed. At the same time, the number of iterations depends on the step size to flow energy from $P_{1,i}^I$ to $P_{2,i}^I$ or from $P_{2,i}^I$ to $P_{1,i}^I$. However, \mathbf{P}_1^I corresponding to the optimal value is not unique in table 1. Assuming that the optimal power allocations of the two SUs are respectively $P_{1,i}^*$ and $P_{2,i}^*$ in the i -th timeslot, which satisfy $P_{1,i}^* g_i + P_{2,i}^* f_i < I$, and then $P_{1,i}^I$ can be arbitrary value within the limit of $P_{1,i}^* \leq P_{1,i}^I \leq \frac{I - P_{2,i}^* f_i}{g_i}$.

Table 1. Proposed iteration water-filling algorithm.

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1. Initialize $P_{1,i}^I = \frac{I}{g_i}$, $P_{2,i}^I = \frac{I - P_{1,i}^I g_i}{f_i}$, $1 \leq i \leq N$, $temp = 0$, the minimum step size s ;
 2. Solve equations (6) and (7) by the DWA with the power upper bounds;
 3. Compute $C = \sum_{i=1}^N \frac{1}{2} W \left[\log_2(1 + \bar{h}_{1,i} P_{1,i}^I) + \log_2(1 + \bar{h}_{2,i} P_{2,i}^I) \right]$;
 4. **while** $C > temp$ **do**
 5. $temp = C$;
 6. **for** $i = 1 : N$ **do**
 7. Let $P_{1,i}^I = P_{1,i}^I - s$, $P_{2,i}^I = P_{2,i}^I + s g_i / f_i$, repeat steps 2 and 3;
 8. As long as the sum-throughput C increases, repeat step 7, otherwise, let $P_{1,i}^I = P_{1,i}^I + s$, $P_{2,i}^I = P_{2,i}^I - s g_i / f_i$;
 9. Let $P_{1,i}^I = P_{1,i}^I + s$, $P_{2,i}^I = P_{2,i}^I - s g_i / f_i$, repeat steps 2 and 3;
 10. As long as the sum-throughput C increases, repeat step 9, otherwise, let $P_{1,i}^I = P_{1,i}^I - s$, $P_{2,i}^I = P_{2,i}^I + s g_i / f_i$;
 11. **end for**
 12. **end while**
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4. Simulation and analysis

The simulation parameters of this paper mainly refer to [10]. The distance between the PU and the BS is 1000m. The Two SUs are uniformly distributed in a circle with a radius of 1000m centered on the mid-point of the PU and the BS. Assuming that the bandwidth of the PU is $W = 2\text{MHz}$, so the bandwidth occupied by the two SUs is $\frac{1}{2}W = 1\text{MHz}$, respectively. The large-scale fading with distance d is approximately $(-31.5 - 30 \log_{10} d)$ dB, and the small-scale fading is modeled as Rayleigh fading. The noise power spectral density is $N_0 = -174$ dBm/Hz.

Let $N = 20$. Then, the proposed algorithm is compared with the maximum capacity transmission algorithm and the optimal power allocation (OPA) of a single SU. The single SU can occupy all the

bandwidth of the PU, but its capacity of energy harvesting is only equivalent to the SU_1 . The basic steps of the maximum capacity transmission algorithm are as follows: First, the harvested energy of the two SUs in each timeslot is respectively allocated to the current timeslot, i.e., $P_{1,i} = E_{1,i}, P_{2,i} = E_{2,i}, 1 \leq i \leq N$. Then, the allocated power of the two SUs is reduced proportionally for the timeslots in which the total interference of the two SUs exceeds the interference threshold, until the total interference is equal to the interference threshold. Suppose that $\bar{P}_{1,i}, \bar{P}_{2,i}$ are the reduced power of the two SUs, then $\bar{P}_{1,i}g_i = \bar{P}_{2,i}f_i$ is satisfied.

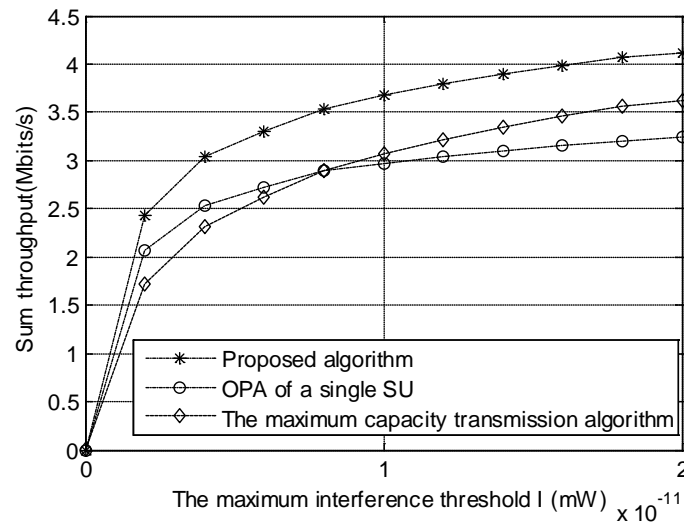


Figure 3. The comparison of the sum-throughput under different I .

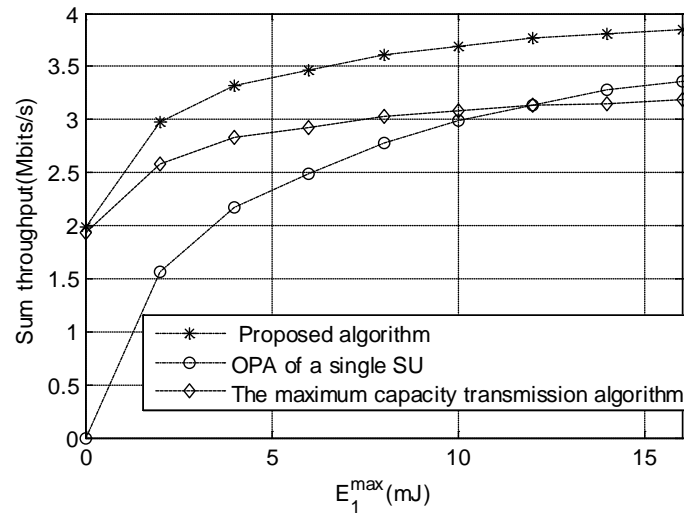


Figure 4. The comparison of the sum-throughput under different E_1^{\max} .

Figure 3 shows the comparison of the sum-throughput under different interference thresholds I . Assuming that the harvested energy of the two SUs in each timeslot is uniformly distributed between $[0, E_1^{\max}]$ and $[0, E_2^{\max}]$, where $E_1^{\max} = E_2^{\max} = 10\text{mJ}$. From the figure we can see that the sum-throughputs of the three algorithms will gradually increase with the increase of the interference

threshold I . After about $I = 0.8 \times 10^{-11}$ mW, the sum-throughput of the maximum capacity transmission algorithm will exceed the OPA of a single SU, because the harvested energy amounts of the two SUs are more than the single SU. Finally, the sum-throughputs of the three algorithms will converge due to the constraint of harvested energy.

Figure 4 shows the comparison of the sum-throughput under different E_1^{\max} . Assuming that $I = 1 \times 10^{-11}$ mW, and the harvested energy of the two SUs in each timeslot is uniformly distributed between $[0, E_1^{\max}]$ and $[0, E_2^{\max}]$, where $E_2^{\max} = 10$ mJ. From the figure we can see that with the E_1^{\max} gradually increases, the sum-throughputs of the three algorithms will gradually increase, too. Note that after $E_1^{\max} = 12$ mJ, the sum-throughput of the OPA of a single SU will exceed the maximum capacity transmission algorithm, because the interference threshold I becomes the main limiting factor for the sum-throughput of the system, and the OPA of a single SU obtains the optimal power allocation, but the maximum capacity transmission algorithm uses the harvested energy to the current timeslot.

5. Conclusions

In this paper, an optimal power allocation algorithm to maximize the sum-throughput in energy harvesting cognitive radio networks is proposed. The algorithm decomposes the interference threshold constraint to the power upper bounds of the two SUs. The power allocation problem within the two SUs can be solved by a DWA with the power upper bounds, respectively. The power allocation problem between the two SUs can be solved by the iteration water-filling between the power upper bounds. Finally, the simulation results verify that the proposed algorithm has obvious advantages.

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