Research on three-phase unbalanced distribution network reconfiguration strategy

Shuang Hu[1], Ke-Jun Li, Yanshun Xu, Zhijie Liu, Jing Guo, Zhuodi Wang

School of Electrical Engineering, Shandong University, Jinan, China

E-mail:15106994270@163.com

Abstract. With the development of social economy, the loads installed in the distribution network become more and more complex which may cause the three-phase unbalance problems. This paper proposes an optimal reconfiguration approach based on mixed integer quadric programming (MIQP) method to address the three-phase unbalance problem. It aims to minimize the total network losses of the system. By using several square constraints to substitute the circular constraint, the original optimization problem is linearized and converted into a mixed-integer linear programming (MILP) model. Then this MILP problem is solved in general algebraic model system (GAMS) software using CPLEX solver. The additional losses caused by three-phase unbalanced are also considered. An IEEE 34 nodes test system is used to demonstrate the feasibility and effectiveness of the proposed method. The results show that the losses and the voltage violation mitigation in the network can be reduced significantly.

1. Introduction

Distribution system reconfiguration is a classical method which is used to optimize objects such as the reliability operation as well as the power quality by changing the switch status or updating the network structure[1]. In this paper, the main objective is to minimize the losses of active power in transmission lines. With the rapid development of city constructions, the investment and update work of city distribution network is relatively backward. The unreasonable network structure and limited network capacity have difficulty to satisfy the electricity consuming demand of city residents. Reconfiguration of distribution network is one of the viable approaches to address these problems.

Various methods for distribution reconfiguration optimization has been studied, such as traditional mathematical algorithm, the optimal load flow algorithm , branch exchange method , simulated annealing algorithm , genetic algorithm, tabu algorithm , differential evolution method, immune algorithms, and family eugenics algorithm etc [2-7]. In [8], a hybrid particle swarm optimization (PSO) approach composed of the binary PSO and discrete PSO was proposed and successfully applied to distribution network reconfiguration problem. Paper [9] studies a comprehensive optimization model that combined reactive power optimization and network reconfiguration to minimize power losses and eliminate voltage violations. Paper [10] proposes a mixed-integer conic programming formulation for the minimum losses distribution network reconfiguration problem. Even if load balance is taken into consideration in the above studies, the research is usually only measured by the relationship between the apparent power of each branch and its capacity, but little attention is paid to the balance among the three phases in distribution systems.

Although the mentioned methods above have achieved success in distribution configuration, as the consuming demands in distribution network have been more and more complex problems, such as unbalance in the three-phase voltages, over-current in the neural point, the decline in the distribution transformer output and the increased active power losses become more serious than before. Balancing the three-phase loads by reconfiguration becomes more demanding. This paper utilize mixed integer quadric programming (MIQP) model to optimize the distribution network taking comprehensive consideration of three-phase unbalance problem to further raise the efficiency and reducing the computing time. Also to be more practical, the reconfiguration should not only study static optimal network structure but also take consideration of the global optimization under the load changing situation in several continuous time periods.

2. The necessity of three-phase balance in distribution network

With the large increase in power demand, the active losses, voltage drop in system nodes and three-phase unbalance problems become increasingly serious. These problems have already become the serious problems that should be overcome for better distribution management, especially the three-phase unbalance problem. This is because the loads in the low-voltage distribution networks are dominated by single-phase power consuming equipment and there still exist some two-phase power consuming equipment. The unbalance also cause problems including three-phase nodal voltage unbalance, lower output of distribution transformers and increased active losses of distribution transformers, losses in transmission lines. These influence, on the other side, not only causing low quality electricity supply with low reliability but also make it difficult to recover power supply and load transfer [11]. This makes a negative effect on the power system operating security. Besides, unbalance in distribution network would make over-current problems in neutral phase and feeders. These problems need to be solved by vast investment to update the system, but this will bring economic problems. Therefore conducting reconfiguration optimization to balance the three-phase is of necessity.

3. The mathematical model of three-phase unbalance degree

3.1. The objective function

To guarantee the feasibility of the model, this paper chooses the minimum system losses as the target, and penalty terms are introduced to include the three-phase active power unbalance and the three-phase reactive power unbalance. The objective function is as follows:

$$\min\sum_{l=1}^{L}\sum_{ph\in\{a,b,c\}}R\left((P_{ij}^{ph})^2+(Q_{ij}^{ph})^2\right)/V_1^{\,2}+\sum_{i=1}^{I}\sum_{ph\in\{a,b,c\}}(d_i^{ph}+q_i^{ph}) \tag{1}$$

Where V_1 are 1.0 pu in the objective function; *L* represents the total number of branches in the system[7], *i* and *j* represent the serial number, *R* represents the line resistance, P_{ij}^{ph} represents the active power in transmission line *i-j* whereas Q_{ij}^{ph} represents the reactive in line *i-j*; the term d_i^{ph} is introduced to represent the degree of three-phase unbalanced active power; q_i^{ph} is for the unbalanced reactive power, respectively.

3.2. The constraints

To guarantee the security operation of distribution network, the reconfiguration of distribution network should include the radial operation constraints, the output constraints at root nodes, power balance constraints, the voltage limitation constraints and the transformer capacity constraints, and etc.

1) Radial network constraints

$$g\in G \tag{2}$$

2) Feeder output constraints

$$\underline{P}_g^{ph}\le P_g^{ph}\le\overline{P}_g^{ph},\forall g,ph\in\{a,b,c\} \tag{3}$$

$$\underline{Q}_g^{ph} \le Q_g^{ph} \le \overline{Q}_g^{ph}, \forall g, ph \in \{a,b,c\} \tag{4}$$

3) Power flow equality constraints:

$$\sum_{i:(k,i)\in W} P_{ki}^{ph} = \sum_{i:(i,j)\in W} P_{ij}^{ph} + P_i, \forall i \in N, ph \in \{a,b,c\} \tag{5}$$

$$\sum_{i:(k,i)\in W} Q_{ki}^{ph} = \sum_{i:(i,j)\in W} Q_{ij}^{ph} + Q_i, \forall i \in N, ph \in \{a,b,c\} \tag{6}$$

4) Branch capacity constraints:

$$(P_{ij}^{ph})^2 + (Q_{ij}^{ph})^2 \le z_{ij} \cdot \overline{S}_{ij}^2, \forall ij \in L \tag{7}$$

5) Voltage limitation constraints:

$$v^{ph} = (V^{ph})^2, \forall t, ph \in \{a,b,c\} \tag{8}$$

$$\underline{V}_i^2 \le v_i^{ph} \le \overline{V}_i^2, ph \in \{a,b,c\} \tag{9}$$

6) Transformer capacity constraints:

$$S_t \le S_{t\max} \tag{10}$$

Where Z_{ij} is a binary variable representing the status of the corresponding branch switch, and note that 1 is represent closed, 0 represent the switch is off; S_t and S_{tmax} are the apparent power and the allowable maximum power of a transformer; g is the current network structure whereas G is the aggregation of all the allowable radial network structure, P_g^{ph}、$\underline{P}_g^{ph}$、$\overline{P}_g^{ph}$ are the real-active power output, the bottom-limit and the upper-limit of active power output; Q_g^{ph}、$\underline{Q}_g^{ph}$、$\overline{Q}_g^{ph}$ are the real-reactive power output, the bottom-limit and the upper-limit of reactive power output; V_i^{ph}、$\underline{V}_i^{ph}$、$\overline{V}_i^{ph}$ are the value and limitations for nodal voltage.

4. The solution of the model forecasting

Heuristic algorithm, mathematical optimization algorithm and artificial intelligence algorithm can be choose for the reconfiguration problems of distribution network [12]. Among them mathematical optimization algorithm has obvious advantages over the others in adaptability and optimizing ability and it is regarded as the most frequently used method. Mixed integer quadratic programming method as the most simple and efficient mathematical optimization method has been developing fast for its fast convergence, stability and high efficiency.

4.1. Three phase transformation of the objective function

The objective function can be rewritten as follow:

$$\begin{aligned} \min \sum_{(i,j)\in W} (r_{ij}^{aa} + r_{ij}^{ab} + r_{ij}^{ac}) \cdot ((P_{ij}^a)^2 + (Q_{ij}^a)^2) / V_1^2 + \sum_{(i,j)\in W} (r_{ij}^{ba} + r_{ij}^{bb} + r_{ij}^{bc}) \cdot ((P_{ij}^b)^2 + (Q_{ij}^b)^2) / V_1^2 + \\ \sum_{(i,j)\in W} (r_{ij}^{ca} + r_{ij}^{cb} + r_{ij}^{cc}) \cdot ((P_{ij}^c)^2 + (Q_{ij}^c)^2) / V_1^2 \end{aligned} \tag{11}$$

Where r_{ij}^{aa} represents the self-impedance of the a-phase between node *i* and node *j*, r_{ij}^{ab} represents the mutual impedance between phase a and phase b in transmission line *i*-*j*, r_{ij}^{ac} represents the mutual impedance between phase a and phase c in transmission line *i*-*j*; whereas, $r_{ij}^{ba}, r_{ij}^{bb}, r_{ij}^{bc}$ represents the mutual impedance between phase b and three-phase in transmission line i-j respectively; $r_{ij}^{bc}, r_{ij}^{cb}, r_{ij}^{cc}$ represents the mutual impedance between phase c and three-phase in transmission line i-j respectively.

4.2. Linearization of the branch capacity constraints

This paper adopts mixed integer quadratic programming method for distribution configuration. To be convenient, linearizing uses quadratic constraint linearization method [13] which is as shown in Figure 1. This paper uses several square constraints to substitute the circular constraint. For general case, by increasing the number of square constraints, the accuracy will be improved. After linearizing, the problem becomes a mixed integer linear programming problem.

In this paper all constraint equations are linear, except the constraint equation (7), which is called circular constraint [13]. Therefore, two nonlinear constraints equations (12), which is called square constraint [13], are employed to substitute for (7), providing an adequate level of precision for engineering applications.

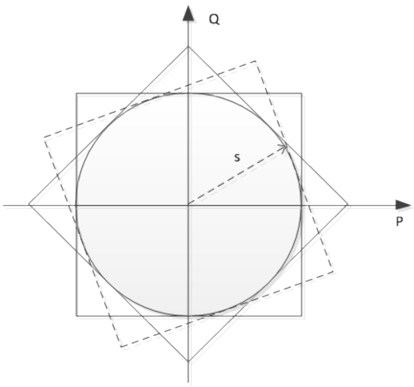

Figure 1. Circular constraint linearization method.

$$\begin{cases} -z_{ij}\bar{S}_{ij} \le P_{ij}^{ph} \le z_{ij}\bar{S}_{ij}, -z_{ij}\bar{S}_{ij} \le Q_{ij}^{ph} \le z_{ij}\bar{S}_{ij} \\ -\sqrt{2}z_{ij}\cdot\bar{S}_{ij} \le P_{ij}^{ph} + Q_{ij}^{ph} \le \sqrt{2}z_{ij}\cdot\bar{S}_{ij}, -\sqrt{2}z_{ij}\cdot\bar{S}_{ij} \le P_{ij}^{ph} - Q_{ij}^{ph} \le \sqrt{2}z_{ij}\cdot\bar{S}_{ij}, \\ \forall(i,j)\in l, ph\in\{a,b,c\} \end{cases} \tag{12}$$

4.3. Linearization of the power balance equations

For the power balance equation, this modification is justified because the squared p.u. resistance and reactance renders this term significantly smaller than the others. The approximation is obtained by replacing (5) and (6) with the following constraints:

$$\begin{cases} v_j^{ph} - v_i^{ph} \le m_{ij} + 2(r_{ij}^{ph}P_{ij}^{ph} + x_{ij}^{ph}Q_{ij}^{ph}),\ \ v_j^{ph} - v_i^{ph} \ge -m_{ij} + 2(r_{ij}^{ph}P_{ij}^{ph} + x_{ij}^{ph}Q_{ij}^{ph}) \\ m_{ij}^{ph} = (1-z_{ij})M,\ \ \forall(i,j)\in l,\ \ ph\in\{a,b,c\} \end{cases} \tag{13}$$

Equation (13) describes the power flow equations, and a big M is introduced to cancel the constraints in disconnected branches. A detailed description is given in [13]. For power balance equations, taking considerations of the three-phase unbalance, each of the three phases must be deduced separately considering the distribution network of three-phase imbalance.

5. The case study

The program is written in GAMS software and the solution is obtained by using CPLEX solver. The case study is the IEEE 34 nodes example. Single line diagram is shown in Figure 2. The original diagram contains 34 nodes. By adding 5 extra transmission branches (numbered by 34, 35, 36, 37 and 38) altogether become 38 branches. At initial state, all the connecting lines are open while the others are closed.

By three-phase load-flow analysis, the three-phase load flow information is derived. The voltage profile is demonstrated in Figure 1, and the unbalance phenomenon is obvious seen from the voltage distributions. The maximum of discrepancy between three-phase voltages for each node is up to 0.054p.u and the minimum nodal voltage is 0.9p.u.

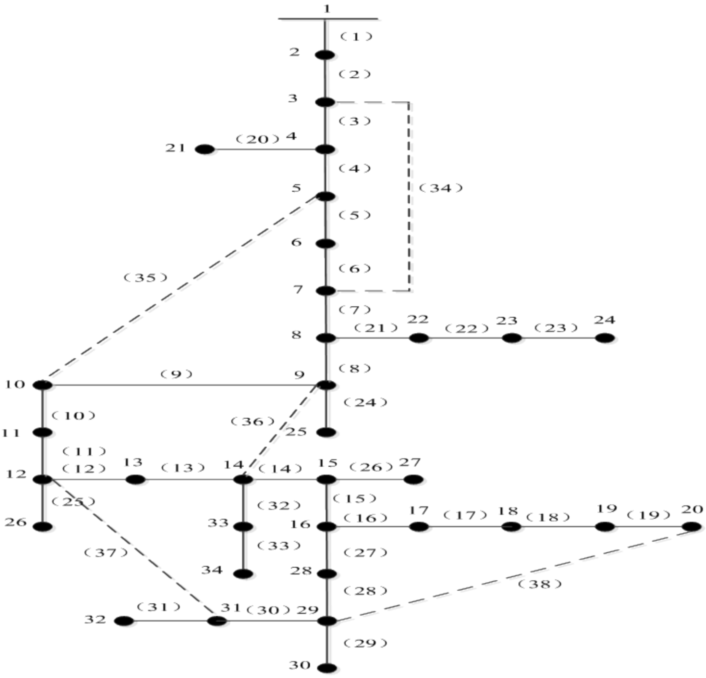

Figure 2. IEEE34 node test feeder

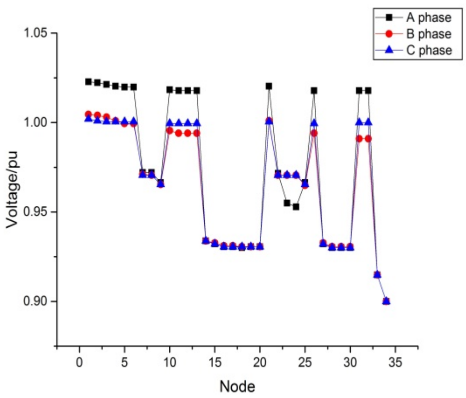

Figure 3. Node voltage before reconfiguration.

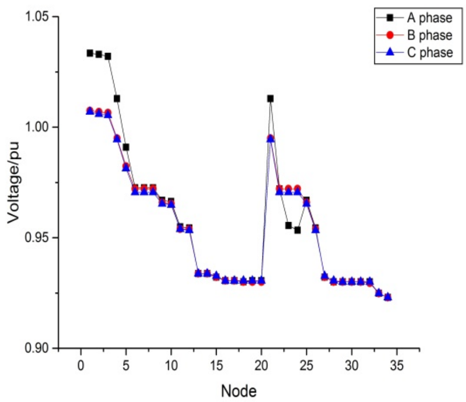

Figure 4. Node voltage after reconfiguration.

Table 1. Comparison between the result before and after reconfiguration.

	Turns on a collection of switches						
Before	2-6	4-9	8-13	11-30	19-28		
After	7-8	8-22	10-11	22-23	12-26	14-15	15-27

Figure 3 and Figure 4 show the node voltage before and after reconfiguration. Using the method proposed in this paper to optimize the three-phase unbalanced system reconstruction, the reconstruction program is shown in Table 1. The switch before reconfiguration has only five tie-switches, after reconfiguration, the contact switches are closed and open the 7 switches as follow: 6-7, 7-21, 9-10, 21-22, 11-25, 13-14, 14-26.

Table 2. Comparison between the result before and after reconfiguration.

	Degree of currents unbalance	Maximum voltage discrepancy(pu)	System losses(kW-hr)
Before	3.135	0.054	205
After	2.517	0.037	180

The distribution of nodal voltages is as shown in Table 2. By comparison, it is concluded that the three-phase currents unbalance is reduced by 19.7% from 3.135 to 2.517 resulting lower level of asymmetry. The maximum voltage discrepancy decreases from 0.054pu to 0.037pu. The objective function is to minimize the system losses, and the losses of the system before reconfiguration is 205 kW-hr while the losses of the system after the reconfiguration optimization is 180 kW-hr.

Table 3. Comparison of nonlinear load flow and linear load flow.

	System losses(kW-hr)	Operation time(ms)
Nonlinear Constraint	198	0.025
Linear Constraint	180	0.016

Table 3 shows the power flow calculation, using nonlinear constraints and linear constraints respectively, in a 34-node system. Considering that the constraint equation (7) is nonlinear, this paper uses equation (12) to substitute for equation (7) in linear constraint power flow calculation, while other constraint equations stay unchanged. The system loss using nonlinear constraint equation is 198 kW-hr, while using linear constraint equation is 180 kW-hr, which is decreased by 9%. The study is performed on a computer with i-3 CPU, 4GB ram, WIN 10 64-bit operating system GAMS version 8.2 and the operation time reduces to 0.016ms from 0.025ms, the calculation time is reduced by 0.051ms.

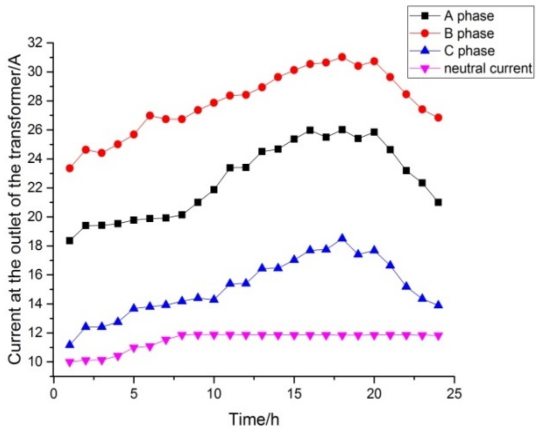

Figure 5. The three-phase currents at the terminal of transformer before reconfiguration.

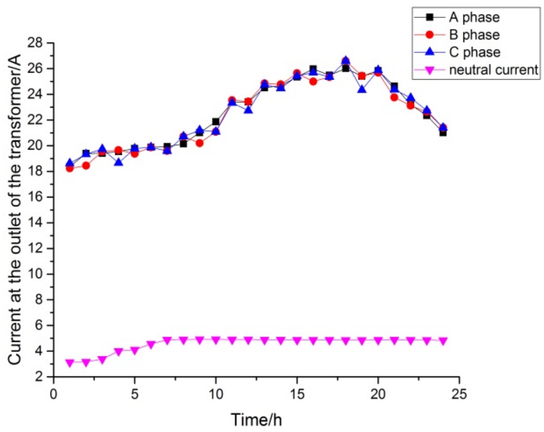

Figure 6. The three-phase currents at the terminal of transformer after reconfiguration.

Figure 5 and Figure 6 show the three-phase current and the current at the neural point at the transformer terminals before and after the reconfiguration optimization respectively. Comparing with the results before and after the reconfiguration, it is obvious that the proposed method can greatly mitigate the unbalance current and nodal voltage.

To demonstrate the advantages of the optimization scheme proposed in this paper, Cases using other methods are simulated including dynamic programming, genetic simulation algorithm, tabu algorithm, immune algorithm and family eugenics algorithm and get the simulation results for comparison which is shown in table 4 below. From the comparison in table 4, this paper can draw the conclusion that the active losses of reconfiguration using hybrid integer second order programing method is only a little more than the losses using hybrid particle swarms method and this method is better than the other four. In this method the minimum nodal voltage is only lower than the result obtained from tabu algorithm and obviously better than the other methods. Theses sufficiently prove the validity of the algorithm.

Table 4. Comparison between the result before and after reconfiguration.

Results	Hybrid integer second order programming	Family eugenics algorithm	Immune algorithm	Fuzzy genetic simulation method	Tabu algorit hm	Hybrid particle swarm algorithm
Losses(kW-hr)	180.08	181.01	181.10	180.81	183.86	180.06
The minimum node voltage(pu)	0.923	0.911	0.911	0.913	0.928	0.910

6. Conclusion

A mixed-integer quadratic programming model for the problem of reconfiguration of electric distribution systems is presented in this paper. To address the three-phase unbalance problem, this paper proposes an optimal reconfiguration approach based on mixed integer quadric programming (MIQP) method. It aims to minimize the total network losses of the system. By using several square constraints to substitute the circular constraint, the original optimization problem is linearized and converted into a mixed-integer linear programming (MILP) model. The additional losses caused by three-phase unbalance are also considered in the paper. And then this MILP problem is solved in general algebraic modeling system (GAMS) software using CPLEX solver. The following conclusions can be drawn from this paper：(a) The proposed method can effectively balance the three-phase voltage asymmetry.(b) The method can reduce the network losses and improve the stability of the system.(c) The linearized processing method reduces the computation time.

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