# Evaluation and Prediction of Water Resources Based on AHP

**Shuai Li**[1,2] **and Anqi Sun**[1,3]

[1] College of Communication Engineering, Jilin University, Changchun, China 130012

[2] luckyappleLS@163.com    [3] 18166880717@163.com

**Abstract.** Nowadays, the shortage of water resources is a threat to us. In order to solve the problem of water resources restricted by varieties of factors, this paper establishes a water resources evaluation index model (WREI), which adopts the fuzzy comprehensive evaluation (FCE) based on analytic hierarchy process (AHP) algorithm.
After considering influencing factors of water resources, we ignore secondary factors and then hierarchical approach the main factors according to the class, set up a three-layer structure. The top floor is for WREI. Using analytic hierarchy process (AHP) to determine weight first, and then use fuzzy judgment to judge target, so the comprehensive use of the two algorithms reduce the subjective influence of AHP and overcome the disadvantages of multi-level evaluation. To prove the model, we choose India as a target region. On the basis of water resources evaluation index model, we use Matlab and combine grey prediction with linear prediction to discuss the ability to provide clean water in India and the trend of India's water resources changing in the next 15 years. The model with theoretical support and practical significance will be of great help to provide reliable data support and reference for us to get plans to improve water quality.

## 1. Introduction

### 1.1. Background
Water shortage has become a global problem to be solved, more and more countries and regions has been caught in moderate or severe water shortages. In the 21st century, the scarcity of water resources is becoming a kind of precious resources, water resources is not only a resource problem, more be related to the national economy, sustainable development and social stability and security of major strategic issues. Therefore, when the human develop the water resources, must according to the law of the water cycle, and make full use of water resources reasonably.

### 1.2. Conspectus
We need a model to measure the ability of a region to provide clean water, the mathematical model can be transformed into a model of water resources evaluation index. We set up the index as the WREI (Water resources evaluation index), which is used to judge if the region clean water is abundant. Considering there are many factors and the influences of water resources are different, we use the analytic hierarchy process (AHP) to help the establishment of the model. We are going to regard WREI as the target layer, considering the water resource, population and water use and capacity will determine the final value of the WREI, so put the three index as criterion layer. And all details of factors as the bottom layer .Considering when using the simple analytic hierarchy model to judge the water abundance degree, larger proportion of subjective factors may cause large deviation, so we can

use fuzzy comprehensive evaluation (FCE) method to remove subjective views [1]. Through the multilayer analysis method to determine the weights, and then through the fuzzy comprehensive evaluation to evaluate target, Combining with the two models will reduce the subjective influence of repeatedly using AHP, and overcome the disadvantages of multi-level evaluation.

The aim of the model is to get the final equation of water evaluation, determine how various factors affect the value of their weights of evaluation index. The conclusion of this model can be used for a particular area, water design intervention programs to change the value of the factors that tend to better water resources evaluation index value, which is to improve the water situation.

## 2. Model establishment and implementation

### 2.1. Model assumptions

- Ignore the secondary factors and the influence factors of water resources is only related to the main factors
- The search data on www.fao.org/nr/water/aquastat/water_res/index.stm is the theoretical data
- Per Capita GDP is roughly on behalf of the economic ability of regional water purification
- A region's supplement and demand of water resources distribute equally, i.e., ignoring the allocation of resources

### 2.2. Symbol definition

- WREI :water resources evaluation index, the target value in the model
- $u_i$ :on behalf of the elements at all levels
- $W_n$ :the weight matrix of target element n
- A:weight matrix
- R:membership matrix

### 2.3. Establishment of an AHP Model
The model is judged by water resources, population and water use and capacity [2].They are provided as u1,u2 and u3.Water resources depend on national rainfall index, internal renewable water resources and External renewable water resources called as u11, u12 and u13 respectively as shown in Figure1. Similarly, we also let u2 and u3 be stratified.

*2.3.1. Data stratification.* WREI as the target layer, criterion layer includes three factors, namely the WSI = {u1, u2, u3}, solution layer includes seven factors, namely the u1 = {u11, u12, u13}, u2 = {u21, u22}, u3 = {u31, u32}, the relation between each factor hierarchy is shown in Figure 1.
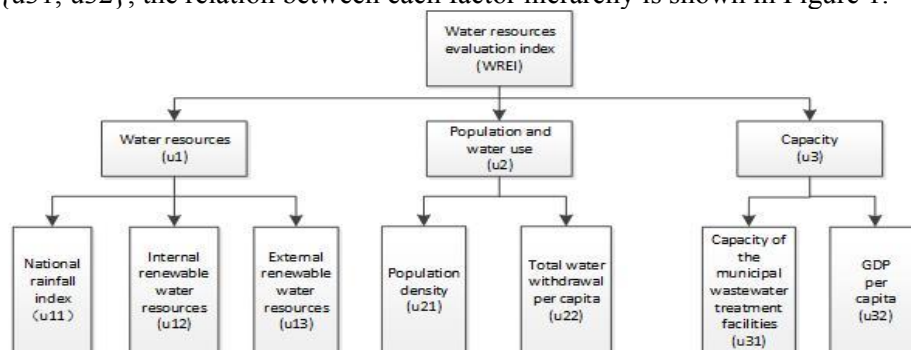


**Figure 1.** Hierarchy diagram

The WREI and index value range from 0 to 1; the bigger the index value is, the better water security situation will be. In general, the mathematical expression of water security index is

$$WREI = \sum_{i=1}^{n} w_{x,i} X \tag{1}$$

WREI refers to the water abundance index value of a certain region; W refers to the weight of sub-index X of the WREI.

*2.3.2. Analytic hierarchy process*. We use Satty's 1-9 Scale method to make thinking quantitative [3].Then we determine the index weight and have a consistency check.

We use water resources, population and water use, capacity in the second layer to solve three weights as an example.

2.3.2.1. Determination of judgment matrix derived from Satty's 1-9 Scale method: $P = \begin{pmatrix} 1 & 1 & 7 \\ 1 & 1 & 7 \\ \frac{1}{7} & \frac{1}{7} & 1 \end{pmatrix}$

*2.3.2.2. Feature vector solution*. To solve the characteristic vector $W_{WREI}$, according to the square root method [4].

1) Calculating the elements' product of each line in judgment matrix P

$$P(1) = 1*1*7 = 7; P(2) = 1*1*7 = 7; P(3) = \frac{1}{7}*\frac{1}{7}*1 = \frac{1}{49};$$

2) Calculate the three square root $W_i$ of P (I): $W_1 = 1.9129; W_2 = 1.9129; W_3 = 0.2733;$

3) Do normalization and regularization processing for W = (W1, W2, W3),

$$W_i' = \frac{W_i}{\sum_{i=1}^{3} W_i} \tag{2}$$

Get: $W_1' = 0.4667, W_2' = 0.4667, W_3' = 0.0666$

The feature vector: $W_{WREI} = (0.4667, 0.4667, 0.0666)$

*2.3.2.3. Consistency check*. We will need a consistency check of judgment matrix to test above characteristic vector weights allocation is reasonable, the method is as follows:

1) Calculate the maximum eigenvalue $\lambda_{max}$ of judgment matrix

$$\lambda_{max} = \sum_{i=1}^{3} \frac{(PW_{WREI})_i}{nW_i'} = \frac{1}{n} \sum_{i=1}^{3} \frac{(PW_{WREI})_i}{W_i'} \tag{3}$$

(n refers to the amount of transversal vector )

In the formula, $(PW_{WREI})_i$ represent the element i of $PW_{WREI}$, and n=3

$$PW_{WREI} = \begin{pmatrix} (PW_{WREI})_1 \\ (PW_{WREI})_2 \\ (PW_{WREI})_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 7 \\ 1 & 1 & 7 \\ \frac{1}{7} & \frac{1}{7} & 1 \end{pmatrix} \begin{pmatrix} 0.4667 \\ 0.4667 \\ 0.0666 \end{pmatrix}, \text{ Get: } \lambda_{max} = 3$$

2）Consistency check, use the test formula:

$$CR = \frac{CI}{RI}, CI = \frac{\lambda_{\max} - n}{n - 1} \tag{4}$$

With n = 3, RI = 0.58, $\lambda_{\max} = 3$, get: CR = 0.7 < 0.1

Show a satisfactory consistency judgment matrix P, so each component

$W_{WREI} = (0.4667, 0.4667, 0.0666)$ can be used as a weight.

*2.3.2.4. Determination of the weights of every index.* According to the above method, can calculate the indexes' judgment matrix , weight and consistency check results as Table 1,Table 2,Table 3,Table 4, you can see the consistency of judgment matrix is satisfied, illustrating the effectiveness of the judgment matrix of each index.

**Table 1.** Judgment matrix WREI_u1, $W_{WREI\_u1} = \{0.7778, 0.1111, 0.1111\}$

| u1 | u11 | u12 | u13 | Weight WWREI_u1 | Consistency check |
|----|-----|-----|-----|-----------------|-------------------|
| u11 | 1 | 7 | 7 | 0.7778 | λmax=3 |
| u12 | 1/7 | 1 | 1 | 0.1111 | CI=0 |
| u13 | 1/7 | 1 | 1 | 0.1111 | CR=0<0.1 |

**Table2.** Judgment matrix WREI_u2

$W_{WREI\_u2} = \{0.8333, 0.1667\}$

| u21 | u22 | Weight WWREI_u2 | Consistency check |
|-----|-----|-----------------|-------------------|
| 1 | 5 | 0.8333 | λmax=2 CI=0 |
| 1/5 | 1 | 0.1667 | CR=0<0.1 |

**Table3.** Judgment matrix WREI_u3

$W_{WREI\_u3} = \{0.8571, 0.1429\}$

| u3 | u31 | u32 | Weight WWREI_u3 | Consistency check |
|----|-----|-----|-----------------|-------------------|
| u31 | 1 | 6 | 0.8571 | λmax=2 CI=0 |
| u32 | 1/6 | 1 | 0.1429 | CR=0<0.1 |

| WSI | u1 | u2 | u3 | Weight WWREI | Consistency check |
|-----|----|----|----|--------------|-------------------|
| u1 | 1 | 1 | 7 | 0.4667 | λmax=3 |
| u2 | 1 | 1 | 7 | 0.4667 | CI=0 |
| u3 | 1/7 | 1/7 | 1 | 0.0666 | CR=0<0.1 |

**Table 4.** Judgment matrix WREI, $W_{WREI} = \{0.4667, 0.4667, 0.0666\}$

*2.4. Establishment of a fuzzy comprehensive evaluation model (FCE)*

*2.4.1. Principle of a fuzzy evaluation model.* Fuzzy evaluation factors is mainly composed of matrix of relation, factor sets and judgment sets.

1) Factor sets: $U = \{U_1, U_2, U_3, \ldots, U_k\}$ $(k = 1,2,3\ldots)$ ;

2) Judgment sets: $V = \{V_1, V_2, V_3, \ldots, V_k\}$ $(k = 1,2,3\ldots)$ ;

Fuzzy rating constitute a set V, and the number should be same as the number of levels of factors' division.

*2.4.2. The single factor evaluation.* R refers to the single factor evaluation

matrix:
$$R = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{pmatrix}$$

*2.4.3. Comprehensive evaluation.* Choose the fuzzy matching model M, for weight $A = (a_1, a_2, a_3, \ldots, a_j)$ $(j = 1,2,3, \ldots)$

According to the actual data situation, determine the plus or minus characteristic of elements in weight A and get the weight A, and then we can get comprehensive evaluation.

$$WREI = A' \cdot R = (wrei_1, wrei_2, wrei_3, \ldots, wrei_m) \tag{5}$$

The bigger WREI value is, the greater the regional water security situation will be, that is, the region can provide more clean water.

## 3. Model validation and prediction

We choose India as an example of explanation because it is known that the water resources was of severe or moderate shortage in most regions of India. We will use the data of India to prove the AHP-FCE Model and predict the water resources evaluation index in future.

*3.1. Data and Data Processing*

*3.1.1. Data.* We found and organized the required data on the reference website. Because the model is of fuzzy evaluation based on analytic hierarchy process (AHP), according to the water resources, population and water use, capacity, we divide the data into three parts.

We take part 1 as an example. We list the survey data about water resources on www.fao.org/nr/water/aquastat/water_res/index.stm in Table 5 as follow:

**Table 5.**Data of Water resources

(mm/year)/(10^9 m3/year)/(10^9 m3/year)

| Factors / Years | National Rainfall Index | Internal renewable water resources | External renewable water resources |
|---|---|---|---|
| 1958-1962 | 1703 | 1446 | 464.9 |
| 1963-1967 | 1704 | 1447 | 465.9 |
| 1968-1972 | 1705 | 1448 | 466.9 |
| 1973-1977 | 1706 | 1449 | 467.9 |
| 1978-1982 | 1707 | 1450 | 468.9 |
| 1983-1987 | 1708 | 1451 | 469.9 |
| 1988-1992 | 1709 | 1452 | 470.9 |
| 1993-1997 | 1710 | 1453 | 471.9 |
| 1998-2002 | 1711 | 1454 | 472.9 |
| 2003-2007 | 1712 | 1455 | 473.9 |
| 2008-2012 | 1713 | 1456 | 474.9 |
| 2013-2017 | 1714 | 1457 | 475.9 |

*3.1.2. Data processing to get WREI.* (Table 5 is an example for complete process, the other two parts are in the same way.)

We need to normalize the element of each year in the table, because the way to deal with these three tables is same. We take the Table 5 as an example to write the specific process of solution $W_{WREI\_u1}$.

1）Normalize each element in Table 5，then we can get Table 6 as follow:

**Table 6.** Normalized matrix of Water resources

(mm/year)/(10^9 m3/year)/(10^9 m3/year)

| Factors / Years | National rainfall index | Internal renewable water resources | External renewable water resources |
|---|---|---|---|
| 1958-1962 | 0.471236061 | 0.400121752 | 0.128642187 |
| 1963-1967 | 0.471121679 | 0.400066355 | 0.128811966 |
| 1968-1972 | 0.471007486 | 0.40001105 | 0.128981464 |
| 1973-1977 | 0.470893483 | 0.399955836 | 0.12915068 |
| 1978-1982 | 0.470779668 | 0.399900714 | 0.129319617 |
| 1983-1987 | 0.470666042 | 0.399845683 | 0.129488275 |
| 1988-1992 | 0.470552603 | 0.399790743 | 0.129656654 |
| 1993-1997 | 0.470439352 | 0.399735894 | 0.129824754 |
| 1998-2002 | 0.470326287 | 0.399681135 | 0.129992578 |
| 2003-2007 | 0.470213409 | 0.399626466 | 0.130160125 |
| 2008-2012 | 0.470100716 | 0.399571887 | 0.130327396 |
| 2013-2017 | 0.469988209 | 0.399517398 | 0.130494392 |

2) After transferring them, we get the single factor evaluation matrix:

$$R_{WREI\_u1} = \begin{pmatrix} 0.4712 & 0.4711 & 0.4710 & 0.4709 & 0.4708 & 0.4707 & 0.4706 & 0.4704 & 0.4703 & 0.4702 & 0.4701 & 0.4700 \\ 0.4001 & 0.4001 & 0.4000 & 0.3999 & 0.3998 & 0.3998 & 0.3998 & 0.3997 & 0.3997 & 0.3996 & 0.3996 & 0.3995 \\ 0.1286 & 0.1288 & 0.1290 & 0.1292 & 0.1293 & 0.1295 & 0.1297 & 0.1298 & 0.1300 & 0.1302 & 0.1301 & 0.1305 \end{pmatrix}$$

3) Weight table available from before: $W_{WREI\_u1} = (0.7778, 0.1111, 0.1111)$

4) For the choice of the fuzzy model, we think a balanced consideration of weights in all factors, so we choose the weighted average model, namely, using the fuzzy algorithm formula for judging value of $WREI_{u1}$ in each time period:

$$WREI_{u1} = W_{WREI\_u1} \cdot R_{WREI\_u1} \qquad (6)$$

Using Matlab to solve more complex matrix operations, and the result is：

$WREI_{u1} = (0.4252, 0.4252, 0.4251, 0.4251, 0.4250, 0.4249, 0.4249, 0.4247, 0.4246, 0.4246, 0.4245, 0.4244)$

As the same theory, we can see single factor evaluation matrix of u2 and u3 are respectively

$$R_{WREI\_u2} = \begin{pmatrix} 0.1927 & 0.2117 & 0.2262 & 0.2538 & 0.2708 & 0.2895 & 0.3319 & 0.3322 & 0.3662 & 0.3689 & 0.3794 & 0.4067 \\ 0.8073 & 0.7883 & 0.7738 & 0.7462 & 0.7292 & 0.7105 & 0.6681 & 0.6678 & 0.6338 & 0.6311 & 0.6206 & 0.5933 \end{pmatrix}$$

$$R_{WREI\_u3} = \begin{pmatrix} 0.0001 & 0.0004 & 0.0012 & 0.0013 & 0.0014 & 0.0017 & 0.0038 & 0.0047 & 0.0046 & 0.0034 & 0.0031 & 0.0034 \\ 0.9999 & 0.9996 & 0.9988 & 0.9987 & 0.9986 & 0.9983 & 0.9962 & 0.9953 & 0.9954 & 0.9966 & 0.9969 & 0.9966 \end{pmatrix}$$

The weight table of u2 and u3 are respectively

$$W_{WREI\_u2} = \{0.8333, 0.1667\}, \quad W_{WREI\_u3} = \{0.8571, 0.1429\}$$

Using the fuzzy algorithm formula for judging value of $WREI_{u2}$, $WREI_{u3}$ in each time period:

$WREI_{u2} = (0.2952, 0.3078, 0.3175, 0.3359, 0.3472, 0.3597, 0.3879, 0.3881, 0.4108, 0.4126, 0.4196, 0.4378)$

$WREI_{u3} = (0.1430, 0.1432, 0.1438, 0.1438, 0.1439, 0.1441, 0.1456, 0.1463, 0.1462, 0.1453, 0.1451, 0.1453)$

5) Make a comprehensive evaluation of factor sets

$U_{WREI} = \{U_{WREI\_u1},\ U_{WREI\_u2},\ U_{WREI\_u3}\}$ , so as to get the following data in Table 7:

**Table 7.** Data of WREI

| Factors \ Years | Water resources （WREIu1） | Population and water use （WREIu2） | Capacity （WREIu3） |
|---|---|---|---|
| 1958-1962 | 0.4252 | 0.2952 | 0.143 |
| 1963-1967 | 0.4252 | 0.3078 | 0.1432 |
| 1968-1972 | 0.4251 | 0.3175 | 0.1438 |
| 1973-1977 | 0.4251 | 0.3175 | 0.1438 |
| 1978-1982 | 0.425 | 0.3472 | 0.1439 |
| 1983-1987 | 0.4249 | 0.3597 | 0.1441 |
| 1988-1992 | 0.4249 | 0.3597 | 0.1456 |
| 1993-1997 | 0.4247 | 0.3881 | 0.1463 |
| 1998-2002 | 0.4246 | 0.4108 | 0.1462 |
| 2003-2007 | 0.4246 | 0.4126 | 0.1453 |
| 2008-2012 | 0.4245 | 0.4126 | 0.1451 |
| 2013-2017 | 0.4244 | 0.4378 | 0.1453 |

6) Normalize each element for Table 7.

7) After transferring them, we get the single factor evaluation matrix:

$$R = \begin{pmatrix} 0.4925 & 0.4853 & 0.4796 & 0.4796 & 0.4639 & 0.4575 & 0.4568 & 0.4428 & 0.4326 & 0.4322 & 0.4322 & 0.4212 \\ 0.3419 & 0.3513 & 0.3582 & 0.3582 & 0.3790 & 0.3873 & 0.3867 & 0.4047 & 0.4185 & 0.4199 & 0.4201 & 0.4345 \\ 0.1656 & 0.1634 & 0.1622 & 0.1622 & 0.1571 & 0.1552 & 0.1565 & 0.1525 & 0.1489 & 0.1479 & 0.1477 & 0.1442 \end{pmatrix}$$

8) Weight table available from before: $W_{WREI} = (0.4667, 0.4667, 0.0666)$

9) Considering the water resources and the capacity are the positive effects of water resources while population and water use is the opposite effects of water resources, so we get the real weight matrix: $W_{WREI'} = (0.4667, -0.4667, 0.0666)$

The evaluation of estimate of each factors of WREI in the period

$$WREI = W_{WREI}{}' \cdot R \tag{7}$$

So the value of WREI is

$WREI = (0.0813, 0.0734, 0.0675, 0.0675, 0.0501, 0.0431, 0.0431, 0.0279, 0.0165, 0.0156, 0.0155, 0.0034)$

*3.2. Grey prediction*

Substitute the value of WREI into the grey forecasting model, we can get accumulated value formula of once fitting formula x1(t+1)=-0.49986exp(-0.17854t)+(0.58116) and accumulated value formula of twice fitting formula x1(t+1)=-0.51421exp(-0.17854t)+(0.59075)，and their prediction results of WREI value is as follows in Figure 2 and Figure 3:
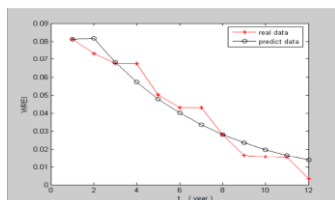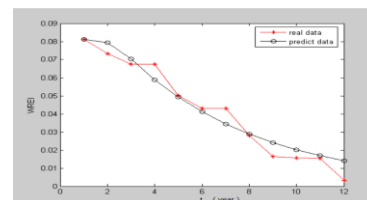


**Figure 2.** Once fitting prediction results



**Figure 3.** Twice fitting prediction results

*3.3. Improvement of Grey Prediction for Linear Prediction*

*3.3.1. Problem analysis.* By above Figure 2 and Figure 3, although the predicted values of the grey prediction is relatively close to the actual value, but in the process of variable t increase, predicted values will tend to be 0, which doesn't conform to the actual shortage situation, so if we chose linear prediction model [5], the effect will be better. And then we get the following equation and Figure 4 by Matlab.

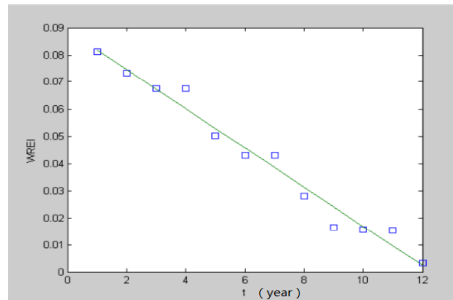WREI=-8315950251235889/1152921504606846976*x+3205089029355199/3602879



**Figure 4.** Linear prediction results

*3.3.2. Data and image of the predicted results.* According to the relational expression, I get the predictions in the next 15 years (2016-2030) of water situation:-0.0048  -0.0120  -0.0192  -0.0264  -0.0337 -0.0409 -0.0481 -0.0553 -0.0625 -0.0697 -0.0769 -0.0842 -0.0914 -0.0986 -0.1058

*3.3.3. Prediction conclusion.* Obviously, according to the prediction of water resources, short supply will occur in 2016.That is to say India's water consumption is greater than the sum of natural water and the water provided by human intervention. The situation becomes increasingly serious in the future, so if there are no special reasons within the next 15 years, India's water resources will become increasingly scarce, bringing significant pressure to economic development and people's life.

## 4. Conclusion

In this paper, we established the model of fuzzy comprehensive evaluation based on AHP to evaluate regional capability of providing clean water. Under the condition of the support of reliable data, we can calculate WREI, the greater the value is, the more abundant water resources are. By fitting analysis of WREI, we can get the trend of water resources in the next few years. The change factor value by having a plan can ultimately increase the value of WREI, namely to improve water quality

Despite our evaluation model include several layers, the modelling domain presented by this problem is vast, and there is a large amount of room for improvement. We believe that the work we have presented here is a significant and successful attempt at solving this problem.

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