

# Stability analysis of the governor-turbine-hydraulic system of pumped storage plant during small load variation

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**Abstract:** Governor-turbine-hydraulic (GTH) system is complex because of strong couplings of hydraulic, mechanical and electrical system. This paper presents a convenient mathematical model of the GTH system of a pumped storage plant (PSP) during small load variation. By using state space method and eigenvalue method, the stability of the GTH system is analyzed and the stable regions of the system can be given as well, which would help to optimize system design or the turning of governors. The proposed method is used to analyze the stability of a practical pumped storage plant during small load variation, which is also simulated in time domain on the basis of characteristics method. The theoretical analysis is in good agreement with numerical simulations. Based on the proposed method, the effect of the system parameters and operating conditions on the stable regions is investigated. These results are useful for the design of the GTH system of pumped storage plants.

## 1. Introduction

Pump storage power stations (PSP) are important as they are the most reliable and affordable energy storages for accommodating intermittent renewable generators in the power grid, also providing ancillary services as readily adjustable stand-by and running reserve for nuclear and other thermal generators [1]. However, unstable fluctuations due to the small disturbance are always happen during its operations [2], which can slow down the process of synchronization and power oscillation, etc. So it is an important issue on the design of a reliable hydraulic turbine regulating system to ensure its stability and regulating quality, especially for low head PSP.

Hydraulic turbine regulating system is complex, which is an application area with an interesting set of problems for control engineering people [3-4]. The early papers [5-7] are important in emphasizing the need to understand the design of control action. These literatures contributed towards establishing general guidelines for the selection of control parameters. Thorne et al. [8] are the first to apply the state space representation and eigenvalue analysis to examine the effects of the governor parameters-proportional, integral gains, system damping and unit loading on the stability boundaries of single hydro unit connected to a large power system. Dhaliwal et al. [9] have used the state space to study the effect of derivative gain and other governor parameter on the stability of single machine supplying an isolated load.

In this paper, the mathematical models for describing the small fluctuations in the GTH system



are presented, and a new method was proposed to analyze the stability of the GTH system of a practical PSP under islanding conditions. The process is simulated in time domain on the basis of characteristics method [10], and the results are compared with the proposed method as well. With this method, the stability region of the system is given, and the effect the cross-sectional area of the surge tank, the derivative time constant  $Tn$  and operating conditions on the stable regions of the GTH system in the PSP is investigated respectively.

## 2. Mathematical models

### 2.1 Unsteady flow Equation in pipeline system

As the changes in the dependent variables are limited to small values during small fluctuation, rigid water column theory and quasi-steady friction terms are applied to analyze the unsteady flow in pipeline. The unsteady flow for a simple pipeline is described by

$$\frac{L}{gA} \frac{dQ}{dt} = H_u - \alpha Q^2 - H_d \quad (1)$$

in which  $L$  = the length of the pipe,  $A$  = the cross-sectional area of the pipe,  $\alpha$  is the head loss coefficient along the pipe,  $Q$  = the flow in the pipe,  $H_u$  = the upstream piezometric head; and  $H_d$  = the downstream piezometric head.

### 2.2 Turbine and generator equations

Whenever there is a change in the power demand of the electrical system, a different level of power is absorbed by the generator and an unbalanced torque exists on the turbine, which gives rise to a speed change. The basic equation for speed change is

$$I \frac{d\omega}{dt} = T - T_g \quad (2)$$

Here  $I = GD^2 / 4$  is the polar moment of inertia of rotating fluid and mechanical parts in the turbine-generator combination,  $\omega$  is the angular speed of the unit,  $T$  is the torque produced by water flowing through the unit, and  $T_g$  is the resistant torque from the generator. The dimensionless form of the torque equation is

$$T_m \frac{d\varphi}{dt} = p - p_G = p - x - s_p \varphi \quad (3)$$

in which  $T_m = \frac{[GD^2]n_0^2}{365P_0}$  = the mechanical starting time,  $\varphi$  = dimensionless turbine speed variation,  $n$  = the rotating speed in rpm, and  $P$  = the power in kilowatts. The power absorbed by the generator,  $P_G$ , depends on the change in the external load,  $X$ , and on the speed variation. Defining  $p_G = \frac{P_G - P_{G0}}{P_{G0}}$ ,

$x = \frac{X - X_0}{X_0}$ ,  $s_p = \frac{\partial p_G}{\partial \varphi}$  gives  $p_G = x + s_p \varphi$ .

The flow passing through the turbine and the power produced by the turbine are given by

$$Q = D_1^2 Q_{11} \sqrt{H} \quad (4)$$

$$P = T_{11} D_1 H \times \frac{n\pi}{30} \quad (5)$$

where  $Q$  = the discharge,  $H$  = the head across the turbine,  $P$  = the turbine output,  $n$  = the rotating speed,  $D_1$  = the runner diameter, and  $T$  = the torque. Subscript 11 refers to the unit value. Both  $Q_{11}$  and  $T_{11}$  are functions of the unit speed,  $n_{11}$ , and the wicket gate position,  $\tau$ , as given by the characteristic curves of the turbine. By using Taylor series expansion, Eqs. (4) and (5) can be linearized in dimensionless form:

$$q = s_1\varphi + s_2\mu + \frac{1}{2}(1-s_1)\xi \quad (6)$$

$$p = (1+s_3)\varphi + s_4\mu + [1-\frac{s_3}{2}]\xi \quad (7)$$

in which  $\varphi = \frac{n-n_0}{n_0}$ ,  $\mu = \frac{\tau-\tau_0}{\tau_0}$ ,  $q = \frac{Q-Q_0}{Q_0}$ ,  $p = \frac{P-P_0}{P_0}$ ,  $\xi = \frac{H-H_0}{H_0}$ , and the parameters  $s$  are given by the turbine characteristic curves and the initial operating point, which are shown in case study.

### 2.3 Governor equation

Turbine governors are of different types, which usually exhibit control features as a combination of proportional, differential, and integral control (PID). Normally, the speed or acceleration is sensed and corrective action is taken as necessary. By neglecting some smaller terms, the governor equation of the PID-controller for turbine system can be expressed as

$$(b_t + b_p)T_d \frac{d\mu}{dt} + b_p\mu = -T_d T_n \frac{d^2\varphi}{dt^2} - (T_n + T_d) \frac{d\varphi}{dt} - \varphi \quad (8)$$

where  $\mu$  is dimensionless turbine gate opening variation,  $\varphi$  is dimensionless turbine speed variation;  $T_d$  is the dashpot time constant;  $T_n$  is the derivative time constant,  $b_t$  is the temporary speed droop, and  $b_p$  is the permanent speed drop.

### 2.4 Surge tank equation

$$\frac{dZ_{su}}{dt} A_{su} = Q_{su} \quad (9)$$

in which,  $Z_{su}$  = the water level of the surge tank,  $A_{su}$  = the cross-sectional area of the surge tank and  $Q_{su}$  = the water flow into the surge tank.

## 3. Stability criteria and coefficient matrix of state equations

The stability of the GTH system of a hydropower station depends on the eigenvalues of coefficient matrix  $\mathbf{A}$  ( $\lambda_n = \sigma_n + i\omega_n$ , where  $\sigma$  = the attenuation factor,  $\omega$  = the natural frequencies of free vibrations in the system and  $n$  = number of the state variables). If all the real part of eigenvalues are negative, i.e.  $\sigma_n < 0$ , the system is stable, otherwise is unstable. The GTH system is in a critical stable state if  $\sigma_{\max} = 0$ . The eigenvalues are found easily by means of a computer with standard subroutines using QR transformations, so the coefficient matrix  $\mathbf{A}$  of the state equations is essential for stability analysis.

Combined with the governing equations above, the state equations for the GTH system are yielded and may be expressed in matrix form

$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y} + \mathbf{B}\mathbf{x} \quad (10)$$

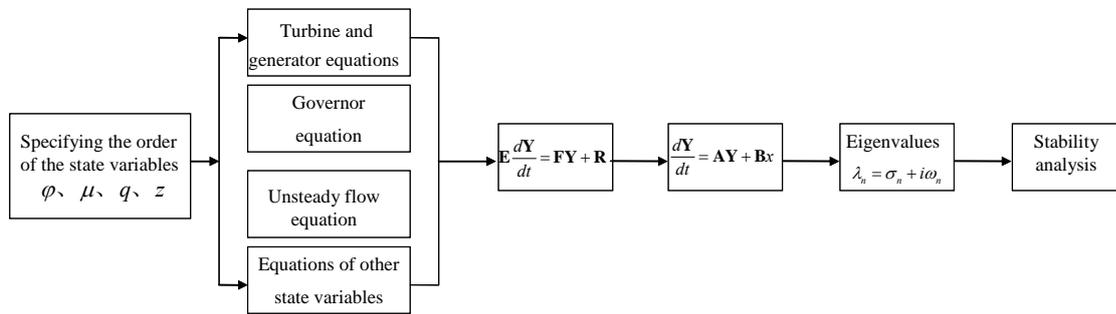
in which  $\mathbf{Y}$ =the state vector,  $\mathbf{A}$ = the coefficient matrix of state equations,  $\mathbf{B}$  = input matrix, and  $\mathbf{x}$  is the relative disturbance.

Unless the numbers of the state variables of the system are few, the coefficient matrix  $\mathbf{A}$  can be deduced directly, it is difficult to solve out the state equations. By left multiplying a coefficient matrix  $\mathbf{E}$  on both sides of equation (10), the equation (10) can be transformed as

$$\mathbf{E} \frac{d\mathbf{Y}}{dt} = \mathbf{F}\mathbf{Y} + \mathbf{R} \quad (11)$$

in which,  $\mathbf{E}$ ,  $\mathbf{F}$  and  $\mathbf{R}$  are coefficient matrixes of the system ( $\mathbf{F} = \mathbf{E}\mathbf{A}$  and  $\mathbf{R} = \mathbf{E}\mathbf{B}\mathbf{x}$ ). Compared with matrix  $\mathbf{A}$ , it is easier to give matrix  $\mathbf{E}$ ,  $\mathbf{F}$  and  $\mathbf{R}$ . These three matrixes are regular and easier for programming. When matrix  $\mathbf{E}$ ,  $\mathbf{F}$  and  $\mathbf{R}$  are solved out, equation (10) can be deduced directly by left multiplying by the inverse matrix of the coefficient  $\mathbf{E}$  on both sides of equation (11). The procedure of

the present method is schematically shown in Fig.1.

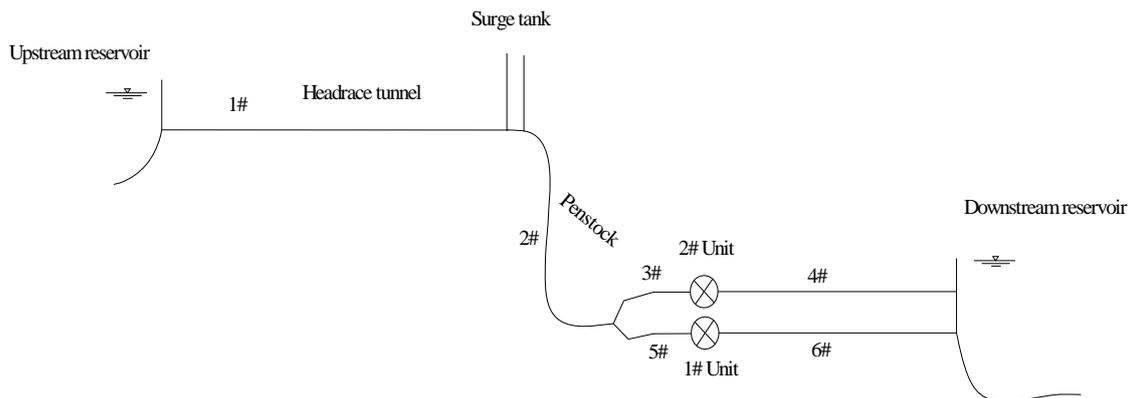


**Fig. 1 Flowchart of the present method**

**4. Case Study**

**4.1 Project introduction**

The model is used to analyze the stability of the GTH system in the practical PSP in China. As shown in Fig. 2, it has two 225 MW pump-turbines and every two-units share a common waterway system, which consists of the intake, the gate shaft, the headrace tunnel, the throttle surge tank, penstock, the tailrace tunnel and so on. The system parameters are presented in tables 1 and 2.



**Fig.2 Schematic diagram of the PSP in case study**

**Table 1 Pipe system data**

Pipe	<i>L</i> (m)	<i>D</i> (m) <sup><i>a</i></sup>	<i>a</i> (m/s)	<i>N</i>	<i>n</i> <sup><i>b</i></sup>
1#	703.7	8.5	1100	136	0.012
2#	289.3	7.8	1100	53	0.012
3#	102.9	4.2	1100	19	0.012
4#	346.3	6.8	1100	72	0.012
5#	109.1	4.2	1100	23	0.012
6#	346.3	6.8	1100	72	0.012

<sup>*a*</sup> *D* is equivalent diameter and *N* is number of reaches for each pipe.

<sup>*b*</sup> *n* is roughness which is used in Manning equation.

**Table 2** Parameters of surge tank and unit

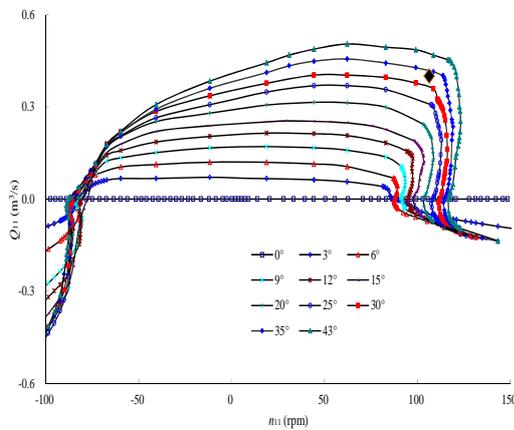
Surge tank					Unit				
$D_s$	$D_{orf}$	$\zeta_{in}$	$\zeta_{out}$	$P_R$	$H_R$	$N_R$	$Q_R$	$D$	$WD^2/g$
15 (m)	4.5 (m <sup>2</sup> )	0.6	0.8	229.6 (MW)	175 (m)	250 (r/min)	147.8 (m <sup>3</sup> /s)	5.2 (m)	11500 (t·m <sup>2</sup> )

According to Fig. 2, the state variables of the GTH system of this hydropower station are defined as  $\varphi_1, \varphi_2, \mu_1, \mu_2, q_1, q_2, q_{t1}$  and  $z_{su}$ . By using the method above, the coefficient matrix **A** of the state equations is derived conveniently, which is used to judge the stability of the GTH system. Due to space limitations, the derivation process is not presented.

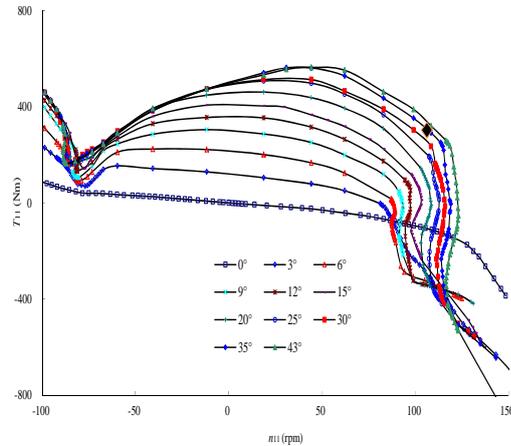
In order to investigate the stability of the GTH system, one of the critical operating conditions is used where the upstream water level is 239.0m, the downstream water level is 81.0m. The initial parameters of the turbine are known which are shown in table 3 and Fig. 3. Substituting these parameters into equation (11) and left multiplying the inverse matrix of the coefficient **E** on both sides, the coefficient matrix **A** of the state equations is solved out. The stable region in  $b_t-T_d$  plane for fixed  $T_n=1.0$  are shown in Fig. 4.

**Table 3** Parameters of initial operating condition

Turbine	$H$	$Q$	$\tau$	$n_{11}$	$Q_{11}$	$T_{11}$	$S_1$	$S_2$	$S_3$	$S_4$
1#	149.6	132.7	33.5°	106.3	0.4	302.8	-0.426	0.901	-2.348	1.196
2#	149.2	133.0	33.5°	106.4	0.4	303.5	-0.421	0.904	-2.337	1.201

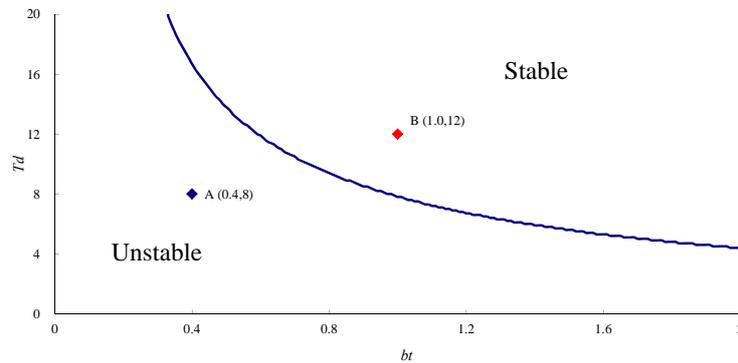


(a) unit discharge



(b) unit torque

**Fig.3** Characteristic curves of pump turbine (a) unit discharge; (b) unit torque



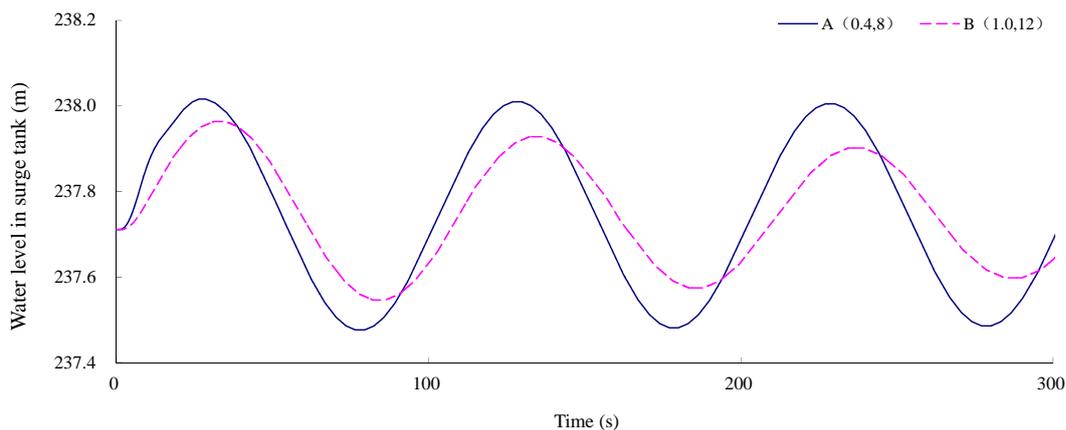
**Fig. 4 Stable region in  $b_r$ - $T_d$  plane for fixed  $T_n=1.0$**

#### 4.2 Verification with numerical simulation

To verify the proposed method, the hydraulic transient model of hydropower system on the basis of characteristics method is used to simulate the small load disturbance in the GTH system. The computer model of the hydraulic transients in the hydropower station is encoded in the FORTRAN programming language. The system as shown in Fig.2 is divided to 375 reaches, and a time interval of 0.0049 s is used. The characteristic data of the turbine is obtained from characteristic curves as shown in Fig. 3.

Two groups of the parameters  $T_d$  and  $b_r$  for the governor are selected randomly, where A=(0.4,8) in the unstable region and B=(1.0,12) in the stable region. The operating condition is the same as before, and the load disturbance is -2%.

By using these two groups of governor parameters, the small load disturbance condition is simulated. As shown in Fig.5, the variation of water level in the surge tank is divergent when the governor parameter A is used; when the governor parameter B is used, the result is convergent. The theoretical analysis is in good agreement with numerical simulations. So it demonstrates that the proposed method is correct for the GTH system.

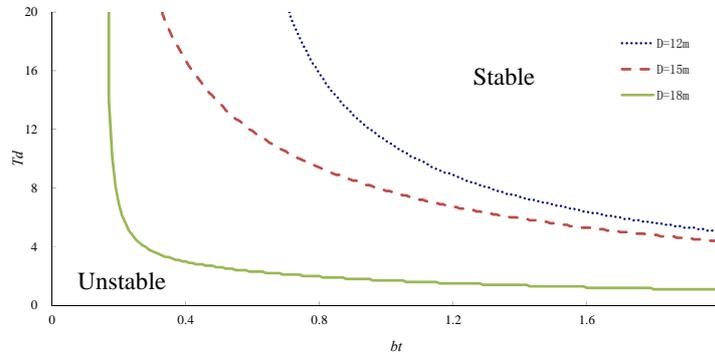


**Fig.5 Variation of water level in surge tank with two different governor parameters**

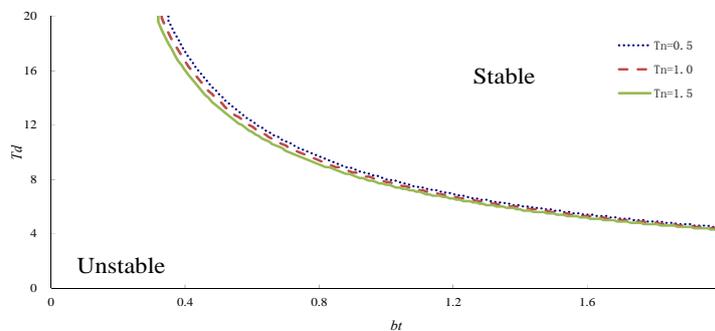
#### 4.3 Sensitive analysis of the system parameters on the stable regions

Based on the proposed method, the effect of the system parameters and operating conditions on the stable regions of the GTH system in the PSP is investigated. The effect of the cross-sectional area of the surge tank on the stable regions is shown in Fig. 6. It is shown that the larger area the surge tank has, the more stable the GTH system will be. In order to keep good regulating quality, the

cross-sectional area of the surge tank should be big enough. As shown in Fig. 7, the bigger  $T_n$ , the more stable the GTH system is, but the influence is not so sensitive.

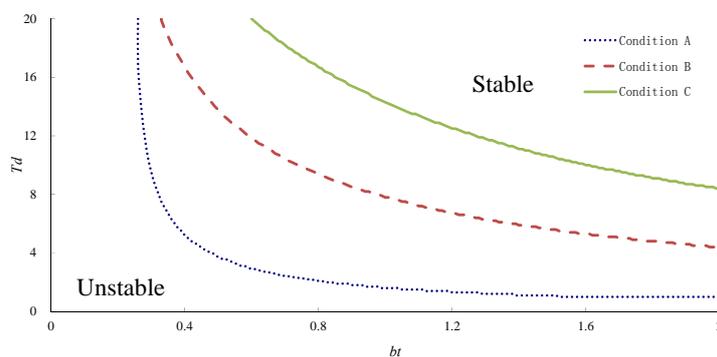


**Fig.6 Influence of surge tank area on stable regions**



**Fig.7 Influence of derivative time constant  $T_n$  on stable regions**

As shown in Fig. 8, when the operating conditions are different, the stable regions are different as well. Condition A is the lowest head condition, and the pump turbine is operating on speed-no-load. Condition B is the lowest head condition as well, while the pump turbine is operating on full load. Condition C is the rated head condition, and the pump turbine is operating on rated load. When the pump turbine operates on speed-no-load, the stable regions of the GTH system are large and the parameters of the governor can be selected with small values, which can accelerate the regulating process and keep the system stable. It is generally considered that the stability of the system will become worse when the turbine head decreases. With respect to condition C, the pump turbine operates under rated head, but the stable regions are smaller than that of condition B on which the pump turbine operates under the lowest head. This is different from the traditionally opinion, which comes from the effect of the pump turbine characteristics. It means that if the characteristics of the turbine are considered, the rated operating condition may become the critical one for stability of the GTH system under small disturbance. In this project, the cross-sectional area of the surge tank is increased to improve the stable regions under rated condition, and keep good regulating quality as well.



**Fig.8 Influence of operating conditions on stable regions**

## 5. Conclusions

The mathematical models for analyzing the stability of the GTH system are presented, and the stable regions of the system can be given as well. The proposed method is used to analyze the stability of the GTH system of an actual PSP during small load variation, and the stability regions are given. The hydraulic transient model on the basis of characteristics method is used to simulate the small load disturbance in the GTH system to verify the proposed method, the agreement of the results between theoretical analysis and numerical simulations is satisfactory, which demonstrates that the proposed method is correct. The influence of the cross-sectional area of the surge tank, the derivative time constant  $Tn$  and operating conditions on the stable regions of the GTH system in the PSP is investigated respectively. It is shown that the larger area the surge tank has, the more stable the GTH system will be. The influence of the derivative time constant  $Tn$  on the stable regions is not sensitive. When the characteristics of the turbine are considered, the rated operating condition may become the critical one for stability of the GTH system under small disturbance. These results are useful for the design of the GTH system of pumped storage plants.

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## Nomenclature

acronyms

GTH	governor-turbine-hydraulic system
PID	proportional, differential and integral

symbols

$A, A_{su}$	area of pipe, cross-sectional area of surge tank
$a$	pressure wave celerity
$b_p, b_t$	temporary speed drop, permanent speed drop
$D, D_1$	equivalent diameter of pipe, turbine diameter
$GD^2$	moment of inertia of unit
$H$	head across turbine
$L$	length of pipe
$n$	rotating speed
$P, P_G$	turbine output, power absorbed by generator
$Q$	discharge
$q$	dimensionless discharge variation

$T, T_d, T_m, T_n$	torque, dashpot time constant, mechanical starting time, promptitude time constant
$x$	dimensionless external load variation
$Z_{su}$	water level of surge tank
$\alpha$	head loss coefficient
$\varphi$	dimensionless rotating speed variation
$\mu$	dimensionless wicket gate opening variation
$\lambda$	eigenvalues of coefficient matrix
$\xi$	dimensionless unit's head
$\tau$	dimensionless wicket gate opening
<i>Subscript</i>	
11	unit value
0	initial steady state
R	rated value

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