

The worst moment of superposed surge wave in upstream series double surge tanks of hydropower station

Y Teng ^{1,2}, J D Yang ¹, W C Guo ^{1,3}, J P Chen ¹

1 State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, 430072, China

2 Sichuan Water Resources and Hydroelectric Investigation & Design Institute, Chengdu, 610072, China

3 Maha Fluid Power Research Center, Department of Agricultural and Biological Engineering, Purdue University, West Lafayette, 47907, USA

E-mail: tengy_w hu@foxmail.com

Abstract: It is a consensus to consider the superposed working conditions when calculating the surge wave in surge tank of hydropower station with long diversion tunnel. For the hydropower station with single surge tank, the method of determining the worst superposed moment is mature. However, for the hydropower station with upstream series double surge tanks, research in this field is still blank. Based on an engineering project, this paper investigated the worst moments and the control superposed working conditions about the maximum surge level and the minimum surge level of upstream series double surge tanks using numerical simulation. In addition, the incidence relations between the worst moment of superposed surge wave and the different areal array and distance between the two surge tanks are also carried out. The results showed that: With the decrease of the distance between auxiliary surge tank and upstream reservoir, the maximum values of the highest surge levels in the two surge tanks always reach close to but a little earlier than the bigger one time when the inflowing discharges of the two surge tanks reach the maximum. It is similar to the minimum values of lowest surge levels in the two surge tanks which also reach close to but a little later than the bigger one time when the outflowing discharges of the two surges reach the maximum. Moreover, the closer the area of auxiliary surge tank to the area of main surge tank is, the closer the worst moment to the bigger one time when inflow or outflow of the two surges reach the maximum will become.

1 Introduction

For hydropower stations with very long diversion tunnels, such as longer than 5 km or even up to 30 km in length [1,2], by having just one surge chamber at the upstream end can cause a number of problems for the surge chamber building. The problems include large area, difficulty in structural lining, high cost, low stability, and so on. Further, in some cases, it may not be possible to build a large surge chamber due to the geological conditions. Under this type of situation, the possibility of building double-chamber surge tank at the upstream need to be considered.

In an upstream double-chamber surge system, due to geological and topographical constraints, the main surge chamber may not be able to have the desired cross-sectional area. The less than desired area in the main surge chamber weakens its function in the transition process to a considerable degree. Thus, an auxiliary surge chamber has to be set up adjacent to the main surge chamber so as to ensure



safe and stable operation of the entire power plant system. After setting up the auxiliary surge chamber, the water level fluctuation in the water diversion system of the power plant becomes more complicated as compared to that of the single surge chamber system. One of the complicating factors is the superposition of fluctuations, including (1) the superposition of mass wave, the reflection and superposition of water hammer wave in the propagation process, the superposition of water hammer wave and mass wave in the main and auxiliary surge chambers; (2) the superposition of mass wave under combined working conditions in the same surge chamber. The other factor is the changes in the layout position and area of the auxiliary surge chamber, which also have an influence on the regulation action during the transition process.

The operation conditions of hydropower stations are usually divided into conventional conditions and combined conditions. The conventional conditions only have one action in the unit load change. The combined conditions have two or more actions. Based on both practical experiences and research, it is commonly known that the highest and lowest surges in surge chambers during the operation of hydropower stations generally occur under the combined conditions [3~6]. In most cases, the highest surge in an upstream surge chamber controls the spiral case's maximum hydrodynamic pressure while the lowest surge is related to the safety problem, such as the 'evacuation' at the bottom of a surge chamber [7]. Thus, it is very important to study the highest and the lowest surges in the main and auxiliary surge chambers in the upstream double-chamber surge system with a very long diversion tunnel.

An existing hydropower station has been used in this study. The transition calculation software TOPsys [3, 8~12] has been used for the numerical simulations under various conditions. The objective of the simulations is to determine the most critical wave superposition moments for the highest and lowest surges under the combined conditions in the hydropower station with the upstream double surge chambers.

2 Calculation model

This study has used the software TOPsys to simulate the transitions in the hydropower station. The calculation methods of this software are briefly introduced in the next paragraph. However, the main calculation methods including the constant flow calculation based on the element method, the hammer simulation based on the method of generalized characteristic curves [13], and the unit boundary conditions based on spatial curved surface [14] are not described in this paper due to limitation of space. As this software takes into account the elasticities of water and the wall, the results are more realistic.

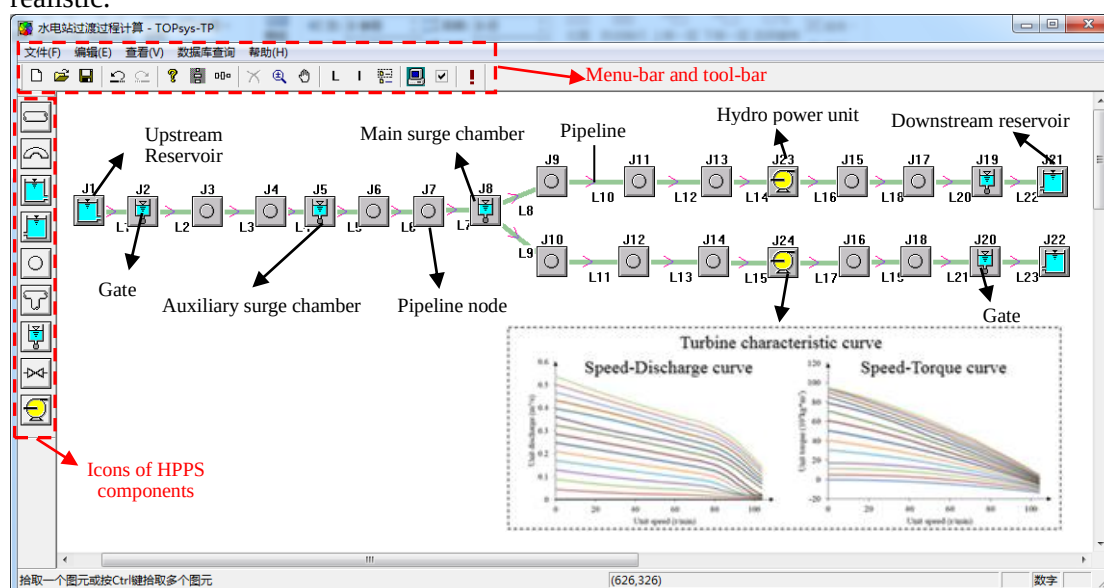


Fig. 1 Graphical user interface of TOPsys and a model of HPP (Case 1 in Section 3).

The governing equations for flow in water pipes are the momentum and continuity equations, as follows:

Momentum equation:

$$V \frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} + \frac{a^2 V}{gA} \frac{\partial A}{\partial x} - \sin \theta \cdot V = 0 \quad (1)$$

Continuity equation:

$$g \frac{\partial H}{\partial x} + V \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + f \frac{V|V|}{2D} = 0 \quad (2)$$

The boundary conditions of the unit, upstream and downstream reservoirs, surge chambers, branch tubes and so on can be found in paper [8]. Fig. 1 shows the interface of TOPsys and the hydropower station model used in this study.

3 Most critical wave superposition moment

In the selected hydropower station, the length of the diversion tunnel is about 17.5 km. There are four hydropower plants, each with two units. The data of these units are shown in Appendix xx. In this study, the hydraulic unit of Jinping Power hydropower station has been modified with a double-chamber surge tank. The size of the impedance of the main and auxiliary surge chambers equals to 20% of the cross-sectional area of the tunnel at the bottom. The pipe connecting the main to the auxiliary surge chamber has a diameter of 11 m. At the initial moment, the cross-sectional area of the main surge chamber is 300 m². This is according to Thomas stable cross-sectional calculation formula, which states that for one main surge chamber, the cross-sectional area should be 407 m². The cross-sectional area of the auxiliary surge chamber is 200 m². The other parameters of the hydropower station remain unchanged.

3.1 Most critical wave superposition moment for highest surge in surge chamber

3.1.1 For fixed areas of main and auxiliary surge chambers and fixed distance between them

After setting the conditions in the double chambers according to the chamber hydropower specification [5], the condition Z1 is for the determination of the most critical wave superposition moment of the highest surge in the double chambers. As the double-chamber surge tank in the hydropower station consists of the main and the auxiliary surge chambers, it is important to consider the influence of the auxiliary surge chamber on the safe and stable operation of the entire hydropower station. As such, ΔT has a significant effect on the water levels in the main and auxiliary surge chambers. In this study, these conditions have been investigated.

Table 1 Specifications of Z1 condition

Calculation condition	Upstream water level /m	Downstream water level /m	Initial conditions	Superposed conditions	Condition to be determined
Z1	1646.0	1330.1	Normal water level in upstream, one unit running, load increasing in the other unit	After ΔT , all units suddenly reject load	Highest surge in the upstream main and auxiliary surge chambers

The maximum flow rates into the main and auxiliary surge chambers can be determined from the initial conditions. In order to determine ΔT , the moments around the maximum flow rates into the main and auxiliary surge chambers have been determined, and the high water levels in the main and auxiliary surge chambers are shown in Fig. 2. The time when the flow into the main surge chamber reaches its peak is 212 s, and the time when the flow into the auxiliary surge chamber reaches its peak is 260 s. There is a difference of 48 s between the two times to peak. Thus, the time interval for

determining the wave superposition moment has been set at 16 s. Specifically, the times for determining the superposition moments are 196 s, 212 s, 228 s, 244 s, 260 s, 276 s, 292 s and 308 s.

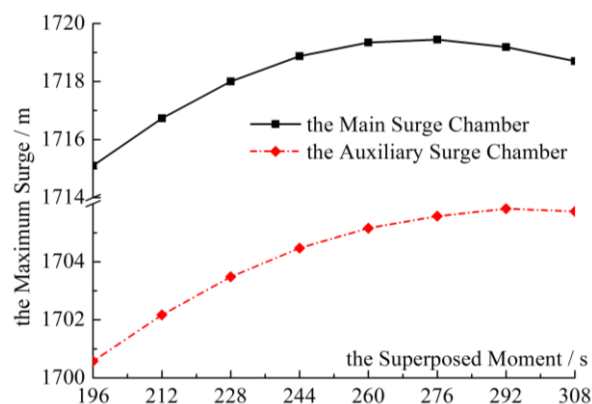


Fig. 2 Variation of surge levels in main and auxiliary surge chambers at various superposition moments

Fig. 2 shows the most critical moment (after the moment when the flow into the auxiliary surge chamber reaches its peak) when the surge reaches its highest level in the main surge chamber. This surge has been maintained in the main and auxiliary surge chambers. The difference between the highest surges is as small as 0.1 m, which is determined from the superposition moment of the most critical moment and the moment of maximum flow into the auxiliary surge chamber. Hence, to predict the most critical condition in real-life power plants, the moment of maximum flow rate into an auxiliary surge chamber can be considered as the most critical moment.

3.1.2 For fixed areas of main and auxiliary surge chambers but change in distance between them change

(1) Change in distance between main and auxiliary surge chambers

While keeping all the other parameters of the hydropower station constant, the auxiliary surge chamber has moved to two new locations, upstream and downstream by 1,000 m. After the moves, the corresponding distances between the main and the auxiliary surge chambers are 3,000 m and 5,000 m, respectively.

When the distance between the main and the auxiliary surge chambers is 5,000 m, based on the initial conditions, it can be determined that the time when the flow into the auxiliary surge chamber reaches its peak is 278.8 s, and the time when the flow into the main surge chamber reaches its peak is 226.4 s. There is a difference of 52.4 s between the two times to peak. Therefore, the time interval for determining the superposition moment has been set at 13.1 s. Specifically, the times for determining the superposition moments are 213.3 s, 226.4 s, 239.5 s, 252.6 s, 265.7 s, 278.8 s, 291.9 s and 305 s. The results are shown in Fig. 3.

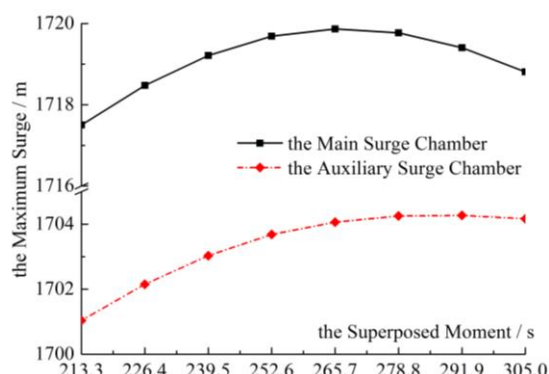


Fig.3 Variation of surge level in main and

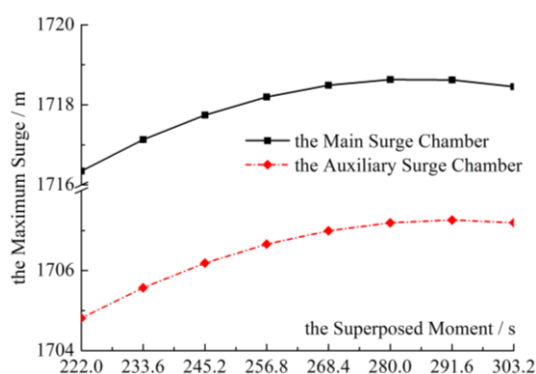


Fig.4 Variation of surge level in main and

auxiliary surge chambers for $L_2 = 5000\text{m}$

auxiliary surge chambers for $L_2 = 3000\text{m}$

When the distance between the main and the auxiliary surge chambers is 3,000 m, based on the initial conditions, it can be determined that the time when the flow into the auxiliary surge chamber reaches its peak is 233.6 s, and the time when the flow into the main surge chamber reaches its peak is 280 s. There is a difference of 46.4 s in the two times to peak. Thus, the time interval for determining the superposition moment has been set at 13.6 s. Specifically, the times for determining the superposition moments are 222.0 s, 233.6 s, 245.2 s, 256.8 s, 268.4 s, 280.0 s, 291.6 s and 303.2 s. The results are shown in Fig.4.

An examination of Figs. 2-4 shows the following:

① When the auxiliary surge chamber has moved upstream, the timing of maximum flow into the auxiliary surge chamber has changed. There is no linear relationship between the change in timing and the length of the pipe connecting the main and auxiliary surge chambers. The surge superposition and the mass wave superposition during that period in the main and auxiliary surge chambers are the fundamental reasons for the changes in timing of the maximum flows into the main and the auxiliary surge chambers.

② The highest surge in the main and the auxiliary surge chambers always occurs near the moment of maximum flow into the main and auxiliary surge chambers. Although the moment of highest surge in the main and auxiliary surge chambers is actually later than the moment of maximum flow into the main and auxiliary surge chambers, the difference between the moments is very small. In practice, the moment of maximum flow into the main and the auxiliary surge chambers can be considered as the most critical moment for the wave superposition of the main and auxiliary surge chambers.

(2) Change in areas of main and auxiliary surge chambers

The individual cross-sectional areas of the main and auxiliary surge chambers have been adjusted. However, according to the initial conditions, the sum of the cross-sectional areas of the main and auxiliary surge chambers remains constant. All the other parameters also remain constant. The cross-sectional area of the auxiliary surge chamber has been increased and decreased by 100 m^2 . With the adjustments, the cross-sectional area combinations of the main and auxiliary surge chambers are $200\text{ m}^2 + 300\text{ m}^2$, and $400\text{ m}^2 + 100\text{ m}^2$.

When the cross-sectional area combination between the main and auxiliary surge chambers is $400\text{ m}^2 + 100\text{ m}^2$, based on the initial conditions, it can be determined that the time when the flow into the auxiliary surge chamber reaches its peak is 212.4 s, and the time when the flow into the main surge chamber reaches its peak is 260 s. As these two times to peak are the same as those under the $200\text{ m}^2 + 300\text{ m}^2$ area combination, the times from the $200\text{ m}^2 + 300\text{ m}^2$ area combination have been used for the determination the superposition moments, except 212 s has been changed to 212.4s. Specifically, the times for determining the superposition moments are 196 s, 212.4 s, 228 s, 244 s, 260 s, 276 s, 292 s, 308 s and 324s. The results are shown in Fig.5.

When the cross-sectional area combination between the main and auxiliary surge chambers is $300\text{ m}^2 + 200\text{ m}^2$, based on the initial conditions, it can be determined that the time when the flow into the auxiliary surge chamber reaches its peak is 270.8 s, and the time when the flow into the main surge chamber reaches its peak is 218 s. There is a difference of 52.8 s between the two times to peak. Thus, the time interval for determining the superposition moment has been set at 13.2 s. Specifically, the times for determining the superposition moments are 204.8 s, 218 s, 231.2 s, 244.4 s, 257.6 s, 270.8 s, 284.0 s, 297.2s and 310.4s. The results are shown in Fig.6.

An examination of Figs.5 and 6 with Fig. 2 shows the following:

① When the cross-sectional area of the auxiliary surge chamber is smaller than that of the main surge chamber, the difference between the times of the highest surge in that main and auxiliary surge chambers and the time when the maximum flow into the auxiliary surge chamber is longer.

② When the cross-sectional areas of the main and auxiliary surge chambers are about the same, the timings of the highest surges in the main and auxiliary surge chambers are close to the moment of

maximum flow into the main and auxiliary surge chambers, albeit it is a little later. In practice, the moment of the maximum flow into the main and auxiliary surge chambers can be considered as the most critical moment for the wave superposition of the main and auxiliary surge chambers.

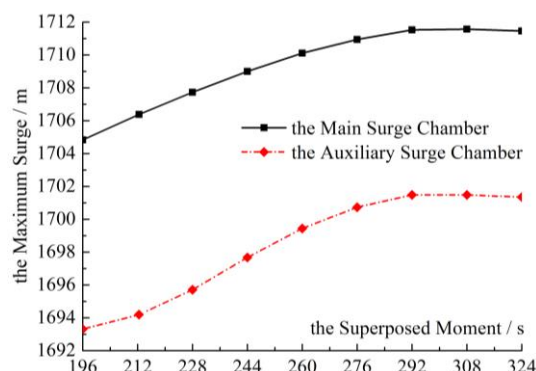


Fig. 5 Variation of levels during highest surges in main and auxiliary surge chambers for $400\text{ m}^2 + 100\text{ m}^2$

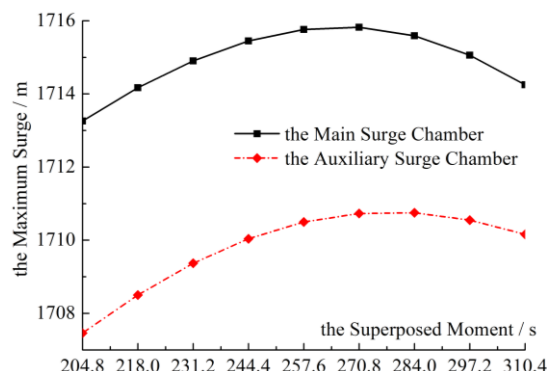


Fig. 6 Variation of levels during highest surges in main and auxiliary surge chambers for $200\text{ m}^2 + 300\text{ m}^2$ areas

3.2 Most critical wave superposition moment for lowest surge in surge chambers

3.2.1 For fixed areas of main and auxiliary surge chambers and fixed distance between them

In addition to the highest surge, it is necessary to consider the lowest surge in the main and auxiliary surge chambers because of the “evacuation” phenomenon in either surge chamber. This phenomenon can cause serious consequences in the stable operation of the units and the stability of the surge chamber structure. Table 2 shows the specifications of the Z2 condition, which is the most critical moment of the lowest surge in the surge chambers of the hydropower station with the upstream double-chamber surge tank. All the units are at the lowest power level. At that moment, there is full load in the surge chamber of the conventional power plants, and the outflow from the surge chamber is at its maximum. This is when the unit starts to increase to the full load conditions from idling.

Table 2 Specifications of Z2 condition

Calculation condition	Upstream water level / m	Downstream water level / m	Initial conditions	Superposed conditions	Condition to be determined
Z2	1640.0	1328.9	Dead water level in upstream, two units running, both units suddenly reject load	After ΔT_1 , load increases in one unit	Lowest surge in the upstream main and auxiliary surge chambers

Fig. 7 shows the relationship between the flow rate into and out from the main and auxiliary surge chambers with time under the initial conditions. The time when the outflow from the main surge chamber reaches its peak is 268.4 s, and the time when the outflow from the auxiliary surge chamber reaches its peak is 247.2 s. There is a difference of 21.2 s in the two times to peak. Thus, the time interval for determining the superposition moment has been set at 10.6 s. Specifically, the times for determining the superposition moments are 226 s, 236.6 s, 247.2 s, 257.8 s, 268.4 s, 279.0 s and 289.6 s. For the superposed conditions, Fig. 8 shows the levels of the lowest surges in the main and auxiliary surge chambers.

From Fig. 8, it can be seen that the superposition moments of the lowest surge in the main and auxiliary surge chambers are different. The most critical wave superposition moment of the main surge chamber is between the moment of maximum outflow from the main surge chamber and the moment of maximum outflow from the auxiliary surge chamber. The most critical superposition moment of the

auxiliary surge chamber is close to the moment of maximum outflow from the main surge chamber, and the difference $\Delta t = 10.6$ s.

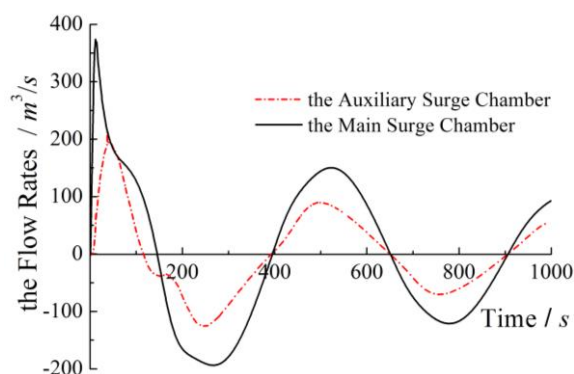


Fig. 7 Variation of flow rates into main and auxiliary surge chambers with time under initial conditions

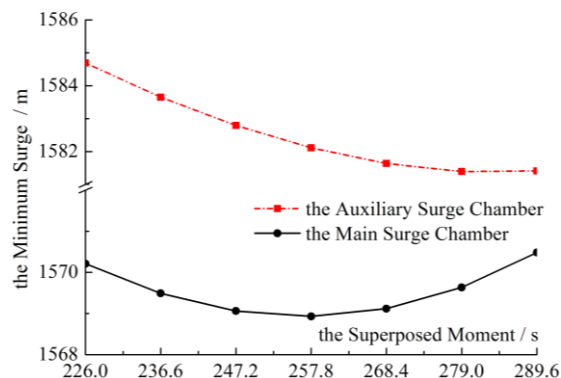


Fig.8 Variation of levels during lowest surges in main and auxiliary surge chambers with time

Assuming that the superposition moments are the same, the difference in the levels of lowest surges in the main and auxiliary surge chambers is 13.18 m, which is very large. During actual production, it is suggested to use this difference as the control value of tunnel gradient between the main and auxiliary surge chambers during the design of upstream double-chamber surge tank. If the slope gradient is greater than the control value, the lowest surge in the auxiliary surge chamber should be used as the control standard. If the slope gradient is smaller than the control value, the lowest surge in the main surge chamber should be used as the control standard.

3.2.2 For fixed areas of main and auxiliary surge chambers but change in distance between them

(1) Change in distance between main and auxiliary surge chambers

While keeping all the other parameters of hydropower station constant, the auxiliary surge chamber has moved to two new locations, upstream and downstream by 1,000 m. After the moves, the corresponding distances between the main and the auxiliary surge chambers are 3,000 m and 5,000 m, respectively.

When the distance between the main and the auxiliary surge chambers is 5,000 m, the time when the outflow from the auxiliary surge chamber reaches its peak is 251.2 s, and the time when the outflow from the main surge chamber reaches its peak is 235.2 s. There is a difference of 16 s between the two times to peak. Thus, the time interval for determining the superposition moment has been set at 16 s. Specifically, the times for determining the superposition moments are 203.2 s, 219.2 s, 235.2 s, 251.2 s, 267.2 s, 283.2s and 299.2s. The results are shown in Fig.9.

When the distance between the main and the auxiliary surge chambers is 3,000m, the time when the outflow from the auxiliary surge chamber reaches its peak is 244.4 s, and the time when the outflow from the main surge chamber reaches its peak is 268.8 s. There is a difference of 24.4 s between the two times to peak. Therefore, the time interval for determining the superposition moment has been set at 12.2 s. Specifically, the times for determining the superposition moments are 232.2 s, 244.4 s, 256.6 s, 268.8 s, 281.0 s, 293.2 s and 305.4 s. The results are shown in Fig. 10.

An examination of Figs. 9 and 10 with Fig. 8 shows the following:

The levels of the lowest surge in the main and auxiliary surge chambers occur near the moment of the maximum outflow from the main surge chamber. Although the moment of the lowest surge in the main and auxiliary surge chambers occurs slightly earlier after the auxiliary surge chamber has moved upstream, the difference between the moments of the lowest surge and the maximum outflow from the main and auxiliary surge chambers is very small. In practice, the moment of maximum outflow from the main surge chamber can be considered as the most critical moment for the wave superposition of the main and auxiliary surge chambers.

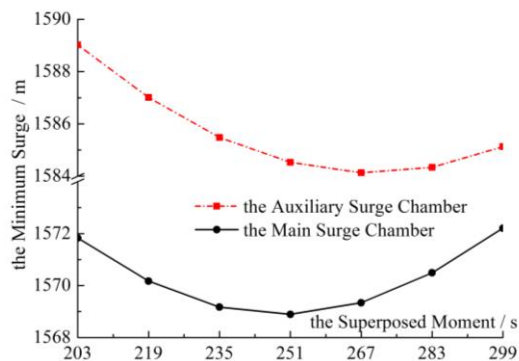


Fig.9 Variation of levels in main and auxiliary surge chambers during lowest surge for $L_2= 5000\text{m}$

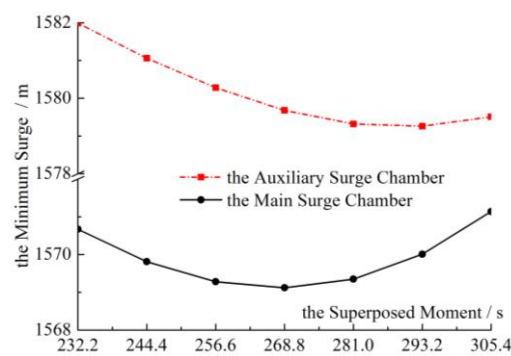


Fig.10 Variation of levels in main and auxiliary surge chambers during lowest surge for $L_2= 3000\text{m}$

(2) Change in areas of main and auxiliary surge chambers

The individual cross-sectional areas of the main and auxiliary surge chambers have been adjusted. However, according to the initial conditions, the sum of the cross-sectional areas of the main and auxiliary surge chambers remains constant. All the other parameters also remain constant. The cross-sectional area of the auxiliary surge chamber has been increased and decreased by 100 m^2 . With the adjustments, the cross-sectional area combinations of the main and auxiliary surge chambers are $200\text{ m}^2 + 300\text{ m}^2$, and $400\text{ m}^2 + 100\text{ m}^2$.

When the cross-sectional area combination between the main and auxiliary surge chambers is $400\text{ m}^2 + 100\text{ m}^2$, based on the initial conditions, it can be determined that the time when the outflow from the auxiliary surge chamber reaches its peak is 205.2 s , and the time when the outflow from the main surge chamber reaches its peak is 258.0 s . There is a difference of 53.8 s between the two times to peak. Thus, the time interval for determining the superposition moment has been set at 13.4 s . Specifically, the times for determining the superposition moments are 191.8 s , 205.2 s , 218.6 s , 232.1 s , 245.5 s , 258.0 s and 271.4 s , 284.8 s , 298.2 s , and 311.6 s . The results are shown in Fig. 11.

When the cross-sectional area combination between the main and auxiliary surge chambers is $300\text{ m}^2 + 200\text{ m}^2$, it can be determined that the time when the outflow from the auxiliary surge chamber reaches its peak is 255.6 s , and the time when the outflow from the main surge chamber reaches its peak is 232.4 s . There is a difference of 23.2 s between the two times to peak. Thus, the time interval for determining the most critical superposition moment has been set at 11.6 s . Specifically, the times for determining the superposition moments are 220.8 s , 232.4 s , 244.0 s , 255.6 s , 267.2 s , 278.8 s , 290.4 s and 302 s . The results are shown in Fig. 12.

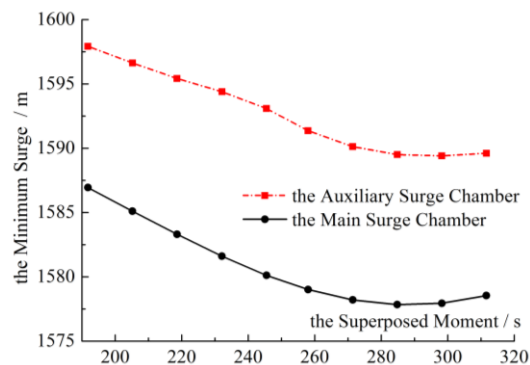


Fig.11 Variation of levels during lowest surges in main and auxiliary surge chambers for $400\text{ m}^2 + 100\text{ m}^2$ areas

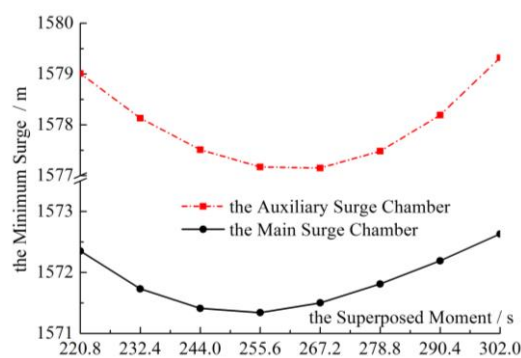


Fig. 12 Variation of levels during lowest surges in main and auxiliary surge chambers for $200\text{ m}^2 + 300\text{ m}^2$ areas

An examination of Figs. 11 and 12 with Fig.8 shows the following:

The levels of the lowest surge in the main and auxiliary surge chambers occur near the moment of maximum outflow from the main surge chamber. The most critical superposition moment is earlier as the ratio of the main surge chamber area and the auxiliary surge chamber area is smaller. Although the moment of the lowest surge in the main and auxiliary surge chambers occurs slightly earlier after the auxiliary surge chamber has moved upstream, the timings between the moment of the lowest surge and the maximum flow into the main and auxiliary surge chambers are very close. In practice, the moment of maximum outflow from the main surge chambers can be considered as the most critical moment for the wave superposition of the main and auxiliary surge chambers.

4 Conclusions

By means of the numerical simulation model developed for the existing hydropower station with a modified upstream double-chamber surge tank and a very long diversion tunnel, the conditions of the load increase and load decrease have been studied. The critical wave superposition moments have been determined for the main and auxiliary surge chambers used in this study. The conclusions are as follows:

(1) Irrespective whether it is the highest or the lowest surge, the superposition moment for the surge in the double-surge chamber system is different from the moment when the flow is maximum into or out from the main and auxiliary surge chambers.

(2) As the auxiliary surge chamber moves upstream, the change in distance between the main and the auxiliary surge chambers and the combination area ratio of the main and auxiliary surge chambers are different, the moment of the highest surge is close to the moment of maximum flow into the main and auxiliary surge chambers. The moment of the lowest surge is close to the moment of maximum outflow from the main and auxiliary surge chambers.

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Appendix

Table 3 Unit parameters for Jinping power plant

Parameters	Jinping Power plant	Parameters	Jinping Power plant
Rated power (MW)	610	Rated flow (m ³ /s)	228.6
Rated head (m)	288.0	Rated speed units (r/min)	228.6

Table 4 List of symbols

a	velocity of pressure wave	g	gravitational constant
A	cross-section area of pipeline	H	piezometric water head in the pipeline
D	inner diameter of the pipe	V	average flow velocity of pipeline section
f	Darcy-Weisbach coefficient of friction resistance	θ	angle between axis of pipeline and horizontal plane
x	position	L_2	Length of the pipeline between main and auxiliary surge chambers

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