Natural frequencies of rotating disk-like structures submerged viewed from the stationary frame

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Abstract. To understand the effect of rotation in the dynamic response of pump-turbine runners, simplified models such as disk-like structures can be used. In previous researches the natural frequencies and mode shapes of rotating disk-like structures submerged and confined have been analysed from the rotating frame. Nevertheless to measure these parameters experimentally from the rotating point of view can be a difficult task, since sensors have to withstand with large forces and dynamic loads. In this paper the dynamic response of rotating disk-like structures is analysed from the stationary frame. For this purpose an experimental test rig has been used. It consists on a disk confined that rotates inside a tank. The disk is excited with a PZT attached on it and the response is measured from both rotating frame (with miniature accelerometers) and from the stationary frame (with a Laser Doppler Vibrometer). In this way the natural frequencies and mode shapes of the rotating structure can be determined from the stationary reference frame. The transmission from the rotating to the stationary frame is compared for the case that the rotating structure rotates in a low density medium (air) and in a high density medium (water).

1. Introduction

Some kind of hydraulic runners (low specific speed Francis, pump-turbines and pumps) are large disk-like structures which are confined and not accessible when they are in operation. Furthermore, they are submerged. Therefore, to measure the natural frequencies of this part in real operating conditions is a difficult task. To perform measurements from the rotating frame, means to install sensors, that have to be submerged, withstand with large pressure fluctuations and with high centrifugal forces. At the same time, in order to not affect the normal operation of the machine, sensors have to be "small" and "light" as possible. Finally, if the acquisition system is located on the stationary frame, the signals have to be transmitted between both systems.

For this reason, it is easier to perform measurements from the stationary frame. In this case the installation of the sensors and the acquisition system is much easier. Nevertheless, this option has the disadvantage that the response from the rotating system has to be interpreted from the stationary system. I.e., generally the response of a rotating structure vibrating at one determined frequency on the rotating frame is different than the frequency that is "observed" from the stationary frame.

The phenomena of the transmission of natural frequencies from the rotating to the stationary frame has been analyzed in many cases for rotating disk-like structures, especially for rotating disk-like structures surrounded by a light density fluid such as air [1-3]. In this case, the relation between natural frequencies in the rotating frame and in the stationary frame is well known. This relation depends on the rotating speed of the disk-like structure and also on the mode shape.

Nevertheless, in case of low specific speed Francis runners and pump-turbine runners, the rotating structure is a disk-like structure totally submerged in a heavy fluid, such as water. In some studies, the effect on the natural frequencies, of water rotating with respect to the rotating structure has been analyzed [4-8]. In some of these studies the disk is supposed to be standing, with water rotating with respect to the disk. In some of them the disk itself is rotating, which creates a relative velocity of the water with respect to the rotating structure. This case is closer to real hydraulic runners which rotate submerged and confined. Nevertheless, in all these studies the response is analyzed from the disk reference frame and the study of how these natural frequencies are transmitted to the stationary frame is not considered.

In this paper the natural frequencies and mode shapes of a rotating disk-like structure confined and submerged are analyzed from the stationary frame. For this purpose the problem is studied from the analytical and experimental point of view. For the analytical model, the equations of the transmission are deduced from the motion characteristic of standing and traveling waves. These waves are inherent in the diametrical modes of rotating disks, which are the only type of modes considered in this study. For the experimental analysis, a rotating disk test rig is used. The disk is excited from the rotating reference frame with a piezoelectric patch. With respect to other studies, in this case the response of the disk has been measured simultaneously from the rotating and from the stationary reference frame.

2. Analytical equations

In this section, the shift between the vibration frequency of a rotating structure considering this vibration from the rotating frame and the frequencies that are observed from the stationary frame are analysed. Firstly, the problem is analysed for a rotating disk-like structure in air and then the same phenomena is analysed for a rotating disk-like structure in water.

In this paper, only the mode shapes with no nodal diameters are considered. These are typically called diametrical modes and are the most similar to hydraulic runners [9-11]. Furthermore this modes are the most prone to be excited by the Rotor Stator Interaction (RSI) [8, 12, 13].

2.1. Air

To analyze how the vibration frequency of a rotating structure is transmitted to the stationary frame, previously it has to be known how this structure is moving on the rotating frame.

Let's consider a rotating disk with angular coordinate θ, rotating at a rotating speed of Ω with respect to the stationary frame. The motion of this disk is observed from the stationary frame. This reference frame has the angular coordinate φ (cylindrical coordinates). For simplicity, the analysis of the diametrical modes is performed at the periphery of the disk and therefore the radial coordinate is considered constant and the same in both systems. Only the transverse vibration w (vibration in the axial direction) is considered (Figure 1).

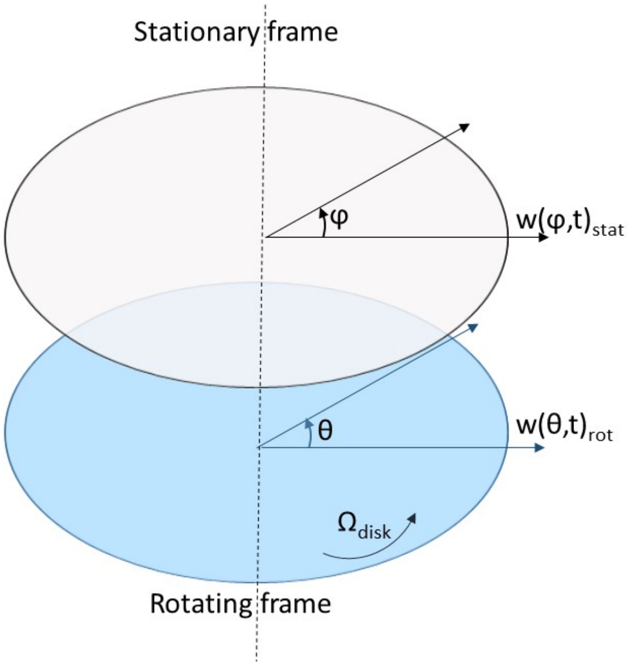

Figure 1: Rotating and stationary frame

According to Figure 1, the relation between coordinates is:

$$\varphi = \theta + \Omega \cdot t \tag{1}$$

A diametrical mode is defined by the number of nodal diameters (nodal circles are 0) [6]. It is considered, that for t=0 the maximum of the deformation is located at θ=0. Therefore, the deformation on the periphery of a diametrical mode with n nodal diameters vibrating with frequency ω can be expressed as:

$$w(\theta, t)_{rot} = A \cdot \cos(n\theta) \cdot \cos(\omega t) \tag{2}$$

It has to be noticed that in this case all the points are moving in phase or counterphase to each other (term ωt is the same for all the points) and with different amplitudes ($A \cdot \cos(n\theta)$). If this deformation is observed from the stationary frame then:

$$w(\varphi, t)_{stat} = A \cdot \cos(n(\varphi - \Omega t)) \cdot \cos(\omega t) \tag{3}$$

Eq.(3) can be also expressed as:

$$w(\varphi, t)_{stat} = \frac{A}{2} \cdot \cos((\omega + n\Omega)t) - n\varphi) + \frac{A}{2} \cos\big((\omega - n\Omega)t\big) + n\varphi) \tag{4}$$

Independently from the point considered on the stationary frame, which is defined by the coordinate φ, the frequencies observed from this reference frame are:

$$\omega_{1-stat} = \omega + n\Omega \; and \;\; \omega_{2-stat} = \omega - n\Omega \tag{5}$$

2.2. Water

The main difference, in case that the rotating disk is surrounded by a heavy fluid, is that for each diametrical mode, two natural frequencies exist [4, 7]. These are traveling waves that rotate in counter direction to each other. The lower natural frequency rotates in the same direction than the disk and the higher natural frequency rotates in the opposite direction [4, 6, 7]. The difference between both frequencies and the parameters that affect this difference can be found in these references.

The lower natural frequency which rotates in the same direction than the disk (positive n) can be expressed as:

$$w(\theta,t)_{rot} = A \cdot \cos(\omega t - n\theta) \tag{6}$$

It has to be noticed that in this case all the points move with the same amplitude and different phase (travelling wave). If the wave is observed from the stationary frame:

$$w(\varphi,t)_{stat} = A \cdot \cos(\omega t - n(\varphi - \Omega t)) \tag{7}$$

This practically means that the natural frequency observed from the stationary frame is:

$$\omega_{1-stat} = \omega + n\Omega \tag{8}$$

In this equation, the sign of n has to be considered. I.e. for the lower natural frequency on the rotating frame the sign + has to be considered and for the higher natural frequency the sign – has to be used.

3. Experimental set-up

3.1. Test set-up

To analyze the transmission of the natural frequencies from the rotating to the stationary frame a rotating disk test rig has been used. This test rig has been also used in previous researches [7] with sensors only on the rotating frame. In the present study the main focus is the transmission of the natural frequencies from the rotating to the stationary frame and therefore a Laser Doppler Vibrometer installed on the stationary frame that points directly the disk, will be used.

The disk is connected to a variable speed motor. It rotates up to *8Hz* and the rotating speed can be adjusted with a precision of *1/300Hz*. When the disk is rotating, it is excited with a Piezoelectric patch (PZT).

On the rotating frame, a miniature and submergible accelerometer (Dytran 3006-A) measures the vibration of the disk. It is checked that after installation of this sensor, the dynamic response of the disk does not change. The signal of this sensor is transmitted to the stationary frame through a slip ring system Michigan S10 which is installed on the tip of the shaft. On the stationary frame a Laser Doppler Vibrometer (LDV) points directly the rotating disk. A view of the test rig used is shown in Figure 2.

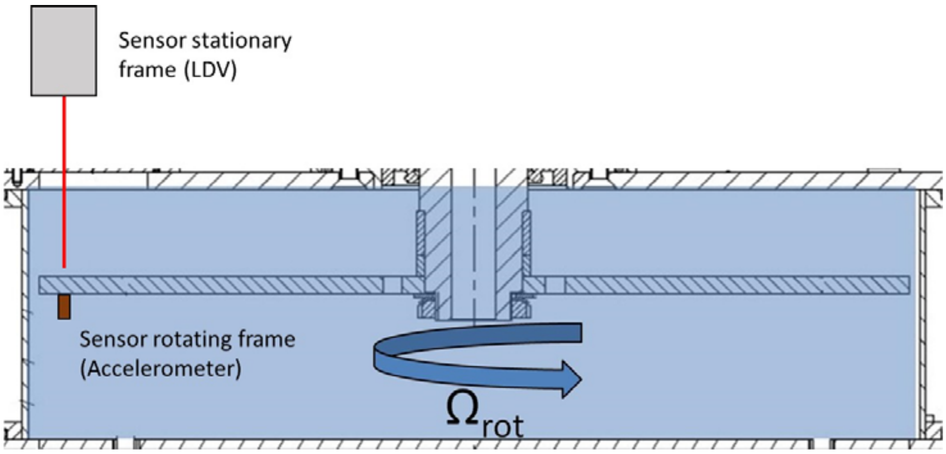

Figure 2: Test rig used

3.2. Tests performed

The excitation created is a slow sweep excitation [11]. The frequency of the excitation signal varies exciting the first diametrical modes n=±2,±3,±4. The response of the disk is measured simultaneously from the rotating and stationary frame.

4. Results

In this section the response measured with the sensor on the rotating frame (AR-0) and the sensor on the stationary frame (LASER) will be compared. Only the structural response of the diametrical mode n=±3 will be shown (in air and in water). For the other diametrical modes analysed the conclusions are equivalent.

4.1. Air

In this case, the disk is rotating surrounded by air. Figure 3 shows the excitation characteristic (a) in a time-frequency plot. Approximately at 590 Hz a resonance on the rotating frame occurs which is detected with the accelerometer on the rotating frame (b). This is the diametrical mode n=±3, which has been determined in previous works [6]. At the same time, the Laser detects two peaks on the stationary frame (c).

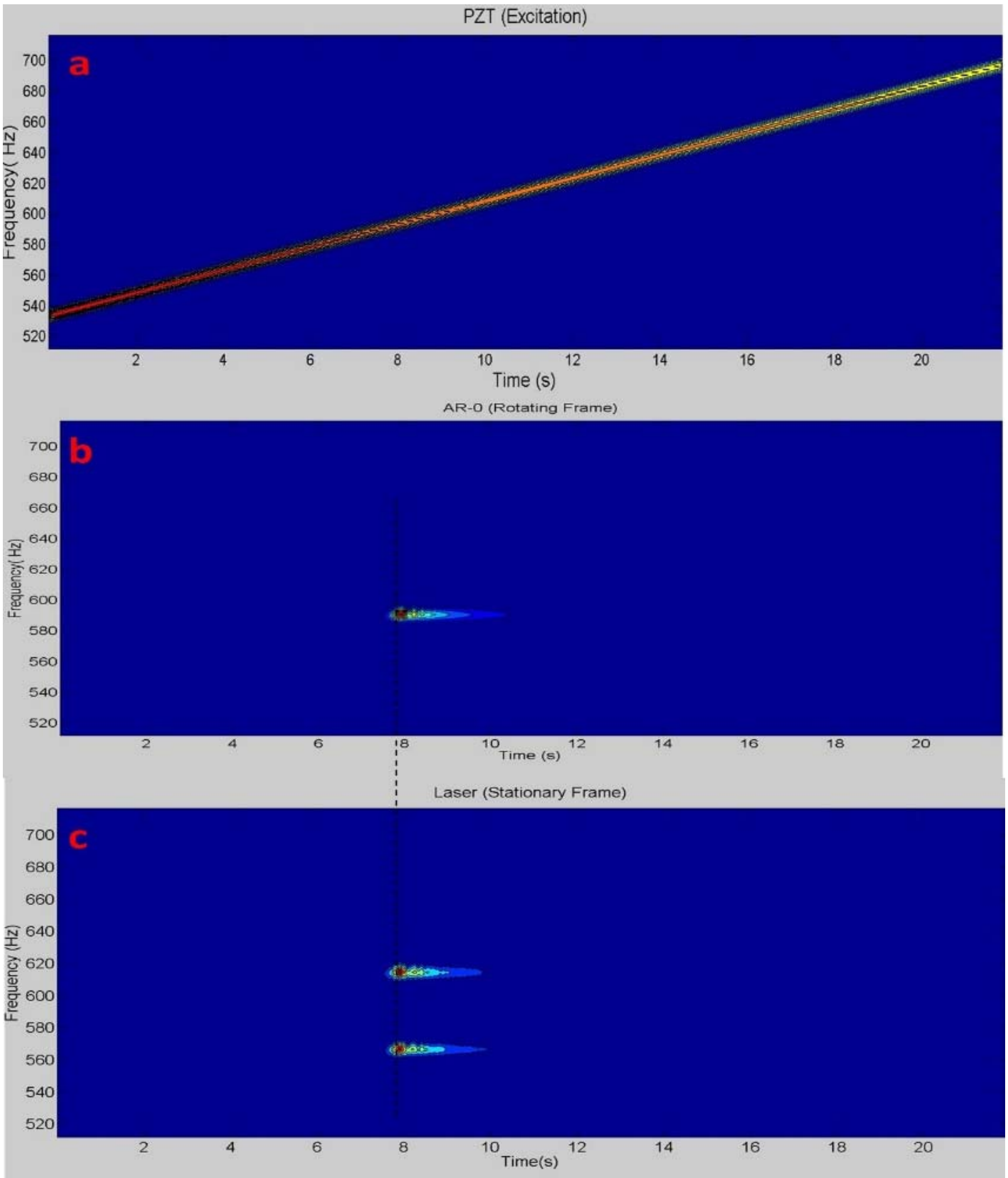

Figure 3: a) Excitation characteristic. b) Response from the rotating frame. c) Response from the stationary frame

The response of the disk is represented along the frequency axis at the time when the resonance occurs (Figure 4). It is shown, that in this case, the sensor on the stationary frame detects two peaks separated $\pm n\Omega$ from the peak detected on the rotating frame, as predicted by the analytical model (Eq.(5)).

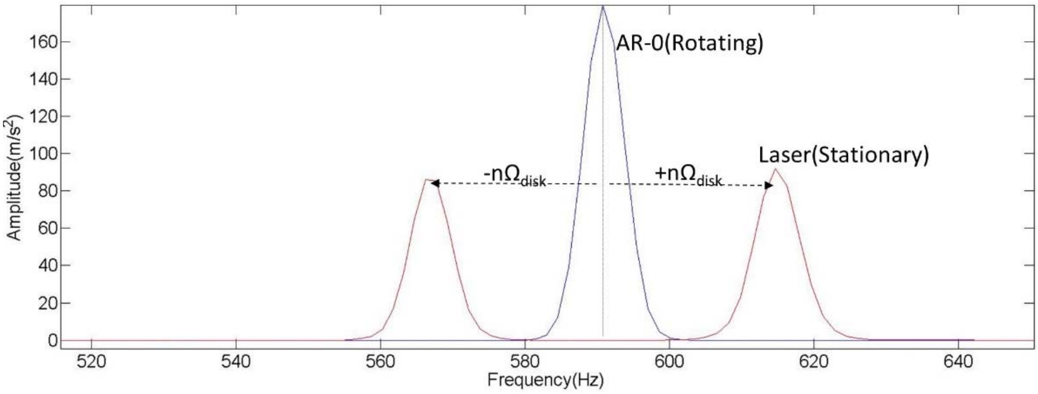

Figure 4: Resonance detected from the rotating and stationary frame

4.2. Water

The same experimentation is performed for the disk surrounded by water. The time-frequency representation of the excitation and response signals is shown in Figure 5.

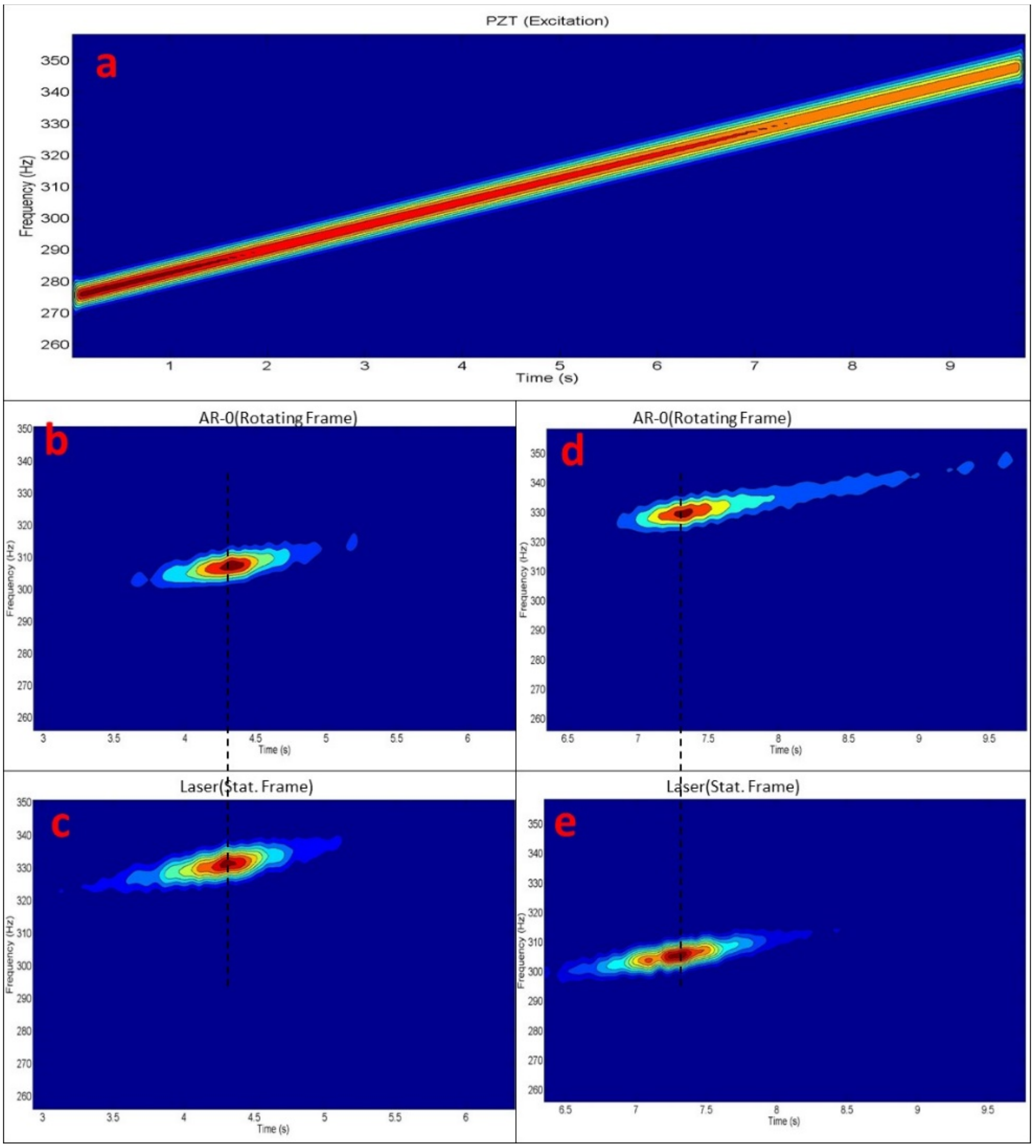

Figure 5: a) Excitation characteristic. b) Mode n=+3 from the rotating frame. c) Mode n=+3 from the stationary frame. d) Mode n=-3 from the rotating frame. e) Mode n=-3 from the stationary frame.

In this case as shown in previous works, the disk has two natural frequencies for each diametrical mode [7]. Firstly, the sweep excitation pass through the resonance of the mode n=+3 which rotates in the same direction than the disk [7] (Figure 5b). At the same time the response of the disk is detected with the sensor on the stationary frame (Figure 5c). Afterwards, the excitation pass through the resonance of the mode n=-3 which is higher in frequency and rotates in the opposite direction than the disk (Figure 5d). At the same time the response of the disk is detected with the sensor on the stationary frame (Figure 5e).

The response of both sensors is represented simultaneously for each of these two resonances along the frequency axis (Figure 6).

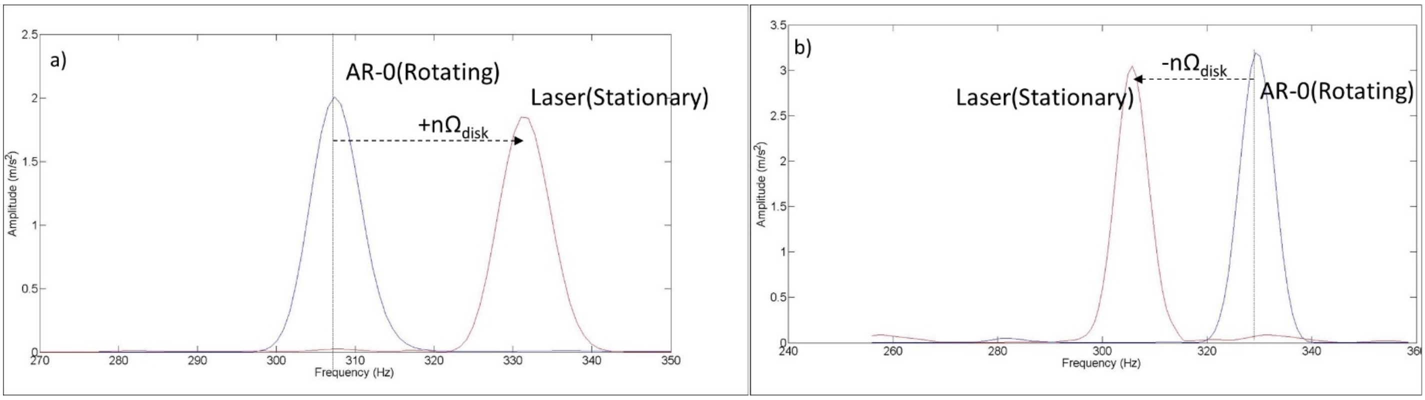

Figure 6: Resonance detected from the rotating and stationary frame. a) Mode n=+3 b) Mode n=-3

As seen in this case, the lower mode (n=+3), which rotates in the same direction than the disk is modulated to a higher value on the stationary frame (Figure 6a) according to the predicted frequency shift in Eq.(8). The higher mode on the rotating frame (n=-3) is shifted to a lower frequency on the stationary frame according to the same equation.

5. Conclusion

In this paper, the transmission of the natural frequencies of rotating disk-like structures from the rotating to the stationary frame is analyzed. The tests are performed in air and in water. Only the diametrical modes, which are comparable to the diametrical modes of some hydraulic runners (low specific speed Francis, centrifugal pump and pump-turbines), are discussed here. This problem is of special interest when it is desired to measure the dynamic response of these kind of hydraulic runners. Sensors installed directly on the runner have to be submerged and have to withstand with high dynamic loads and therefore it may be easier to measure the response of the rotating structure installing sensors only on the stationary reference frame.

The fact, that the diametrical modes in air are standing waves and in water they are pure traveling waves, has an important consequence on the transmission of the natural frequencies between the rotating and stationary frame. When the disk rotates in air, a resonance of a diametrical mode on the rotating frame is observed as two resonances on the stationary frame equally shifted in frequency with respect to the peak on the rotating frame.

When the disk rotates in water, a resonance of a diametrical mode on the rotating frame is observed as one resonance on the stationary frame shifted in frequency. The sign of the shift depends on the travelling wave direction.

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