

Experimental study of the influence of Thoma number and model testing head on pressure fluctuation in draft tube of a Francis turbine

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Abstract. In this paper, the influence of Thoma number and model testing head on pressure fluctuation of Francis turbines was studied through experimental method. Firstly, the influence of model testing head on pressure fluctuation in the draft tube was carried out by varying model testing head at 6 typical operating conditions including no load, deep part load, part load, optimum, rated and overload points. It is found that model testing head has little influence on amplitudes of the pressure fluctuation in the draft tube of Francis turbine within the test range, which represented the influence of similitude number such as Reynolds number, Froude number, Weber number and so on. Then, analysis of the influence of Thoma number on pressure fluctuation amplitudes in the draft tube as well as frequency was performed at the part load and rated load conditions. It shows that the Thoma number not only influences pressure fluctuation amplitude but also the distribution of the frequency components in the draft tube. Finally, comparison of pressure fluctuation with two different cavitation levels was carried out. It is reasonable that selection of guide vane centerline is as cavitation reference level in the pressure fluctuation tests for Francis turbines. Hence, when pressure fluctuation similarity is studied, apart from load condition, the influence of the difference of Thoma number and the selection of cavitation reference level should be considered.

Keywords. Francis turbine, Cavitation, Pressure fluctuation, Thoma number, Head

1. Introduction

In hydropower plants, model testing using a reduced-scale model is well-established practice regulated by international standards [1], which is always used to predict steady state characteristics of prototypes. Pressure fluctuation in the draft tube of Francis turbines is a main source of hydraulic instabilities which will induce vibration, noise and power swing. It is an unsteady flow phenomenon related to

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hydraulic design and cavitation patterns [2-4]. In the hydraulic machine industries, it is still a problem whether or not it can be transferable from the model to prototype. Fisher *et al.* [5-7] directly studied pressure fluctuations in the draft tube in model turbine and homogenous prototype turbine.. Stein *et al.* [8-10] investigated the phenomena through the method of the unsteady two-phase CFD simulation . In summary it mainly focused on experimental or numerical methods [11]. From the research, it shows some similarity for the pressure fluctuation in the draft tube between the model and prototype of a Francis turbine . However, the similarity is not very well at the overload and deep part conditions [12, 13]. Besides the difference of geometry between model and prototype, there is a big difference for the head between the model and prototype, which leads to different Reynolds number, Froude number , Weber number and so on. Furthermore, pressure fluctuation in the model testing was usually carried out at a constant sigma number which corresponding to extreme lowest tail water level in the prototype, but at actual situations tail water of the prototype varies in a range. So a difference of Thoma number between model testing and prototype operation may occur.

In this paper, the influence of model testing head as well as Thoma number on the pressure fluctuation in the draft tube was investigated in a low specific speed model turbine. It aims to find the similarity and difference of pressure fluctuation in the draft tube between model turbine and prototype. In addition, there is a controversy about the cavitation reference selection for pressure fluctuation test at early stage in the industry, which is whether the draft tube cone or runner centerline should be selected as the cavitation reference line. By comparison the model testing results, it shows more reasonable to select the runner centerline as the cavitation reference for pressure fluctuation test in Francis turbines.

2. Model testing setup and model turbine

The investigation of this paper was a low specific speed reduced-scale Francis turbine. The model turbine is shown in the figure 1 and main parameters of the turbine are listed in the table 1.

Table 1 Main parameters of model testing turbine

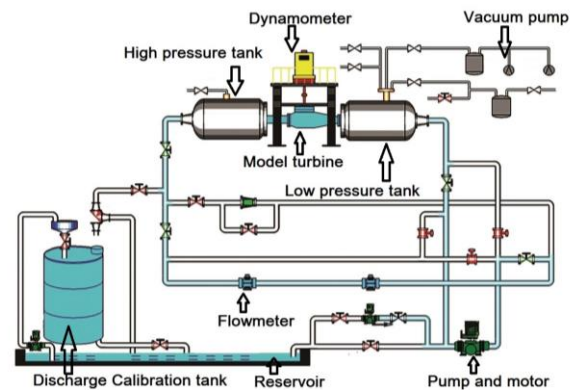
Name	Value
Runner blade number Z_B	17
Guide vane number Z_0	24
Stay vane number Z_s	24
Runner outlet diameter D_2 (m)	0.327
Unit speed at optimum point $n_{11,opt}$ (r/min)	48.5
Unit discharge optimum point $Q_{11,opt}$ (m ³ /s)	0.530
Specific speed n_s (m, kW)	107.4

In the table, the unit speed and unit discharge were defined as

$$n_{11} = n D / H^{0.5} \quad (1)$$

$$Q_{11} = Q / (D^2 H^{0.5}) \quad (2)$$

Where n , Q , and H are the model turbine speed, discharge and head, respectively. D is the runner outlet diameter. n_{11} and Q_{11} are dimensionless variables used to describe operating conditions in China and are calculated according to the test parameters. The specific speed is defined as $n_s = n P^{0.5} / H^{1.25}$. It is calculated at the optimum point. Model testing was carried out at the hydraulic machine test rig 3 in Harbin Electrical Machinery Company. It is an universal test rig, and its test parameters and methods meet the requirements of relevant IEC standards. Schematic diagram of model testing is shown in figure 2.

**Figure 1.** Model testing turbine**Figure 2.** Schematic diagram of model testing rig

For the pressure fluctuation investigation, this paper focused on the location of draft tube cone downstream and $0.3D_2$ from runner outlet, which are always used to characterize the model and prototype pressure fluctuation level for Francis turbines. The test condition was carried out at the optimum unit speed with relative discharge $Q_{11}/Q_{11,0}$ equal to 0.1, 0.5, 0.7, 1.0, 1.15 and 1.2 respectively, which are normally corresponding to no load, deep part load, part load, optimum, rated and overload operating conditions for prototype. Test heads include 20m, 25m and 30m with the variation of Thoma number from 0.3 to 0.025, a big interval at high sigma number and a small interval at low value. The detailed test conditions are summarized in the table 2. In the model testing rig, the test head was adjusted by the service pump speed and sigma value was changed by vacuum pump connected to the tail water tank.

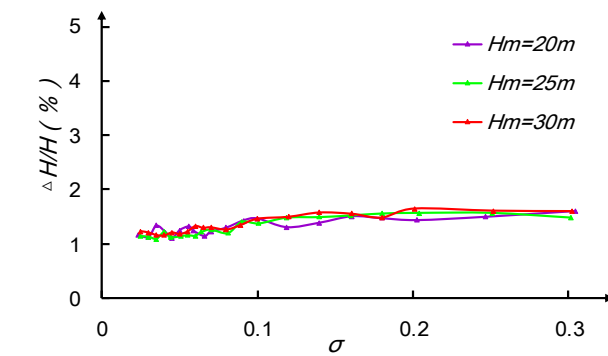
Table 2 Detained operation conditions of model turbine test investigation

$Q_{11}/Q_{11,opt}$	Test head $H(m)$	Thoma number σ
0.10	20m	0.30, 0.25, 0.20, 0.18, 0.16, 0.14, 0.12, 0.10, 0.09,
	25 m	0.08, 0.07, 0.065, 0.06, 0.055, 0.05, 0.045, 0.04,
	30m	0.035, 0.03, 0.025
0.50	20m	0.30, 0.25, 0.20, 0.18, 0.16, 0.14, 0.12, 0.10, 0.09,
	25 m	0.085, 0.08, 0.075, 0.07, 0.065, 0.06, 0.055, 0.05,
	30m	0.045, 0.04, 0.035, 0.03, 0.025
0.70	20m	0.30, 0.25, 0.20, 0.18, 0.16, 0.14, 0.12, 0.10, 0.09,
	25 m	0.085, 0.08, 0.075, 0.07, 0.065, 0.06, 0.055, 0.05,
	30m	0.045, 0.04, 0.035, 0.03, 0.025
1.00	20m	0.30, 0.25, 0.20, 0.18, 0.16, 0.14, 0.12, 0.10, 0.09,
	25 m	0.085, 0.08, 0.075, 0.07, 0.065, 0.06, 0.055, 0.05,
	30m	0.045, 0.04, 0.035, 0.03, 0.025
1.15	20m	0.30, 0.25, 0.20, 0.18, 0.16, 0.14, 0.12, 0.10, 0.09,
	25 m	0.085, 0.08, 0.075, 0.07, 0.065, 0.06, 0.055, 0.05,
	30m	0.045, 0.04, 0.035, 0.03, 0.025
1.20	20m	0.30, 0.25, 0.20, 0.18, 0.16, 0.14, 0.12, 0.10, 0.09,
	25 m	0.085, 0.08, 0.075, 0.07, 0.065, 0.06, 0.055, 0.05,
	30m	0.045, 0.04, 0.035, 0.03, 0.025

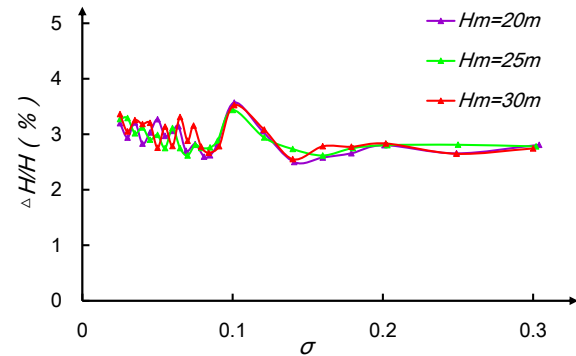
The amplitude of pressure fluctuation in the draft tube is expressed as the ratio of peak to peak value in 97% confidential level and the model testing head in the time domain, $\Delta H/H$. Thoma number σ is the ratio of net positive suction head NPSH and model testing head, $NPSH/H$, which is measured and calculated according to the relevant IEC standard.

3. Influence of model testing head on pressure fluctuation amplitude in draft tube

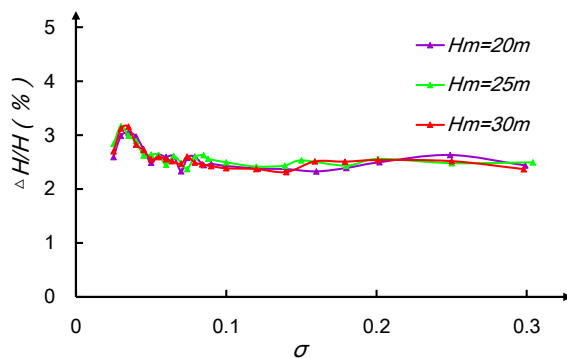
For the scale reduced model turbine test, besides geometry similarity, it mainly is insured hydrodynamic similarity and kinematic similarity between model and homogenous prototype. For other similitude numbers, it is hard to make sure them satisfactory simultaneously. Apart from the runner diameter, there is a big difference between model turbine test head and prototype operating head, which leads to the difference of Reynolds number, Froude number, Weber number and so on. Figure 3 presents pressure fluctuations in the draft tube at different heads with a series of identical Thoma numbers. Investigated points are typical operating conditions of Francis turbines including $Q_{11}/Q_{11,0}$ equal to 0.1, 0.5, 0.7, 1.0, 1.15 and 1.2 respectively which are corresponding to no load, deep part load, part load, optimum, rated and overload operating conditions of the prototype. In the figures, x-axis represents the Thoma number and y-axis is the relative pressure fluctuation amplitude $\Delta H/H$ in the time domain.



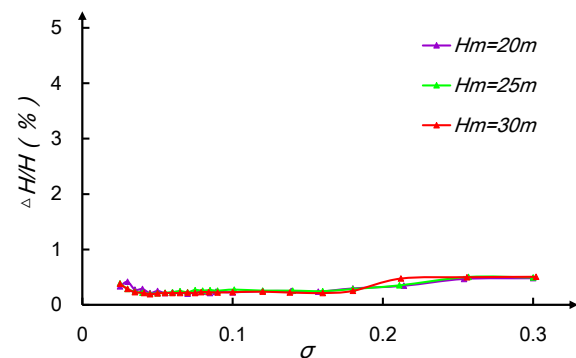
(a) $Q_{11}/Q_{11,0}=0.10$, No load



(b) $Q_{11}/Q_{11,0}=0.50$, Deep part load



(c) $Q_{11}/Q_{11,0}=0.70$, Part load



(d) $Q_{11}/Q_{11,0}=1.00$, Optimum condition

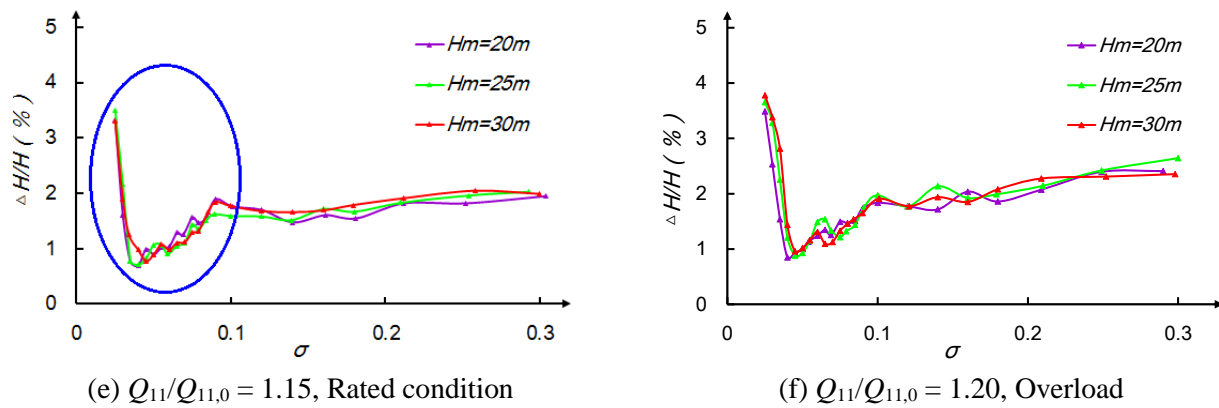


Figure 3 Comparison of pressure fluctuation in draft tube of the model turbine at different heads

From above figures, it is shown that the test head has little influence on pressure fluctuation amplitude in draft tube if the Thoma number is the same at the investigated conditions. Due to the Froude number, Reynolds number and Weber number are defined as

$$Re = Du/v \quad (3)$$

$$Fr = (E/g D)^{0.5} \quad (4)$$

$$We = (\rho L v^2/\sigma^*)^{0.5} \quad (5)$$

Where, E , u and v are directly related to the model turbine test head and the other parameters are constant for model and prototype turbine.

It is well known that the Reynolds number is directly related with the friction loss in hydraulic machines. When carrying out efficiency or output scale-up, the influence should be considered. The degree of atomization of droplets, which influences the windage losses and jet disturbance of Pelton turbine, is dependent on Weber number. In general, these two similitude numbers are not directly related to the pressure fluctuation of Francis turbines. From above test results, it is found that these two similitude numbers have little influence on pressure fluctuation prediction in the draft tube in the research range, which is coincide with previous study.

Generally, Froude number is related with two-phase flow or flow with a free surface in hydraulic machines. It seems that the influence of Froude similarity should be considered when carrying out pressure fluctuation test as it is related to vortex rope in the draft tube which is a two-phase phenomenon. For the investigated model turbine, the corresponding prototype head and runner diameter are 320m and 2.8m respectively. So the Froude number for model testing is from 7.8 to 9.6 and the corresponding prototype value is 10.7. It is close between model turbine and prototype. Combining the test results, it can also be concluded that the Froude number almost has little influence on pressure fluctuation in the draft tube.

4. Influence of Thoma number on pressure fluctuation in draft tube

For the model turbine test, pressure fluctuation test is always carried out at a constant equipment Thoma number. However, for prototype actual operating, the tail water level varies in a wide range according to discharge and environment conditions. The red region marked in figure 4 is the Thoma number range corresponding to the prototype tail water variation limit for the investigated model turbine. In actual situations, it always occurs that the model testing Thoma number is different from the one of the prototype. This paper presents the influence of Thoma number on pressure fluctuation amplitude in a Francis turbine in the time domain as well as the distribution of frequency components in the frequency domain at the part load and rated conditions with 30m model testing head. Figure 4 summarizes the results of the influence of Thoma number on pressure fluctuation in the draft tube amplitude of a Francis turbine at the two typical investigation conditions in the time domain. Figure 5 shows the corresponding influence analysis in the frequency domain, in which x -axis is relative frequency normalized by rotational frequency n and y -axis is the pressure fluctuation amplitude of

frequency component through FFT (Fast Fourier Transformation) analysis. Different colors represent different Thoma number. It is a decreasing direction of Thoma number from bottom to top.

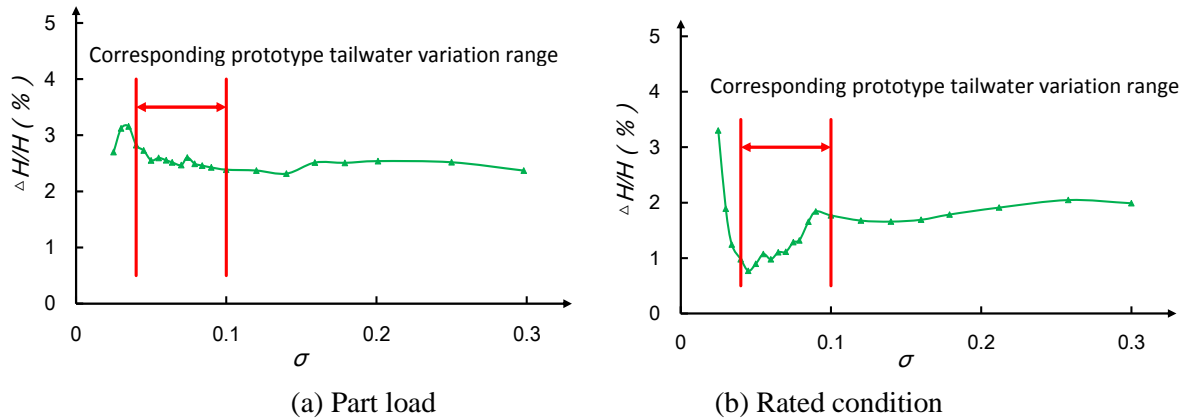


Figure 4 Influence of Thoma number on pressure fluctuation amplitude in draft tube of the Francis turbine

From figure 4, it could be found that Thoma number has a substantial influence on pressure fluctuation amplitude in draft tube of the Francis turbine for the two typical operating points in the time domain. In the following figure 5-a, the Thoma number has little influence on the domain frequency at the part load. But for rated condition, as the decreasing of Thoma number, the distribution of the frequency components moves towards to the low frequency region and the main frequencies become more clearly. In contrast, it is a frequency band in a high Thoma number. From the flow pattern image system in the draft tube cone, it was observed clearly that the processing vortex has just become more thickly as the Thoma number decreasing at the part load condition. But for rated condition, besides the size increasing as Thoma number decreasing, the anti-clockwise rotation vortex becomes axial resonant at the end. In a word, Thoma number not only changes the size of vortex but also the structure at some conditions. Hence, when study the pressure fluctuation similarity between model and prototype, it should consider the influence of Thoma number.

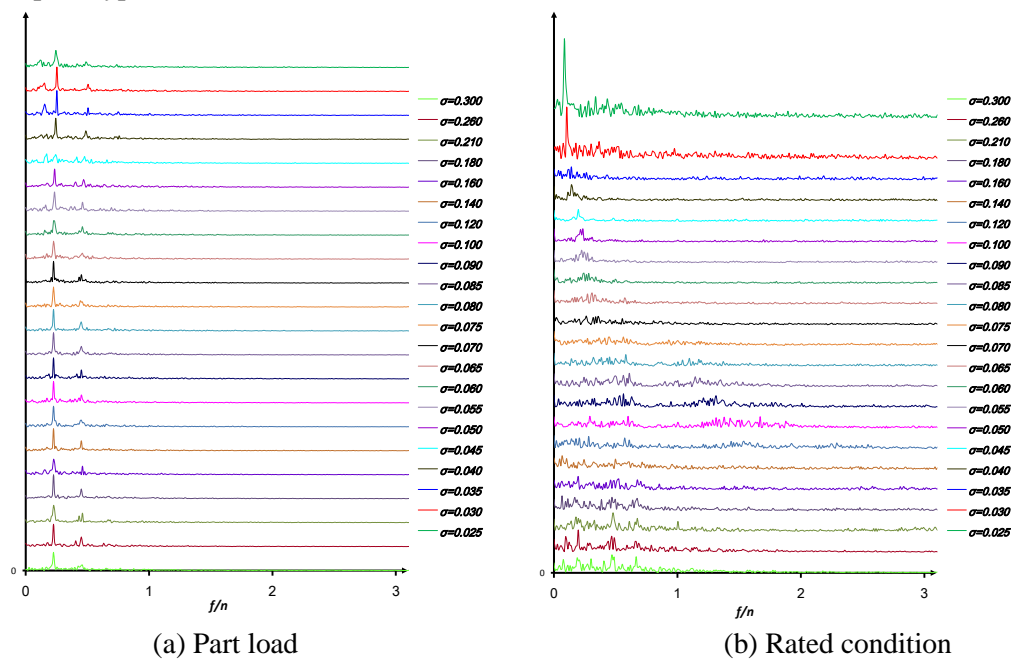


Figure 5 Influence of Thoma number on the distribution of frequency components in draft tube

5. Selection of cavitation level for pressure fluctuation test

There is a dispute in the industry whether the centerline of guide vane or the draft tube or even the elbow should be chosen as cavitation level in pressure fluctuation tests, especially for some early projects in China. This chapter presents the comparison between different methods at the rated points.

According to IEC 60193, guide vane centerline was selected as the cavitation reference and Thoma number was defined as

$$\sigma = NPSH/H = [(p_{abs2} - p_{va}) / \rho g + v_2^2 / 2g + z_2 - z_r] / H \quad (6)$$

If selection the draft tube as the cavitation reference for pressure fluctuation test, there is a height difference between guide vane centerline z_r and draft tube cone z_d . For this model turbine, the relation between z_r and z_d is

$$z_d = z_r - 0.5 \quad (7)$$

So the corresponding Thoma number could be calculated as follows.

$$\begin{aligned} \sigma' &= NPSH'/H = [(p_{abs2} - p_{va}) / \rho g + v_2^2 / 2g + z_2 - z_d] / H \\ &= [(p_{abs2} - p_{va}) / \rho g + v_2^2 / 2g + z_2 - z_r + 0.5] / H \\ &= \sigma + 0.5/H \end{aligned} \quad (8)$$

The following figures are results of comparison of two different cavitation reference selection. They are the partial enlarged curves of figure 3-e, in which the region was marked with blue color.

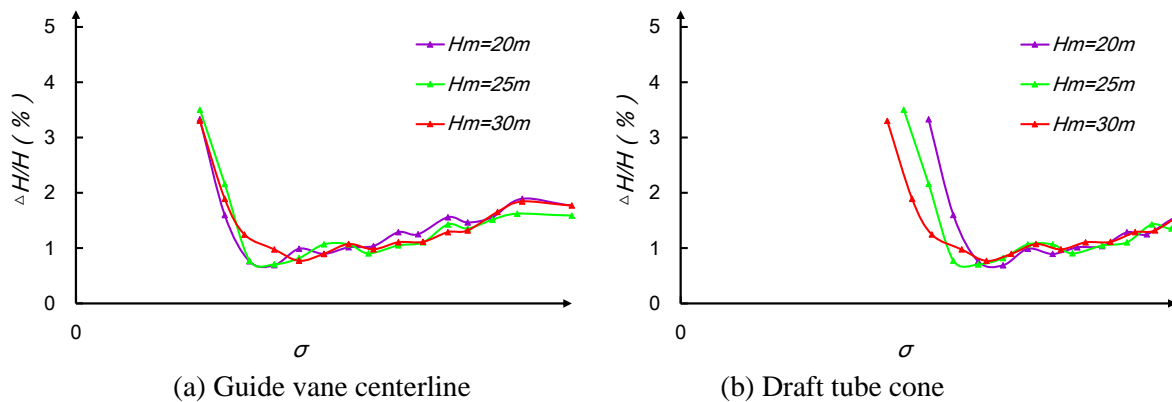


Figure 6 Comparison of pressure fluctuation amplitude in draft tube between different methods of cavitation reference selection

From above curves, if selection the runner centerline is as cavitation reference, the model testing head almost has no influence on pressure fluctuation amplitude in draft tube. But if the draft tube cone is selected, the curves move towards leftward with increasing of the model testing. The criteria about selection of cavitation should make sure the pressure fluctuation irrelevant with the test heads which represent different similitude number. So it is much more reasonably to choose the guide vane centerline as the cavitation reference in the pressure fluctuation tests.

6. Conclusions

- (1) In this paper, it presents the experimental investigation of the influence of model testing head as well as Thoma number on pressure fluctuation in draft tube of the Francis turbine.
- (2) By comparison of pressure fluctuation with the different heads under 6 typical operating conditions, it indicates that model testing head has little influence on pressure fluctuation in draft tube of the studied model turbine. When studying the similarity of pressure fluctuation in draft tube, it should neglect the influence of Reynold number, Froude number, Weber number and so on.
- (3) According to the experimental results, Thoma number has a big influence on pressure fluctuation amplitude as well as the distribution of frequency components the draft tube at some operating conditions.
- (4) It shows that selection of guide vane centerline as cavitation reference is much more reasonable in pressure fluctuation tests.

7. Acknowledgement

This work was supported by the General Administration of Quality Supervision (Project No.201510207).

8. References

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