

# A numerical study of gravity waves in the atmosphere: smooth and steep orography effects

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**Abstract.** Two types of nonhydrostatic meteorological models are applied to modeling of front wave propagation: a finite-difference model based on a terrain-following coordinate transformation and a finite-element model based on triangular elements. The former is used for a relatively smooth orography, and the latter for steep surfaces. The front surface is described in both models by an equation for advection of a scalar substance, which is solved with a third-order semi-Lagrangian procedure. Various meteorological effects, the influence of stratification and the introduction of an inversion layer above an orographic obstacle are investigated and discussed.

## 1. Introduction

Orographic retardation of gravity flows in the atmosphere is a subject of great practical and theoretical interest [1, 4, 5]. Atmospheric phenomena take place on a wide range of horizontal length scales. The scales of the flows of interest range from micro to macroscales. Flows ranging from several to thousands of kilometers are called mesoscale ones.

Atmospheric fronts over complex terrain are examples of mesoscale gravity flows. A terrain-following coordinate system is most often used to describe numerically a local topography of complicated shape. The domain becomes a rectangular one that can be easily discretized into a finite-difference grid. However, the transformed equations are more complicated than the original ones. In addition, it can be shown that the transformation function must satisfy some smoothness restrictions. In the present paper a finite-element model is used as an alternative to the above approach. Specifically, a 3D non-hydrostatic finite-difference meteorological model and a 2D finite-element model are used to simulate the effects of atmospheric front propagation over a hill.

The propagation of an atmospheric front over steep terrain is a phenomenon of great practical importance in meteorology [1-5]. This is also a subject matter of interest for numerical modelers, since atmospheric fronts can be considered as surfaces of discontinuity in the atmosphere. To simulate the deformation of these surfaces by spatial obstacles like mountains and valleys with good accuracy, efficient numerical methods are needed. Two distinct approaches can be recognized in the numerical simulation of front propagation. In one approach, the front to be calculated is considered as a gravity current driven by a cold air source [7-8]. In the other, the front surface is considered as a passive scalar, a tracer to distinguish between warm and cold air masses [10].



In the present paper, a preliminary investigation is carried out to simulate cold front propagation over a steep hill in two dimensions with a finite-element model and in three dimensions with a finite-difference model. Specifically, 3D non-hydrostatic meteorological model is used. The model is based on spatial discretizations that conserve some important quantities of the phenomena under study like momentum and scalars. Also, an efficient procedure is used to calculate the advection of scalars. A 2D finite-element model based on triangular elements is used to simulate the same phenomenon of cold front propagation over an idealized orographic obstacle.

## 2. Governing equations

For the simulations we use a small-scale non-hydrostatic model, which is essentially a form of the Navier-Stokes equations in the Boussinesq approximation. A version of the equations relevant for a compressible fluid is used here. Since sound waves in the atmosphere have small effects on gravity flows like fronts, the compressible version of the Navier-Stokes equations is taken here for pure technical reasons. The equations are discretized with a splitting method in time, and the compressible form of the equations is suitable for an efficient splitting. Sound waves are then eliminated by the use of an Asselin-type filter [4]. The governing equations may be written as follows:

$$\frac{dU}{dt} + \frac{\partial P}{\partial x} = f_1(V - V_g) - f_2W + R_u,$$

$$\frac{dV}{dt} + \frac{\partial P}{\partial y} = -f_1(U - U_g) + R_v,$$

$$\frac{dW}{dt} + \frac{\partial P}{\partial z} + \frac{gP}{C_s^2} = f_2U + g \frac{G^{1/2} \bar{\rho} \theta'}{\theta} + R_w$$

$$\frac{d\theta}{dt} = R_\theta,$$

$$\frac{ds}{dt} = R_s,$$

$$\frac{1}{C_s^2} \frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = \frac{\partial}{\partial t} \left( \frac{\bar{\rho} \theta'}{\theta} \right)$$

$$U = \bar{\rho}u, V = \bar{\rho}v, P = \bar{\rho}p', W = \bar{\rho}w$$

where  $p'$ ,  $\theta'$  are deviations from a reference state for pressure  $\bar{p}$  and potential temperature  $\bar{\theta}$ , respectively,  $s$  is the specific humidity,  $C_s$  is the sound wave speed,  $u_g$  and  $v_g$  are geostrophic wind components representing the synoptic part of the pressure,  $f_1$  and  $f_2$  are the Coriolis parameters,  $g$  is

the gravity constant.  $R_u$ ,  $R_v$ ,  $R_\omega$ ,  $R_\theta$ , and  $R_s$  are terms describing the subgrid-scale processes in the K-theory for turbulent diffusion in meteorology [4].

### 3. Advection of a substance

The problem examined in this study has very small physical diffusion, and the governing equations can be considered as almost inviscid and adiabatic. Special operators of space discretization are used to provide conservation of momentum and scalars in the finite-difference model (for a detailed description see [11-13]). Triangular elements are used in the finite-element model [12]. In this section we consider a suitable scheme for the advection of a scalar, e.g. temperature.

For fully hyperbolic problems explicit schemes have been widely used in the past and will probably continue to be used in the future. Traditionally, problems for which the governing equations are hyperbolic everywhere have been solved by the method of characteristics. Sometimes traditional methods of characteristics are slower than competitive finite difference methods. However, some finite difference schemes use a form of the governing equations such that knowledge of the characteristic locations can be exploited in an efficient way [2].

A flux-corrected transport scheme developed by Smolarkiewicz [4] was successfully applied to simulation of the propagation of an idealized atmospheric front by Schumann [10]. This is an example of a general technique of the predictor-corrector type in which large diffusion is introduced at the predictor stage and an almost equal amount of anti-diffusion is introduced at the corrector stage.

In this paper, the advection of a scalar like temperature is treated with an efficient semi-Lagrangian finite difference scheme [6]. Here the advection of a scalar is calculated in two steps:

1. Determination of the so-called departure point. This is the point from which the point under consideration is reached at the next time step.
2. Interpolation of values of the advected scalar from grid points to the departure point.

$$x_D = x - \int u dt$$

$$f(x, t + \Delta t) = f(x_D, t).$$

Here  $u$  is the velocity vector and  $\Delta t$  is the time step size.

At the first step, interpolation of the velocity vector is performed. The second step is devoted to interpolation of the advected scalar. The procedure of interpolation determines the accuracy of the method. For velocity, linear interpolation is used. At the second step, a third-order accuracy scheme will be used in this paper for reasons to be discussed below.

The total advection of temperature is split into horizontal and vertical components. For simplicity, we present here only a one-dimensional scheme. The scheme is designed as follows: an arbitrary function  $f$  is expanded into a Taylor series up to terms of the fourth order. The free coefficients of this expansion are determined through values of the function at grid points. Denote  $\lambda = (x_D - x_i) / \Delta x$ . Here  $\Delta x$  is the grid size. By solving the system of linear equations for the free coefficients, we finally obtain:

$$f(t + \Delta t) = f_i (1 - \lambda / 2 - \lambda^2 / 2 + \lambda^3 / 2)$$

$$+ f_{i+1} (\lambda + \lambda^2 / 2 - \lambda^3 / 2)$$

$$+ f_{i+2} (-\lambda / 6 - \lambda^2 / 6 + \lambda^3 / 6)$$

$$+ f_{i-1} (-\lambda / 3 + \lambda^2 / 2 - \lambda^3 / 6)$$

Numerical experiments have been performed to compare various semi-Lagrangian schemes. Some conclusions follow:

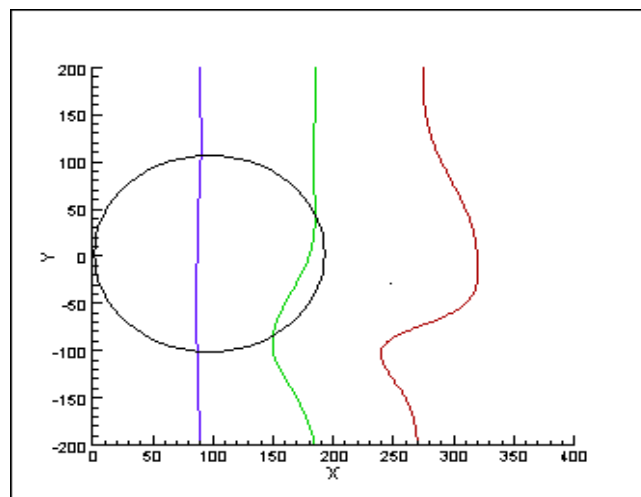
1. The first-order schemes have large numerical diffusion.
2. The second-order schemes are non-monotonic and have a small-scale wavelike structure.
3. In the third-order schemes, the above two effects are essentially reduced.
4. The schemes of order higher than 3 have a significant increase in the operation count but only a small increase in the solution quality.

The above third-order semi-Lagrangian scheme will be used as a reasonable compromise between cost and accuracy.

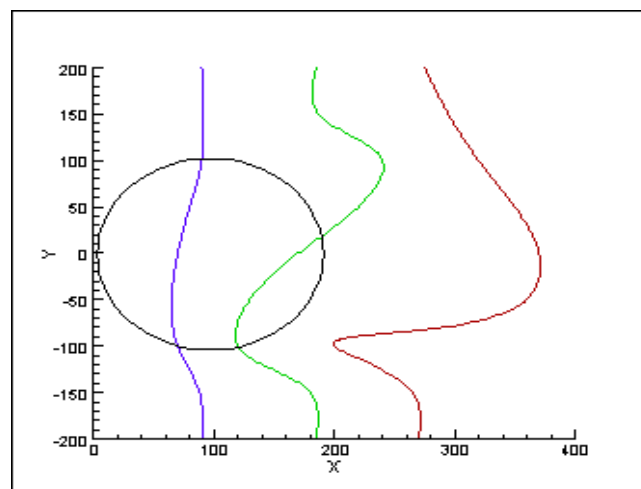
#### 4. Smooth surface orography simulation

In this Section, we apply the above finite-difference model to simulating the propagation of a cold atmospheric front over a hill-type obstacle in three dimensions. The input parameters are taken from [10].

The obstacle is a circular mountain with an axially symmetric sinusoidal height profile of 2 km and a surface diameter of 200 km. The computational domain is 600 x 400 x 6 km.



**Figure 1.** Cold front propagation over a hill. Neutral stratification.



**Figure 2.** Cold front propagation over a hill. Stable stratification.

Figure 1 shows the results of simulations on the propagation of a cold atmospheric front with an asymptotic height of 4 km and a temperature jump of 3K for neutral stratification. Surface isochrones at a sequence of non-dimensional time varying from 0 to 1 are shown. In Figure 2 the same is shown for a stably stratified atmosphere.

The results show acceleration of the front on the northern side and retardation near the mountain center. Examination of the flow also shows strong anticyclonic motion over the mountain, which causes significant front deformation. It follows from Figure 2 that stable stratification greatly increases the effects of acceleration and retardation. A qualitative agreement is observed between the above results and similar temperature patterns obtained in [6] with different schemes.

### 5. Steep surface orography simulation

In this section the effects of introducing an inversion layer into a stably stratified atmosphere over a steep orographic obstacle are studied with the finite-element model described above. The model parameters are taken as in papers [7-8]. In these papers, a cold front propagating over an axially-symmetric obstacle in the form of a bell-shaped hill is simulated with a finite-difference model. The height of the hill is 600 m. The calculation domain is 25x2 km. In contrast to [7-8], where the front is generated by a volume of cold air, in the present study the front is initially given in the form of a step-function of 400 m in height.

An inversion layer of 300 m in height is placed over the isolated hill described above. The atmosphere has the standard stable stratification of 3.5K/100 m, whereas the stratification of the inversion layer is 1K/100 m. The results of a series of calculations of cold front propagation over the hill and a plain are shown in Tables 1 and 2. The calculated values of windward and leeward speeds are given in the last columns.

It can be seen from these tables that the introduction of the inversion layer produces a significant decrease in the front speed both for the currents over the obstacle and those over flat orography. This is in agreement with the results of papers [7-8], where the front was generated by a volume of cold air. As in [7-8], the introduction of the elevated inversion layer causes an intensification of the vertical air motion and increases the mesoscale heat flux. This results in a more intensive entrainment.

**Table1.** Calculated windward and leeward speeds of cold front propagation over an orographic obstacle. Stable stratification.

obstacle height (m)	initial front height (m)	stratification ( K / 100 m)	windward speed (m/s)	leeward speed (m/s)
600	400	no inversion	4.9	2.7
600	400	inversion	4.4	2.2

**Table2.** Calculated windward and leeward speeds of cold front propagation over a plain. Stable stratification.

obstacle height (m)	initial front height (m)	stratification ( K/100 m)	windward speed (m/s)	leeward speed (m/s)
0	400	no inversion	5.1	5.1
0	400	inversion	4.6	4.6

## 6. Conclusions

The above results have been obtained with two types of models: a 3D non-hydrostatic finite-difference meteorological model and a 2D finite-element model. Both models are discretized versions of the Navier-Stokes equations in the Boussinesq approximation written in a compressible form.

The 3D model is based on spatial discretizations that conserve some important quantities of the phenomena under study, atmospheric gravity currents. Also, an efficient procedure is used to calculate the advection of scalars. Section 4 presented an application of the non-hydrostatic version of the small-scale meteorological model to simulating flows that occur due to the retardation of a cold front by a steep mountain in three dimensions. The simulations described above have been carried out to test the specific discretizations for the advection operators mentioned above (see[11-13]). The preliminary simulation results are in qualitative agreement with the existing theoretical insight, observations, and calculations performed by other authors [10].

The 2D finite-element model is based on triangular elements. Section 5 showed an application of the model to simulating cold front propagation over an idealized hill-type obstacle in a stratified atmosphere with an inversion layer over the isolated hill. The study was performed under stable stratification in and beyond the inversion layer. It has been shown that the introduction of the inversion later produces a significant decrease in the front speed both for the currents over the obstacle and those over flat orography.

Although the present studies are of limited scope, these preliminary results show that both the finite-difference and the finite-element model can be used for the simulation of the propagation of gravity flows like atmospheric fronts over steep orographic obstacles. These studies will be extended to more realistic situations in forthcoming papers.

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