

Calculating the Marine Gravity Anomaly of the South China Sea based on the Inverse Stokes Formula

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Abstract Marine gravity field information has a great significance for the resource, environment and military affairs. As a new way to get marine gravity data, the satellite altimetry technique makes up for what ship measuring means lack. The paper carries out the researches on how altimeter data applied for calculating marine gravity anomaly based on inverse Stokes formula. In the article, the editing of 14-track Jason-1 data over South China Sea for 7 years is for collinear processing and cross-point adjustment. The inverse Stokes formula and fast Fourier transform technique are applied to calculate marine gravity anomaly of the region (0°~23°N, 103°~120°E), and to draw gravity anomaly map. Compared with the gravity anomaly by ship observation, RMS is 12.6mGal, and single altimetry satellite has a good precision.

1.Introduction

Satellite altimetry data to calculate marine gravity anomaly can provide abundant marine gravity and seabed geophysical information. It is of great significance in building the high-precision global gravity model, in the study of climate, and in providing information for further exploitation and utilization of marine resources.

So far, the altimetry satellites launched by countries are mainly as follows: Geosat Satellite by the United States, the T/P, Jason-1 and Jason-2 Satellite by the United States and France jointly, and ERS-1, ERS-2, Envisat Satellite by the European Space Agency. Besides, China launched the first radar altimetry satellite HY-2 in 2011. After the successful launching of the altimetry satellites, how to



use satellite altimetry data to calculate marine gravity anomaly becomes a hot topic in the field physical geodesy. Hereby, experts both inside and outside of China carry out the related research. Inverse Stokes formula proceeds inverse operation based on the principle of Stokes formula, which resumes the geoid into a gravity anomaly. This method uses Fast Fourier Transform (FFT) algorithm to get grid-type gravity anomaly rapidly without the global integration. Houze Xu^[1] used inverse Stokes formula to calculate the 30'×30' marine gravity anomaly in China offshore waters by T/P, ERS satellite altimetry data, with the accuracy at 35mgal. Haiying Wang^[2] inverted 30'×30' gravity anomaly in the China Sea using the FFT inversion method of gravity anomaly, on the basis of inverse Stokes formula with T/P altimetry satellite data for 4 years and ERS 1/35 days of satellite altimetry data for 1.5 years, with the accuracy at 10 mgal. Xiaoyun Wan^[3] discussed the accuracy analysis of the remove-restore method in the calculation of inverse Stoke formula^[4-8].

2.Methods

2.1.Inverse Stokes formula

If g_0 represents the gravity at a point on the geoid, and γ_0 represents the normal gravity at this point on the average projection on ellipsoid surface, the difference between g_0 and γ_0 is marine gravity anomaly.

We can push to a spherical approximation of the geoid height by the fundamental formula of Geodesy and Bruns formula, Stokes integral formulas:

$$N = \frac{R}{4\pi\gamma} \iint \Delta g S(\psi) d\sigma \quad (1)$$

In the formula (1), N , R , γ , Δg , ψ respectively represent the geoid height, the earth average radius, the normal gravity value, the gravity anomaly value, and the spherical angle range between Point P and the flow point Q . $S(\psi)$ is the Stokes formula (2) and γ is the formula (3):

$$S(\psi) = \frac{1}{\sin \frac{\psi}{2}} - 6 \sin \frac{\psi}{2} + 1 - 5 \cos \psi - 3 \cos \psi \ln \left(\sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2} \right) \quad (2)$$

$$\gamma = 978.0327(1 + 0.0053024 \sin^2 B - 0.0000058 \sin^2 2B) \quad (3)$$

In the formula (3), B is the latitude.

The formula (2) can be rewritten as:

$$S(\psi) = \frac{1}{s} - 6s - 4 + 10s^2 - (3 - 6s^2) \ln(s + s^2) \quad (4)$$

In the formula (4), $s = \sin \frac{\psi}{2}$

Converting Stokes formula (4) into the formula with latitude and longitude as the independent variables:

$$\sin^2 \frac{\psi}{2} = \sin^2 \frac{1}{2} (\varphi_P - \varphi_Q) + \sin^2 \frac{1}{2} (\lambda_P - \lambda_Q) * [\cos^2 \varphi_M - \sin^2 \frac{1}{2} (\varphi_P - \varphi_Q)] \quad (5)$$

In the formula (5), λ_P and φ_P are respectively the latitude and longitude of calculated gravity anomaly points. λ_Q 、 φ_Q are respectively the latitude and longitude of Integral floating point, φ_M is the average latitude of Point P and Q , it can approximate by an average of integral area of approximate latitude, so the Stokes formula (4) turns into the following form:

$$S(\psi) = S(\varphi_P - \varphi_Q, \lambda_P - \lambda_Q) \quad (6)$$

Taking the formula (6) into formula (1), geoid high integral form can be received:

$$N(\varphi_P, \lambda_P) = \frac{R}{4\pi\gamma} \iint [\Delta g(\varphi_Q, \lambda_Q) * \cos \varphi_Q] S(\varphi_P - \varphi_Q, \lambda_P - \lambda_Q) d\varphi_Q d\lambda_Q \quad (7)$$

Be discrete and the convolution formula is as follows:

$$N(\varphi, \lambda) = \frac{R}{4\pi\gamma} [\Delta g \cos \varphi] * [S(\varphi, \lambda)] \quad (8)$$

In the formula (8), φ is the latitude difference between Point P and Point Q , so its spectrum calculation formula is as follows:

$$F[N(\varphi, \lambda)] = \frac{R}{4\pi\gamma} F[\Delta g \cos \varphi] * F[S(\varphi, \lambda)] \quad (9)$$

If the geoid high N is known, the formula (9) becomes:

$$\Delta g(\varphi, \lambda) = \frac{4\pi\gamma}{R \cos \varphi_M} F^{-1} \left\{ \frac{F[N(\varphi, \lambda)]}{F[S(\varphi, \lambda)]} \right\} \quad (10)$$

In the formula (10), F and F^{-1} represent respectively two-dimension FFT and inverse transform operator, φ_M is the average latitude of integral area.

2.2. Inner circle zone effect

Due to the Stokes formula will encounter integral formula singular problem in the process of the integral, it must be considered the influence of calculation data points in the square, so it need to calculate the inner loop effects:

$$N_{ib}(P) = \frac{\sqrt{ab}}{\gamma\sqrt{\pi}} \Delta \bar{g}_{ib} \quad (11)$$

In the formula (11), $N_{ib}(P)$ is the inner loop effects, $\Delta \bar{g}_{ib}$ is the average inner loop gravity anomaly, $a = R\Delta\varphi$, $b = R\Delta\lambda \cos \varphi$.

3.Experiments

3.1. Study Area and Data

The paper studies on the area of South China Sea ($0^\circ \sim 23^\circ \text{N}$, $103^\circ \sim 120^\circ \text{E}$), as shown in figure 1, using 7-year Jason-1 satellite altimetry data of 14-rail. Jason-1 satellite was launched successfully in December, 2001 E.S.T. Its observation precision in offshore waters is up to 2-3 cm, the orbit height at 1336 km, orbital tilt angle at 66° , orbital repetition period of 9.9 days, with operating range covering 90% of the global ocean.

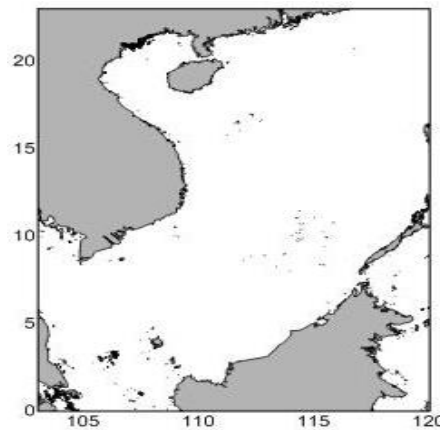


Figure 1. The South China Sea Waters

3.2. Technical Procedures

Data pre-processing includes data editing, collinear processing, cross-over point determining and adjustment, aimed at getting the processed data of the height of sea level.

The pre-processing sea surface height taking off the surface terrain model values is the geoid height. Using geoid height minus the corresponding EGM2008 geoid model values, and then making use of the residual geoid height calculate the residual gravity anomaly based on inverse Stokes formula, and combining with the corresponding EGM2008 gravity anomaly model values, the gravity anomaly will be restored. Finally, ship gravity anomaly is applied to evaluate the accuracy of calculated value. The technical procedures are shown in figure 2.

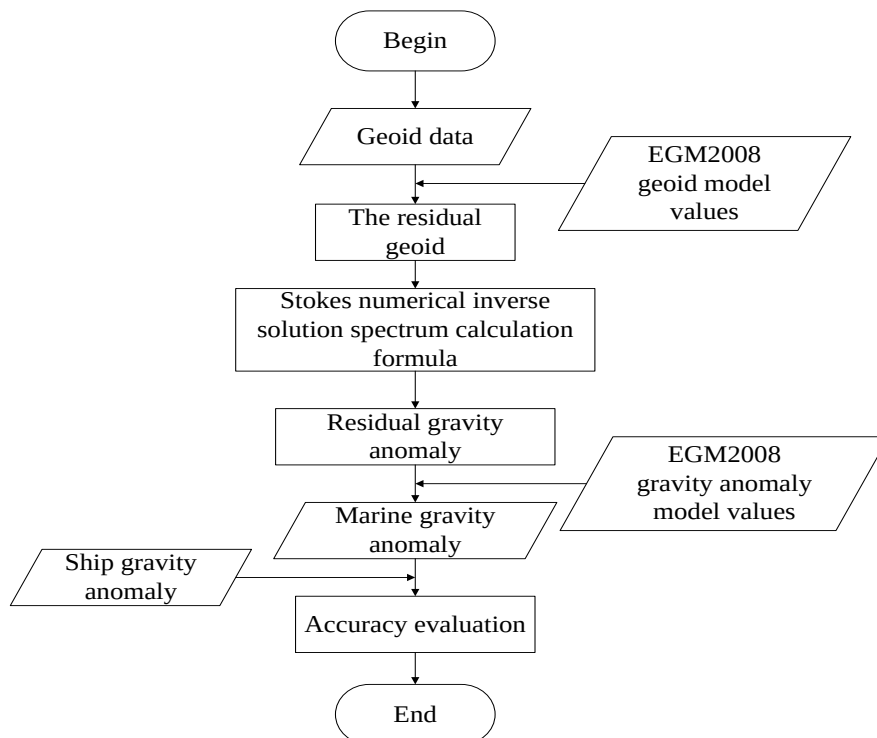


Figure 2. Technical procedures

3.3 Result and Analysis

Figure 3 is a 30'×30' gravity anomaly chart drawn by Surfer software. The gravity anomaly inversion measurement precision is validated through ship gravity data of the South China Sea area. The data is preceded with accuracy control, gross error elimination and grid processing, at the accuracy of 1-3mgal in figure 4. The red dot represents 30'×30'grid of the ship gravity anomaly with the measuring points well-distributed and typical.

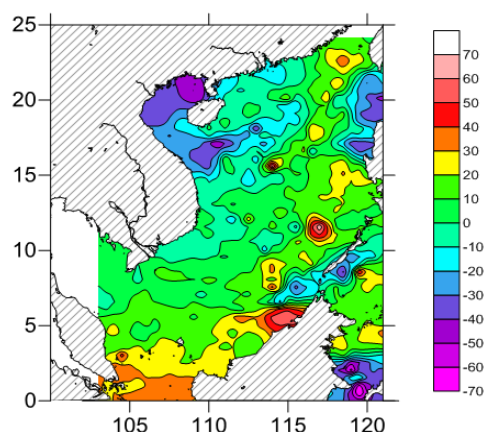


Figure 3. Gravity anomaly in the South China Sea

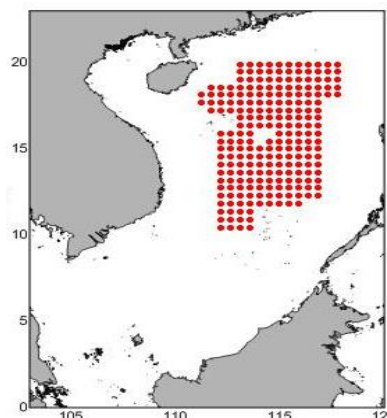


Figure 4. The ship gravity anomaly data distribution

As a result of the ship gravity not exactly corresponding to the calculation result, it needs interpolation fitting out their corresponding gravity anomaly values, by using measuring points around the ship within 4' of the calculated value to interpolate, with 54 efficient points. Using the difference between the calculated value and measured value of the maximum and minimum value, the average difference and RMS four precision indexes to evaluate the precision, the results are shown in table 1.

Table 1. The comparison of calculated value and measured value

	Points	maximum value	minimum value	The average difference	RMS
Calculated value - ship measured values	54	31.21	-21.79	-1.15	12.63

In this table, the mgal is the unit of maximum value, minimum value, the average difference and RMS. Table1 indicates that the calculation accuracy of gravity anomaly is 12.63mgal, higher than the results from Houze Xu. Besides, Haiying Wang used method of gravity anomaly along the path to invert the 2'×2' gravity anomaly of the South China Sea along, at the accuracy up to 10mgal, corresponding with the result in this article.

4. Conclusion

This paper uses inverse Stokes formula to carry out the study on gravity anomaly calculation, by using the 7 year's 14-rail Jason-1 satellite altimetry data to calculate gravity anomaly in the South China Sea, with the precision of 12.63mgal. The huge track interval based on the Jason-1 satellite altimetry data

only will have effect on the accuracy of gravity anomaly to some extent. By contract, the calculation accuracy of gravity anomaly from multi-altimetry satellite data will be higher.

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Acknowledgments

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