

# Optimization of thermal performance of Ranque Hilsch Vortex Tube: MADM techniques

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**Abstract.** Thermal performance of vortex tube is noticeably influenced by its geometrical and operational parameters. In this study effect of various geometrical (L/D ratio: 15, 16, 17, 18; exit valve angle: 30°, 45°, 60°, 75°, 90°; cold end orifice diameter: 5, 6 and 7mm, tube divergence angle: 0°, 2°, 3°, 4°) and operational parameters (inlet pressure: 2 to 6 bars) on the performance of vortex tube have been investigated experimentally. Multiple Attribute Decision Making (MADM) techniques are applied to determine the optimum combination of the vortex tube. Performance of vortex tube was analysed with optimum temperature difference on cold end, COP for cooling. The MADM (Multiple Attribute Decision Making) methods, namely WSM (Weighted Sum Method), WPM (Weighted Power Method), TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and AHP (Analytical Hierarchy Process) are applied. Experimental best performing combinations are obtained for Length to Diameter ratios 15, 16, 17 with exit valve angle as 45°, 75° and 90° at orifice diameter 5mm for inlet pressure of 5 and 6 bar pressure. Best COP, efficiency and cold end temperature difference are 0.245, 40.6% and 38.3K respectively for the combination of 15 L/D, 45° valve angle, 5mm orifice diameter and 2 bar pressure by MADM techniques.

## 1. Introduction

The vortex tube is a device that splits compressed air into two different temperature air streams viz. cold and hot. Vortex tube consists of hollow vortex cavity, exit valve, cold end orifice, hot end and entry nozzles. Vortex cavity can be cylindrical, divergent or convergent. Cold end consists of orifice and nozzles for supplying compressed air. On the hot end, exit valves are placed to vary the temperature and cold mass fraction. The advantages of vortex tube are constructional simplicity, less cost, easy repairs, smaller size, light weight, quick response; capability to reach a mark temperature immediately. Vortex tubes are majorly used for plastic blow molding, spot and panel cooling, vacuum forming, cleaning, drying, separating gas mixtures, DNA application and liquefying natural gas. [1]

Vortex tube invented by Ranque [2] and Hilsch [3], works on compressed air and provides two different temperature streams as outlet. When compressed air enters the vortex cavity through tangential

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nozzles, air expands on its entry and attains high velocity. This high velocity air starts moving towards the hot end inside the tube. This movement of air is called as swirl flow. As air reaches hot end the valve on hot end restricts the air flow. The pressure on valve end increases slightly and the reversal of air flow takes place. The air now flows to cold end of the tube through tube center forming two streams of air. The air stream at periphery is compressed by this central layer and there is energy transfer from central layer to peripheral layer. This way the central air gets cooled and the peripheral layer gets heated and two streams are obtained at different temperatures.

Saidi and Valipour [4] conducted experiments with vortex tubes with different  $L/d$  ratios and tube geometries to investigate the effect of geometry on the operational characteristics of vortex tube, the major investigation was; optimum value of  $L/d$  is in the range  $20 \leq L/d \leq 55.5$ . C. Gao [5] obtained cold air temperature difference of  $28^\circ\text{C}$  at  $L/d$  ratio equal to 64.7, when the tested tubes had  $L/d$  ratio equal to 8, 32.7 and 64.7.

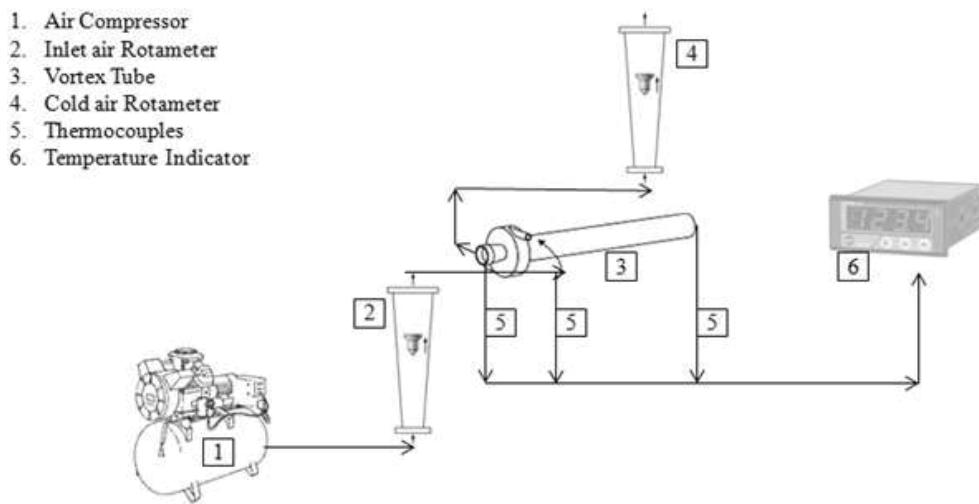
Pourmahmoud and Bramo [6] deduced that the best cold air temperature difference of 43.96 K is obtained when the length to diameter ratio was 9.3 among the experiments on six different tubes of 8, 9.3, 10.5, 20.2, 30.7, and 35  $L/d$  ratio. Aydin et al. [7] Experimented on four different tubes with  $L/d$  10, 20, 30 and 40. It was reported that tube with  $L/d$  30 gives maximum cold air temperature difference of 45.9K. The major intention of study by Kirmaci [8] was to investigate the effect of the nozzle number. It was investigated that using 2 nozzles produce best result. Similarly Polat and Kirmaci [9] conducted experiments with 5 different nozzle no. (2, 3, 4, 5 and 6) and working fluid used was Air, O<sub>2</sub>, N<sub>2</sub> and Ar to conclude that maximum temperature difference is obtained for 2 nozzles with air. Promvong and Eiamsa-ard [10] experimented to investigate effect of orifice diameter. The used orifice diameter was in the range of 0.6d to 0.9d and it was concluded that 0.5d orifice yields the highest temperature reduction. Prabhakaran and Vaidyanathan [11] experimented for orifice diameter and concluded that minimum cold air temperature is obtained for 0.5d. Nimbalkar and Muller [12] investigated experimentally and numerically that cold orifice diameter of 0.5d is responsible for maximum energy separation. For investigation of nozzle diameters Prabhakaran and Vaidyanathan [13] performed experiments, but they couldn't establish relationship between nozzle diameter and tube diameter. Markal et al. [14] Tested the effect of the exit valve angles and concluded that effect of valve angle is generally negligible. Experiments performed by Devade and Pise [15] state that 450 and 600 valve angle produce best cooling and heating respectively. While on the other hand Dincer et al. [16] Reported best performance at angle of 300 and 600. Experiments by Chang et al. [17] were based on the influence of divergence angle on the performance of vortex tube. The experimental result show that performance of vortex tube is enhanced by using a divergent tube and  $4^\circ$  divergent angle yields the highest temperature reduction. CFD analysis was done to consider  $L/D$  ratio as design criteria by Pour Mahmoud et.al [18] with focus on stagnation point and length of tube

Literature review states that performance of vortex tube varies considerably with changes in  $L/d$ ,  $d_o$ ,  $N$ ,  $\phi$ ,  $d_n$  and working fluid. The range of selected geometry as used in experiments is far wide. For ex.  $L/d$  ratio is taken then the ranges used are from 0.6 to hundreds. Close end study has not been conducted; this prompted to make use of  $L/d$  ratio in close range of 15 to 20. The literature also says that valve angle has very limited effect on performance to validate this valve angles are chosen in steps of 150 from 300 to 900. As mentioned in Nimbalkar et al. [12] Optimum performance of tube is at  $d/d_o$  equal to 0.5 to validate this  $d_o$  is selected as 5, 6, and 7 mm where 6 mm represents  $d_o/d$  equal to 0.5. The divergence angle has also been set to 00 and 40. The purpose is to combine most of the parameters from the literature for getting true optimum performance. Vortex tube has the potential to replace conventional refrigeration system and get commercialized to be implemented in number of applications. Optimization studies have also been conducted by many researchers, Ersoyogule et.al [19] used Rule-Based Mamdani-Type Fuzzy modelling for optimization. Suresh Kumar et.al [20] used taguchi approach for optimizing the performance of vortex

tube. ANN is used by Uluer et.al [21] for modelling performance of counter flow vortex tube. Graphical and experimental optimization by Devade and Pise [22] on vortex tube for geometry and operational parameters. Pinar et.al [23] applied taguchi method for assessment of performance of vortex tube. In the present study MADM methods are used for optimization of geometrical combinations.

## 2. Experimental method

The experimental study of selected parameters is done using the setup shown in Figure 1. The experimental setup consists of a compressor, an air reservoir, pressure regulator, Rota-meters for measuring the flow rates of inlet air and cold air, pneumatic pipes, connectors, vortex tube, digital temperature indicator and thermocouples. The details of measuring instruments are as given in table 1. [1]



**Figure 1.** Experimental setup.

**Table 1** Instrument details [1].

Instrument	Range	Accuracy
Rota meters	0 to 500LPM	$\pm 1$ LPM
Pressure Regulator	0 to 10 bar	$\pm 0.1$ bar
RTD	-50 to 150°C	$\pm 0.1$ °C

### 2.1. Data reduction

Cold Mass Fraction (CMF) is the ratio of mass of cold to total mass of air entering the vortex tube.

$$CMF = \frac{m_c}{m_i} \quad (1)$$

Coefficient of Performance (COP) is the ratio of cooling effect produced to the energy input required by the compressor.

$$COP_{act} = \frac{R_E}{W_{comp}} \quad (2)$$

Refrigeration effect/ cooling effect of the vortex tube is the total enthalpy drop in air emerging from cold end.

$$R_E = m_c c_p (t_i - t_c) \quad (3)$$

Work required by compressor is the isothermal work being minimum assuming the compression as isothermal is given by

$$W_{comp} = m_i R t_i \ln \frac{p_d}{p_i} \quad (4)$$

The cold end temperature drop is the difference in temperature at inlet and temperature of air at cold end.

$$\Delta t_c = t_i - t_c \quad (5)$$

The temperature drop without vortex effect, because of pure expansion is static temperature which is given by

$$\Delta t'_c = t_i \left[ 1 - \left( \frac{p_d}{p_i} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad (6)$$

The relation between the static temperature drop and actual temperature drop is given by

$$\Delta t_{rel} = \frac{\Delta t_c}{\Delta t'_c} \quad (7)$$

The adiabatic efficiency is proportional to product of CMF and relative temperature drop as

$$\eta_{ad} = CMF(\Delta t_{rel}) \quad (8)$$

Efficiency of compression is the only input parameter and is calculated as

$$\eta_{comp} = \frac{\ln \frac{p_d}{p_i}}{\frac{\gamma}{\gamma-1} \left[ \left( \frac{p_d}{p_i} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} \quad (9)$$

Theoretical COP can be calculated by

$$COP_{th} = \eta_{ad} \eta_{comp} \left( \frac{p_d}{p_i} \right)^{\frac{(\gamma-1)}{\gamma}} \quad (10)$$

The tube is experimented to record the temperatures and mass flow rates for various combinations as listed in Table 2. Experimental parameters.

**Table 2.** Experimental parameters.

Parameters	
Tube L/D ratio	15,16,17,18
Tube Divergence Angle	0 and 4°
Number of Nozzles	2 No's
Orifice Diameters	5,6,7 mm
Valve Angles	30°, 45°, 60°, 75° and 90°
Pressure at Inlet	2 to 6 bars in step of 1 bar

Experiments are performed systematically with the parameters used is given in table 3. The sets are formed for illustration following the sequence of  $L/d - \phi - d_o - \theta - p_i$ . i.e. 15-0-5-30-2 to 6

**Table 3.** Experiment sets of vortex tube.

L/D-Div. Angle-Orifice Diameter-Valve Angle-Inlet Pressure
15-0-5-30-2 to 6; 15-0-5-45-2 to 6; 15-0-5-60-2 to 6; 15-0-5-75-2 to 6; 15-0-5-90-2 to 6.
15-0-6-30-2 to 6; 15-0-6-45-2 to 6; 15-0-6-60-2 to 6; 15-0-6-75-2 to 6; 15-0-6-90-2 to 6.
15-0-7-30-2 to 6; 15-0-7-45-2 to 6; 15-0-7-60-2 to 6; 15-0-7-75-2 to 6; 15-0-7-90-2 to 6.
15-4-5-30-2 to 6; 15-4-5-45-2 to 6; 15-4-5-60-2 to 6; 15-4-5-75-2 to 6; 15-4-5-90-2 to 6..... For all combinations.

## 2.2. Observations

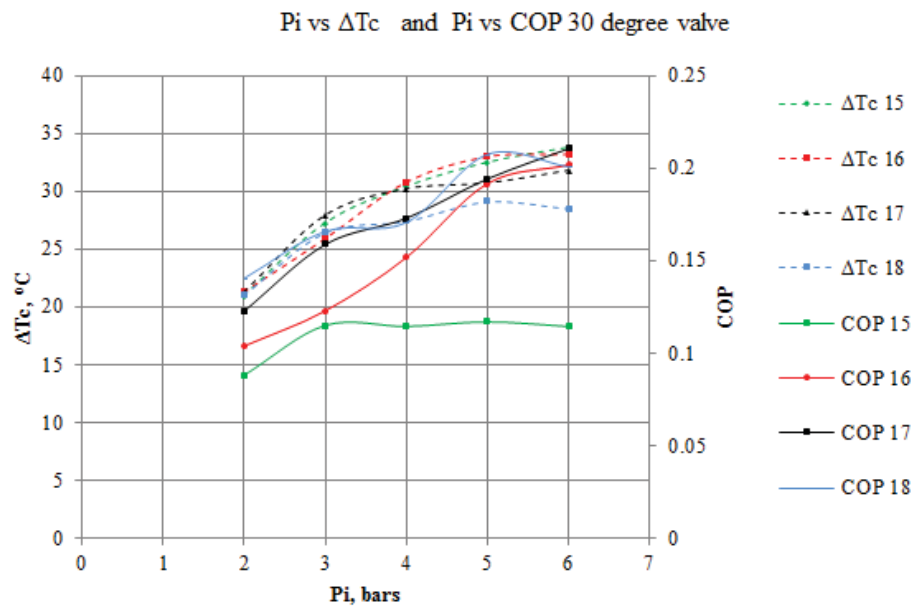
Using the coded system given earlier the observations for temperature, pressure flow meter are recorded for all tubes with various  $L/d$  ratios and divergence angle  $\phi$  for various  $d_o$  and  $\theta$  observations are made for all pressures  $p_i$ . The data is analyzed using the relations provided earlier for  $CMF$ ,  $COP$ ,  $R_E$ ,  $W_{comp}$ . The analytical results are discussed below.

## 3. Results and discussion

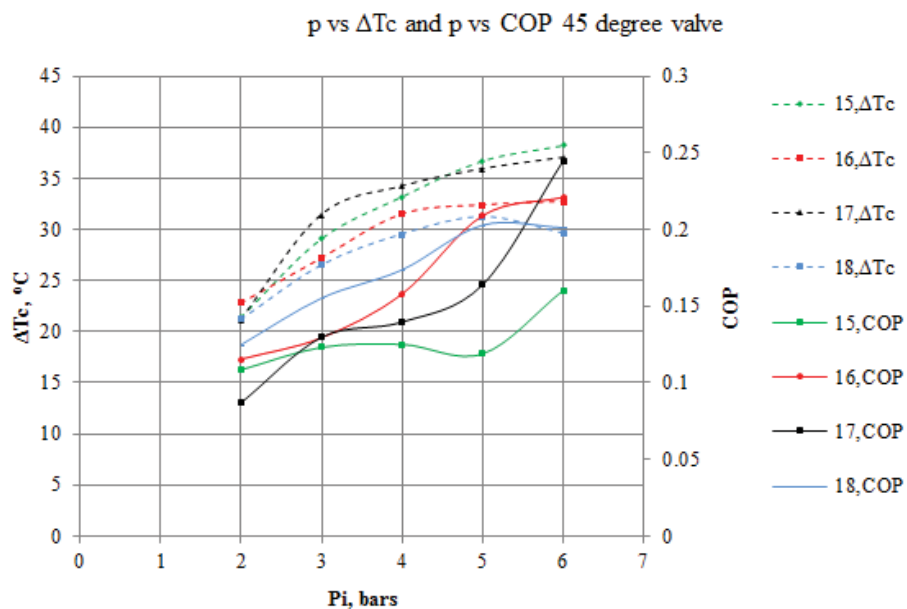
The experiment is performed with the empirical relations provided in literature with some ranges of  $L/d$  to decide the optimum geometrical combinations under provided conditions. The results are presented in terms of effect of  $L/d$ ,  $d_o$  and effect of  $\theta$  on  $COP$  and  $CMF$ .

### 3.1. Effect of $L/D$ at specific $\theta$ on $COP$ and $\Delta t_c$ .

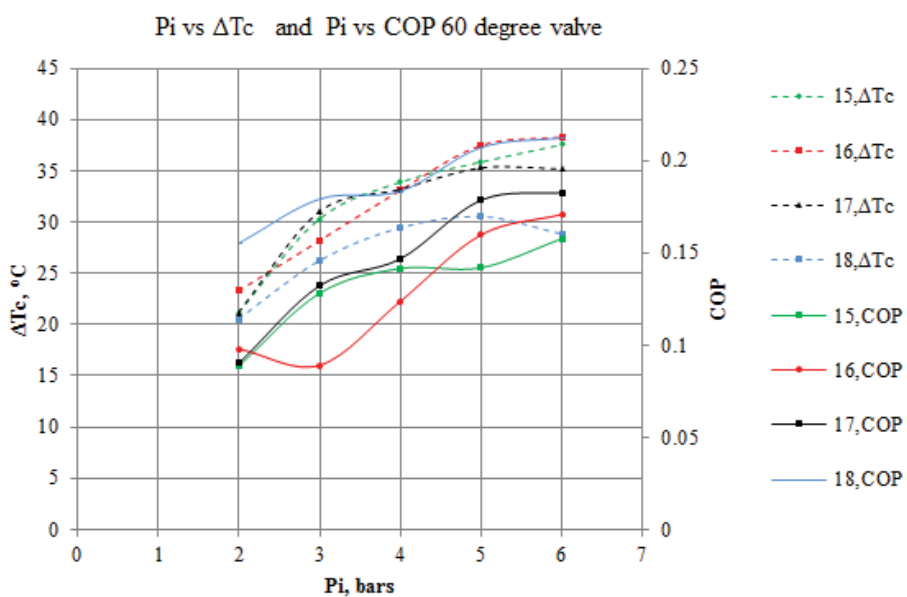
The figure 2-6 show the effect of  $L/d$  ratio at a specific valve angle  $\theta$  on  $COP$  and  $\Delta t_c$ . It can be seen that the combination of  $L/d$  and  $\theta$  has mixed effects, it is observed that  $COP$  increases for some combinations of  $L/d$  and  $\theta$  and decreases for other combinations.



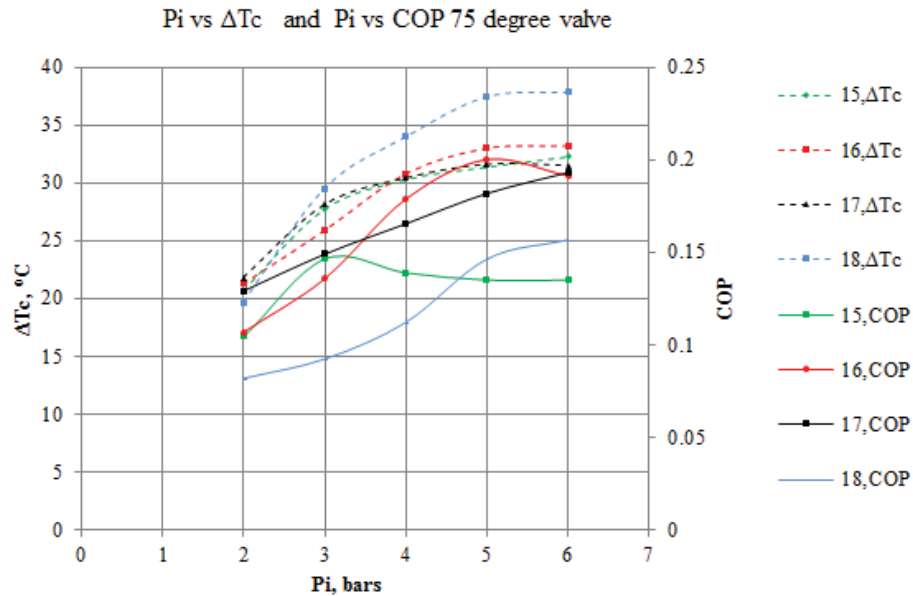
**Figure 2.** Effect of  $L/D$  on  $COP$  and  $\Delta t_c$  for 30° valves.



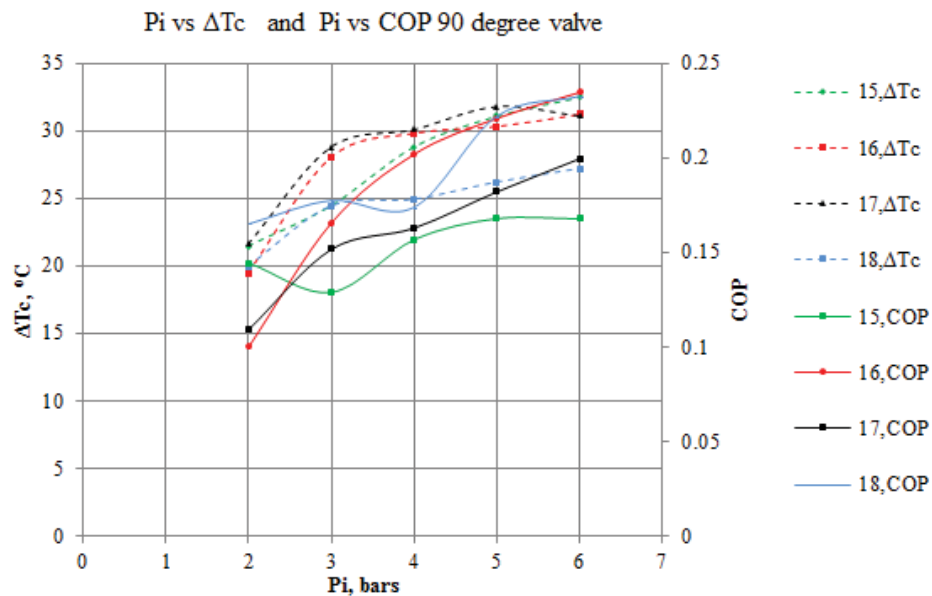
**Figure 3.** Effect of L/D on COP and  $\Delta t_c$  for 45° valves.



**Figure 4.** Effect of L/D on COP and  $\Delta t_c$  for 60° valves.



**Figure 5.** Effect of L/D on COP and  $\Delta t_c$  for 75° valves.



**Figure 6.** Effect of L/D on COP and  $\Delta t_c$  for 90° valves.

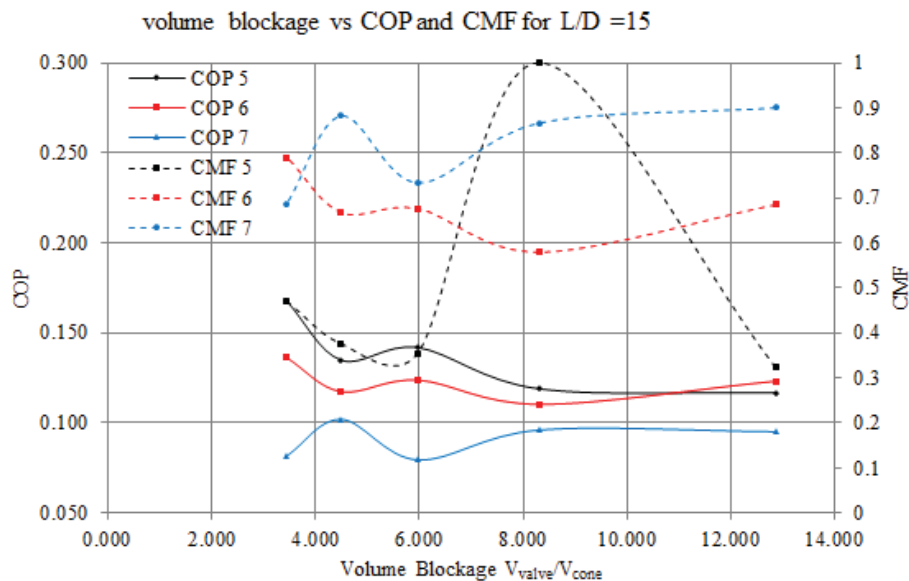
The mechanism of energy separation acts between peripheral vortex flow of air and the reversal of air flow at hot end. On hot end as the flow is restricted by valves based on the angle of valve the flow reversal and mass of air getting reversed changes. For more flat surface of valve more mass of air is reversed but since the peripheral air mass is reduced the energy transfer from central layer to peripheral layer is reduced. As length of tube increases the transaction length for heat exchange also increase between two layers.

The length of tube also has effect on energy separation in this way. But on hot end flow reversal starts after stagnation point. Stagnation point usually gets located inside the tube and it is away from the flat surface of valve. It is logical to say that as length of tube increases the energy transfer should increase, but stagnation point puts limitation on this. With increase in length the velocity of peripheral decreases and this may be the reason that length of stagnation region also increases. Because of this the mass of air getting reversed gets hampered. This might be the reason for mixed results getting produced. Thus it cannot be said that a definite combination of  $L/d$  and  $\theta$  produces better result. Experimental results show that as  $p_i$  increase all tubes produce considerably good  $COP$  but still the  $COP$  obtained is less than unity. The reason for this is changes in cold mass of air.

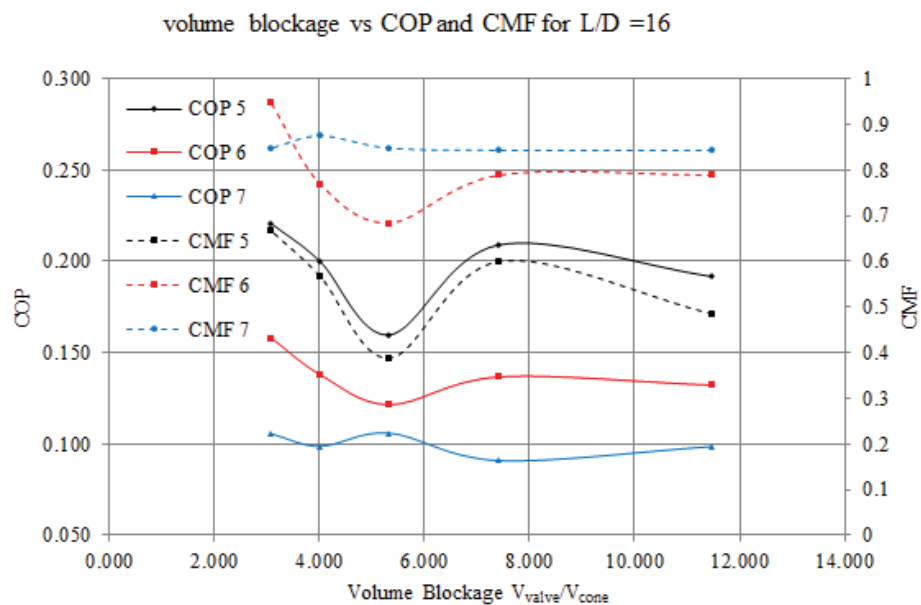
### 3.2. Effect of orifice diameter $d_o$ :

To analyze the effect of orifice diameter  $d_o$  as well as valve angles on  $COP$  and  $CMF$ , a ratio called as filling ratio is defined. This ratio considers the volume blocked by the valve and volume available inside the cone. The blocked volume is calculated as ratio of valve volume  $V_{valve}$  to cone volume  $V_{cone}$ . This represents blocked volume or filled volume in percentage.

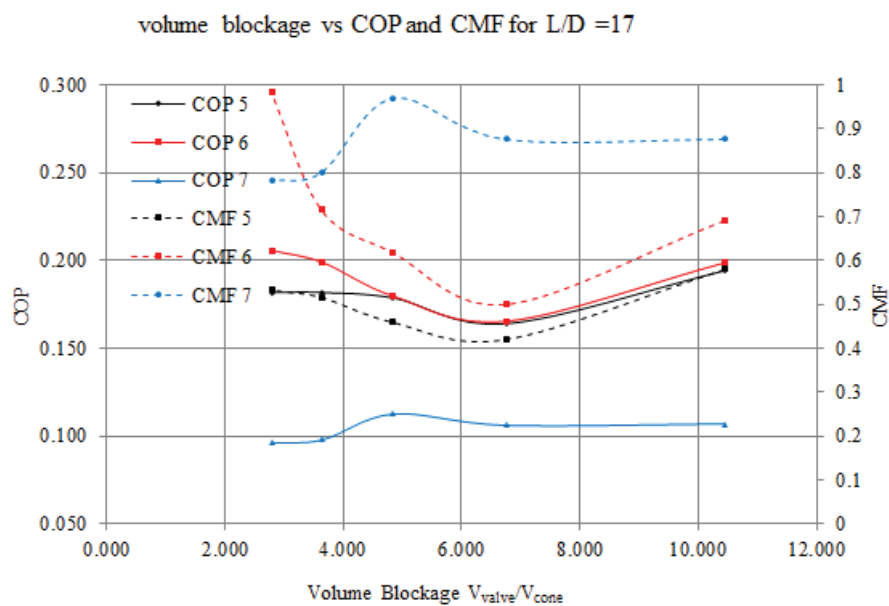
Figures 7-10 show the Effect of orifice diameter  $d_o$  on  $COP$  and  $CMF$  for various  $L/d$  ratios it can be observed that as  $d_o$  increases the  $COP$  and  $CMF$  also increases. This confirms the results of Nimbalkar *et.al.* [12]



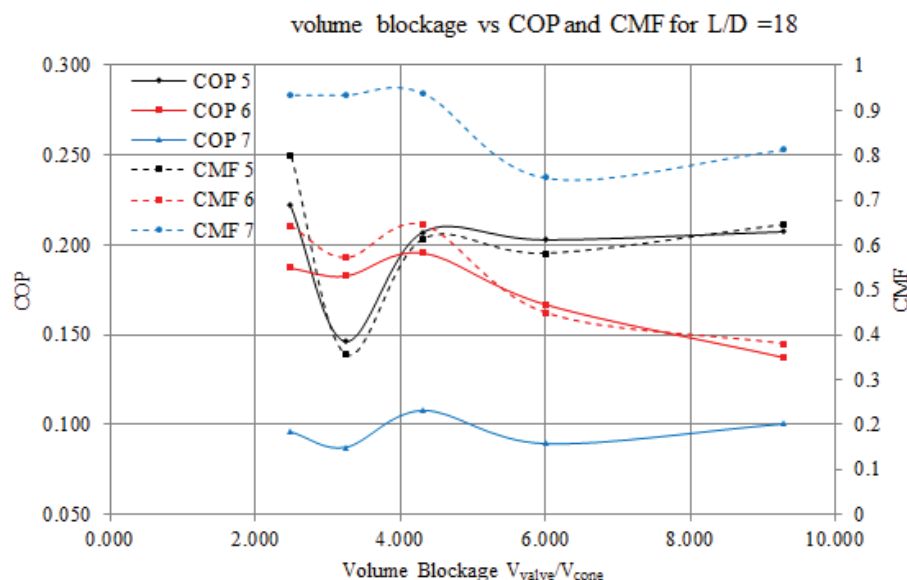
**Figure 7.** Effect of Orifice diameter on COP and CMF for  $L/D = 15$



**Figure 8.** Effect of Orifice diameter on COP and CMF for L/D =16



**Figure 9.** Effect of Orifice diameter on COP and CMF for L/D =17



**Figure 10.** Effect of Orifice diameter on COP and CMF for  $L/D = 18$

The major reason is when the ratio of  $d_o/d$  is less than 0.5 the cold mass of air at periphery is restricted at the walls of orifice. Because of this restriction the cold air gets mixed with the fresh air. This also produces secondary circulation on orifice end. As  $d_o/d$  is 0.5 or more back flow is reduced and cold air comes out of the orifice without reversal. This helps to increase mass of cold air at exit and thus improves  $COP$  and  $CMF$ . This is the probable reason that increase in orifice diameter is addressed and that to in relation of  $d_o/d \geq 0.5$ . The results also show the change in  $d_o$  with  $L/d$  gives mixed results i.e. for some  $L/d$  ratios  $d_o/d \leq 0.5$  produces good result for  $COP$  and  $CMF$  while with change in  $L/d$  for  $d_o/d \geq 0.5$  produces good for  $COP$  and  $CMF$ . This mixed nature of the results prompt to make use of MADM methods to select the best combination for getting better  $COP$  and  $CMF$ .

#### 4. MADM (Multiple attribute decision making)

MADM is a decision making method when there are multiple parameters affecting the end output. MADM methods are discrete with a limited number of pre specified alternatives. These methods require both intra and inter- parametric comparisons for unbiased judgment or decision for the considered problem. [24]

The decision making system in MADM methods is based on four main components, namely: alternatives (Set of experiments), attributes (resulting parameters), weight (index of influence for variation), or strength of each resulting parameter (attribute) and measure of performance of each set of experiment (alternative) with respect to the others. Some of the commonly used methods under MADM are Weighted Sum Method (WSM), Weighted Power Method (WPM), Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). [24]

##### 4.1. Weighted sum method (WSM)

WSM was introduced by Fishburn in 1967 [25]. This is the simplest method. Here each resulting parameter (attribute) is given a weight and sum of all weights must be equal to 1. Each set of experiment (alternative) is assessed with regard to every resulting parameter (attribute). The data presented for each

set of experiment is normalized based on beneficial or non-beneficial parameter. The composite score of a set of experiment is given by [26]

$$C_i = \sum_{j=1}^N w_j(n_{ij}) \text{norm} \quad (11)$$

#### 4.2. Weighted Power Method (WPM)

This method is similar to WSM. The main difference is that instead of addition, there is multiplication.

This method is introduced by Miller and Star [27]. The composite score is given by

$$C_i = \prod_{j=1}^N [(n_{ij}) \text{norm}]^{w_j} \quad (12)$$

#### 4.3. Analytic hierarchy process (AHP)

It is one of the popular techniques introduced by Satty [28]. It decomposes a decision making problem into an organized orders of objectives, resulting parameter (attributes). AHP can proficiently deal with objective as well as subjective features. The main procedure of using AHP using the radical root method (Geometric Mean Method, GMM) is as follows.

- Step 1: Determine the objectives and evaluation attributes
- Step 2: Determine relative importance of different attributes. This step comprises of relative comparison of attributes against attributes this becomes matrix A1. The relative importance weights are given as designed by Satty [28] for the attributes comparison.

$$\begin{matrix} & \begin{matrix} b_1 & b_2 & b_3 & b_4 \end{matrix} \\ \begin{matrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{matrix} & \begin{bmatrix} 1 & b_{12} & b_{13} & b_{14} \\ b_{21} & 1 & b_{23} & b_{24} \\ b_{31} & b_{32} & 1 & b_{34} \\ b_{41} & b_{42} & b_{43} & 1 \end{bmatrix} \end{matrix} = B_{N \times N} \quad (13)$$

- Step 2.1 Find relative normalized weight of each attribute by calculating mean of i-th row and as

$$GM_j = [\prod_{i=1}^N b_{ij}]^{1/N} \quad (14)$$

- Step 2.2 Normalise the geometric mean of rows in the comparison matrix

$$w_j = GM_j / \sum_{j=1}^N GM_j \quad \text{..This becomes matrix A2.} \quad (15)$$

- Step 2.3 Calculate matrices A3 and A4 such that

$$A3 = A1 * A2 \text{ and } A4 = A3 / A2$$

- Step 2.4 Determine the maximum Eigen value  $\lambda_{max}$  that is the average of the matrix A4.
- Step 2.5 Calculate consistency Index CI as

$$CI = (\lambda_{max} - N) / (N - 1) \text{ The smaller the value of CI the smaller is the deviation.}$$

- Step 2.6 Calculate consistency ratio CR as

$$CR = CI / RI, \text{ RI is random index, given by Satty.}$$

- Step 3: compare the alternatives pair wise with respect to each attribute.
- Step 4: Obtain overall or composite scores for the alternatives by using WSM, WPM method. [24]

#### 4.4. Technique for order preference by similarity to ideal solution (TOPSIS)

TOPSIS was developed Hwang and Yoon [29]. This method is based on the concept that the chosen alternative should have the shortest Euclidean distance from the ideal solution and the farthest negative ideal solution. The main procedure of the TOPSIS for selection of best alternative from the available is as follows [30, 31]:

- Step 1: determine objectives and evaluation attributes.
- Step 2: Establish decision matrix comprised of attributes and alternatives.
- Step 3: Establish normalized matrix as

$$R_{ij} = n_{ij} / (\sum_{j=1}^N n_{ij}^2)^{1/2} \quad (16)$$

- Step 4: Decide relative importance weights using either AHP or any other suitable method
- Step 5: obtain normalized weighted matrix as

$$B_{ij} = w_j R_{ij} \quad (17)$$

- Step 6: obtain ideal best and worst solutions as  $B^+$  and  $B^-$
- Step 7: obtain separation measures as Euclidean distance (the distance of the solution from ideal or worst solution) from the ideal solution as

$$S_i^+ = \left[ \sum_{j=1}^N (B_{ij} - B^+)^2 \right]^{1/2} \quad (18)$$

$$S_i^- = \left[ \sum_{j=1}^N (B_{ij} - B^-)^2 \right]^{1/2} \quad (19)$$

- Step 8: The relative closeness of a particular solution from ideal solution is given by the Euclidean distance. (Here as  $S_i^-$  appears in numerator the higher the distance from the worst solution the closer is the solution to ideal one.)

$$p_i = (S_i^- / S_i^+ + S_i^-) \quad (20)$$

- Step 9: A set of alternatives is generated in the descending order in this step to indicate most preferred and least preferred solutions. [32]

## 5. Application of MADM

The decision matrix in the given problem consists of total 375 combinations presented as  $L/d - \phi - d_o - \theta - P_i$ . Where  $L/d$  takes values as 15,16,17,18  $\phi$  varies as 0 and  $d_o$  varies from 5, 6, 7mm  $\theta$  varies in steps of  $15^\circ$  in the range 30 to 90 and inlet pressure varies in step of 1 bar from 2 to 6. Thus in total there were many alternatives to choose the best from. The attributes are cold end temperature  $t_c$ , COP and CMF. MADM methods are used since the experimental results do not lead to predict the combination of parameters producing best result. The results are conflicting.

The decision matrix for illustration is given in table 4. The table provides Final set of values selected from the best performance of each  $L/d$  ratio. The selection is based on MADM methods. Here Out of the best performing combinations MADM will be used to order the tubes in hierarchy. The range of all parameters is as shown in table 2.

**Table 4.** Decision Matrix for Illustration.

	TC	CMF	COP
15-0-6-45-2	66.4	0.923076923	0.103644
15-0-7-90-4	52.2	0.896551724	0.090148
15-0-7-45-3	62.7	0.846153846	0.093995
15-0-7-90-6	49	1	0.083057
15-0-7-90-5	50.6	0.941176471	0.083068
15-4-5-45-2	48.1	1	0.305182
15-4-5-45-3	55.5	1	0.230179
15-4-5-45-4	59.9	1	0.190488
15-4-5-45-5	63.4	1	0.169185
15-4-5-45-6	64.8	1	0.149352
16-4-6-90-5	46.9	0.947368421	0.106654
16-4-6-90-4	45.1	0.8	0.104375

16-4-6-30-6	54.5	0.925	0.09264
16-4-7-75-4	41.8	0.928571429	0.095466
16-4-6-45-6	54.5	0.857142857	0.088922
17-4-6-90-2	40.7	0.756756757	0.14284
17-4-6-90-3	44.5	0.904761905	0.122115
17-4-6-90-4	45.5	0.979591837	0.102501
17-4-6-90-5	47.2	0.98245614	0.092424
17-4-7-60-5	40.4	0.967741935	0.093052
18-4-5-90-2	41	0.5	0.139263
18-4-7-90-3	31	0.916666667	0.100402
18-4-5-45-2	50.4	0.380952381	0.116153
18-4-6-60-6	44.8	0.806451613	0.087671
18-4-5-45-5	59	0.580645161	0.083825

### 5.1. Using AHP method

AHP method is used to find out relative importance weight of each attribute. This is done by comparing the attributes against attributes and giving scale of importance. The A1 matrix is formed by comparing attributes  $t_c$ ,  $COP$  and  $CMF$  against each other and allocating relative importance like when  $t_c$  is compared with  $COP$  then  $COP$  is more important than  $t_c$ , and when  $t_c$  is compared with  $CMF$  then  $CMF$  is more important than  $t_c$  but less important than  $COP$ .

$$A1 = \begin{matrix} & \begin{matrix} t_c & CMF & COP \end{matrix} \\ \begin{matrix} t_c \\ CMF \\ COP \end{matrix} & \begin{bmatrix} 1 & 1/3 & 1/5 \\ 3 & 1 & 1/3 \\ 5 & 3 & 1 \end{bmatrix} \end{matrix}$$

The Geometric mean matrix of the above relative importance is given by Equation. 14

$$GM_j = \begin{bmatrix} 0.4055 \\ 1 \\ 2.4662 \end{bmatrix}$$

The weights of the attributes which makes the Matrix A2 is given by Equation. 15

$$A2 = w_j = \begin{bmatrix} 0.1047 \\ 0.2583 \\ 0.6370 \end{bmatrix}$$

The matrix A3= A2\*A1 is given by,

$$A_3 = \begin{bmatrix} 0.3182 \\ 0.7848 \\ 1.9355 \end{bmatrix}$$

The matrix A4=A3/A2 is given by

$$A_4 = \begin{bmatrix} 3.0385 \\ 3.0385 \\ 3.0385 \end{bmatrix}$$

The maximum Eigen value  $\lambda_{max}$  that is the average of the matrix A4 is 3.0385. Consistency index is = 0.0192, for R=0.52 as given by Satty [28] for number of attributes =3 the consistency ratio is  $CR = 0.03703$ , the consistency ratio is much less than 0.1 hence the weights are acceptable for the analysis and the decided weights are,  $w_{tc} = 0.10147$   $w_{cmf} = 0.2583$   $w_{cop} = 0.6370$

Now we apply the MADM methods to sort among the decision matrix for the best alternative. For doing this the normalized matrix is formed on the basis of beneficial and non-beneficial attributes. Here all  $T_c$ ,  $COP$  and  $CMF$  are beneficial attributes i.e. we have to have the maximum values for all attributes. The normalized matrix is calculated and is given as,

$$\begin{array}{l} 15-0-6-45-2 \\ 15-0-7-90-4 \\ 15-0-7-45-3 \\ . \\ . \\ . \\ 18-4-6-60-6 \\ 18-4-5-45-5 \end{array} \begin{bmatrix} 1 & 0.9231 & 0.3396 \\ 0.7861 & 0.8966 & 0.2954 \\ 0.9443 & 0.8462 & 0.3080 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0.6747 & 0.8065 & 0.2873 \\ 0.8886 & 0.5806 & 0.2747 \end{bmatrix}$$

The normalized data is used to calculate composite scores of each alternative. For this WSM, WPM and TOPSIS is used the TOPSIS normalized matrix is calculated using Equation. 16.

$$\begin{array}{l} 15-0-6-45-2 \\ 15-0-7-90-4 \\ 15-0-7-45-3 \\ . \\ . \\ . \\ 18-4-6-60-6 \\ 18-4-5-45-5 \end{array} \begin{bmatrix} 0.2597 & 0.2078 & 0.1556 \\ 0.2041 & 0.2019 & 0.1354 \\ 0.2452 & 0.1905 & 0.1412 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0.1752 & 0.1816 & 0.1317 \\ 0.2307 & 0.1307 & 0.1259 \end{bmatrix}$$

TOPSIS normalized weighted matrix is as follows for TOPSIS using Equation. 17.

$$\begin{array}{l} 15-0-6-45-2 \\ 15-0-7-90-4 \\ 15-0-7-45-3 \\ . \\ . \\ . \\ 18-4-6-60-6 \\ 18-4-5-45-5 \end{array} \begin{bmatrix} 0.02719 & 0.05368 & 0.09914 \\ 0.02137 & 0.05214 & 0.08623 \\ 0.02568 & 0.04921 & 0.08991 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0.01834 & 0.0469 & 0.08386 \\ 0.02416 & 0.03376 & 0.08018 \end{bmatrix}$$

### 5.2. WSM method

The overall or composite score of an alternative for WSM method is given by equation 11 and the alternatives are ordered in descending order.

### 5.3. WPM method

The overall or composite score of an alternative for WPM method is given by equation 12 and the alternatives are arranged in descending order.

### 5.4. TOPSIS method

Using the TOPSIS normalized weighted matrix presented above and calculating the separators as to distinguish the alternatives on the basis of Euclidean distances, equation 18 and 19 are used to check how far the solution is from the worst and the ideal solution. Equation 20 gives the Euclidean distance from the worst solution. Hence the maximum the value the close the solution is to ideal solution. Hence the values are arranged in descending order. Composite scores for WSM, WPM and TOPSIS methods for the

nonlinear data are calculated and the results are arranged descending order. Table 5 shows the results and composite scores in descending order. The results give the best possible combination for getting optimized solution from the experimental data.

**Table 5.** MADM Comparison of results.

Preference	$L/d-\phi-\theta-\pi$	TOPSIS	SAW	WPM
1	15-4-5-45-2	0.966408	15-4-5-45-2	0.966798
2	15-4-5-45-3	0.669491	15-4-5-45-3	0.820008
3	15-4-5-45-4	0.499512	15-4-5-45-4	0.732705
4	15-4-5-45-5	0.411276	15-4-5-45-5	0.683445
5	15-4-5-45-6	0.332415	15-4-5-45-6	0.632708
6	17-4-6-90-2	0.281886	15-0-6-45-2	0.545072
7	18-4-5-90-2	0.251621	17-4-6-90-3	0.521417
8	17-4-6-90-3	0.216663	17-4-6-90-2	0.492343
9	16-4-6-90-5	0.175486	16-4-6-90-5	0.486726
10	15-0-6-45-2	0.171441	17-4-6-90-4	0.482264
11	17-4-6-90-4	0.17059	17-4-6-90-5	0.477168
12	17-4-6-90-5	0.152734	16-4-6-30-6	0.457684
13	18-4-7-90-3	0.152292	15-0-7-45-3	0.44964
14	18-4-5-45-2	0.150331	15-0-7-90-6	0.449229
15	17-4-7-60-5	0.149248	17-4-7-60-5	0.448787
16	15-0-7-90-6	0.147381	16-4-7-75-4	0.445805
17	16-4-7-75-4	0.145738	15-0-7-90-4	0.444682
18	16-4-6-30-6	0.144357	15-0-7-90-5	0.441718
19	16-4-6-90-4	0.143586	16-4-6-90-4	0.435972
20	15-0-7-90-5	0.136321	18-4-7-90-3	0.429129
21	15-0-7-45-3	0.135814	16-4-6-45-6	0.42283
22	15-0-7-90-4	0.134333	18-4-5-90-2	0.417699
23	16-4-6-45-6	0.125981	18-4-6-60-6	0.410126
24	18-4-6-60-6	0.109951	18-4-5-45-2	0.409235
25	18-4-5-45-5	0.071188	18-4-5-45-5	0.376862

Thus the best combination revealed by all these methods is 15-4-5-45-2, 15-4-5-45-3, 15-4-5-45-4, 15-4-5-45-5, and 15-4-5-45-6. Thus it can be concluded that the  $L/d$  15 at 4 degree divergence angle and 5 mm orifice diameter at 45 degree conical valve angle at all inlet pressures performs best.

## 6. Conclusion

Vortex tube is tested with reference to the literature data experimentally for optimum performance under the tested condition. It has produced lowest cold end temperature of  $-14.8^{\circ}\text{C}$  and COP 0.305 with maximum CMF as 1. The tube has shown best results for  $4^{\circ}$  divergence angle as compared to plain tube tested at  $0^{\circ}$ . The Nimbalkar relation of  $d_o/d \geq 0.5$  is also verified for obtaining maximum CMF. Application of MADM methods like WSM, WPM and TOPSIS have come up with the optimum tube combinations as,  $L/d$  15,  $\phi - 4$ ,  $d_o - 5$ ,  $\theta - 45$ , at all inlet pressures  $p_i$ . The limitations of the study so far

are the tube has been tested with only 2 entry nozzles, hence in future study can also be conducted with increasing number of nozzles,  $N_n$  and the close range of divergence angle,  $\emptyset$ . If the results can be optimized with these vortex tube may find wider range of applications in the commercial field.

## Nomenclature

### Capital Letters

A	matrix	
B	Ideal and worst values,	
b	row column element	
C	composite score	
c	specific heat	[KJkg <sup>-1</sup> K <sup>-1</sup> ]
CI	Consistency Index	[Non Dimensional]
CMF	Cold Mass Fraction	[Non Dimensional]
COP	Coefficient of Performance	[Non Dimensional]
CR	Consistency Ratio	[Non Dimensional]
d	Diameter of Tube	[mm]
GM	Geometric Mean	
L	Length of Tube	[mm]
m	Mass of air	[Kg]
N	Matrix Size	
P	Pressure	[Kpa]
R	Row element	
RE	Refrigeration effect	[KJsec <sup>-1</sup> ]
RI	Random Index	
S	Euclidean Separation distance	
T	Temperature	[K]
$\Delta T$	Temperature Difference [K]	
V	volume	[m <sup>3</sup> ]
W	Compressor Work	[KJsec <sup>-1</sup> ]
w	weightage	

### Greek Letters

$\emptyset$	Divergence angle
$\Theta$	Conical valve angle
$\lambda$	maximum Eigen value
$\eta$	efficiency

### Subscripts

ad	adiabatic
c	cold end
cone	divergent cone
comp	compressor
d	discharge condition
i	inlet condition, i-th column
j	j-th row
max	maximum
n	number

norm	normalized value
o	orifice
rel	relative
valve	hot end conical valve
1	relative importance value matrix
2	weight matrix
3	geometric mean matrix
4	operation matrix
Superscripts	
+	Highest value in range of data
-	Lowest value in range of data
‘	Stagnation value

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