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A mathematical procedure to predict optical performance of CPCs

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Abstract. To evaluate the optical performance of a CPC based concentrating photovoltaic system, it is essential to find the angular dependence of optical efficiency of compound parabolic concentrator (CPC-θ_e) where the incident angle of solar rays on solar cells is restricted within θ_e for the radiation over its acceptance angle. In this work, a mathematical procedure was developed to calculate the optical efficiency of CPC-θ_e for radiation incident at any angle based radiation transfer within CPC-θ_e. Calculations show that, given the acceptance half-angle (θ_a), the annual radiation of full CPC-θ_e increases with the increase of θ_e and the CPC without restriction of exit angle (CPC-90) annually collects the most radiation due to large geometry (C_t); whereas for truncated CPCs with identical θ_a and C_t, the annual radiation collected by CPC-θ_e is almost identical to that by CPC-90, even slightly higher. Calculations also indicate that the annual radiation on the absorber of CPC-θ_e at the angle larger than θ_e decrease with the increase of θ_e but always less than that of CPC-90, and this implies that the CPC-θ_e based PV system is more efficient than CPC-90 based PV system because the radiation on solar cells incident at large angle is poorly converted into electricity.

1. Introduction

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In recent years, solar photovoltaic (PV) have been widely used for electrical generation over the world due to emerging issues of conventional energy shortage [1]. However, the application of PV system is limited due to high cost of electricity generation as compared to the conventional electrical generation technologies [2]. Therefore, to lower the cost of electricity generating from PV systems is a unique solution to expanding applications of PV technology. Apart from seeking for new materials and production techniques of solar cells, the use of cheep optical concentrator is regarded as an effective way to cost reduction of electricity generation from PV systems.

Concentrating PV systems (CPV) are generally classified into high concentrating PV system (HCPV) and low concentrating systems (LCPV). CPVs seem simple in the mechanism but are difficult to implement, especially HCPVs, which require expensive and complex sun-tracking device, cooling technique of solar cells and specially designed solar cells [3, 4]. Thus LCPV is more attractive due to no need of sun-tracking system. In the past two decades, many theoretical and experimental studies were performed to employ compound parabolic concentrators (CPC) for concentrating solar radiation on solar cells [5-9]. Compared to CPCs, the V-trough concentrator shares the advantages of easy construction and more uniform solar irradiation on the base, but the increase in power output from V-trough based CPVs is limited [10-12]. To make the irradiation on solar cells of CPC based CPV uniform, Hatwaambo tested a CPV system where semi-diffuse reflective materials were used, and found that the increase of the fill factor of the photovoltaic system was insignificant [13, 14]. However, recent study by Yu and Tang found that the use of semi-diffuse reflectors in CPVs would lead the collectible radiation decrease greatly[15]. For a CPV system, the incident angle of solar rays reflecting from the lower part of reflectors is considerably large, thus can not be efficiently converted into electrical power due to poor solar absorption [16,17]. To increase the absorption of solar radiation by solar cells of CPC based PV system, CPCs with a restricted exit angle (CPC-θ_e) was first proposed [5,18], but it is in recent years that several researches on its performance are found in the literature [17,19,20,21]. For such CPC, all solar rays over its acceptance angle (θ_a) arrive on the absorber at the angle less than the desired value θ_e, whereas for solar radiation beyond its acceptance angle, a fraction of incident radiation will arrive on the receiver at the angle larger than θ_e, but it was not considered in previous works of the authors [20,21], leading its performance underestimated. In this work, a general mathematical procedure to determine the angular dependence of optical efficiency of CPC-θ_e with any θ_e was developed, and effects of θ_e on its performance was theoretical investigated in terms of annual collectible radiation.

2. Mathematical procedure to calculate the optical efficiency of CPC-θ_e

2.1. Equation of reflectors

As seen from figure 1, the reflectors of CPC-θ_e consist of upper parabola and lower plane mirror, and the plane mirror is tangent to the lower end of parabolic reflector. To be convenient analysis, the width of the absorber is set to be 1, thus, parabolic reflector can be expressed by:

$$\begin{cases} x = \dfrac{(\sin\theta_e + \sin\theta_a)\sin\theta}{1-\cos(\theta+\theta_a)} - 0.5 \\ y = \dfrac{(\sin\theta_e + \sin\theta_a)\cos\theta}{1-\cos(\theta+\theta_a)} \end{cases} \quad (\theta_t \le \theta \le \theta_e) \tag{1}$$

where θ is the polar angle at the absorber end; θ_e is the maximum exit angle for radiation within its acceptance angle; and θ_t is the edge-ray angle of CPC- θ_e . For full CPC- θ_e , $\theta_t = \theta_a$ and $C_t = \sin\theta_e / \sin\theta_a$; whereas for truncated CPCs with a given C_t , θ_a and θ_e , the θ_t can be calculated based on equation (1) [20]. Obviously, CPC-90, the CPC without restriction of exit angle, is a special case of CPC-θ_e for the case of $\theta_e = 90^\circ$.

It should be noted that the maximum θ_t should be less θ_e otherwise CPC-θ_e is reduced into a V-trough concentrator. The coordinate of lower end *D* of the parabola can be determined based on equation (1) by setting $\theta = \theta_e$ as follows:

$$\begin{cases} x_D = \dfrac{(\sin\theta_e + \sin\theta_a)\sin\theta_e}{1-\cos(\theta_e+\theta_a)} - 0.5 \\ y_D = \dfrac{(\sin\theta_e + \sin\theta_a)\cos\theta_e}{1-\cos(\theta_e+\theta_a)} \end{cases} \tag{2}$$

The plane mirror is expressed by:

$$y = c\tan\gamma_D (x - 0.5) \qquad (0.5 \le x \le x_D) \tag{3}$$

The tilt angle of plane mirrors (γ_D) relative to y-axis is given by:

$$\gamma_D = 0.5(\theta_e - \theta_a) \tag{4}$$

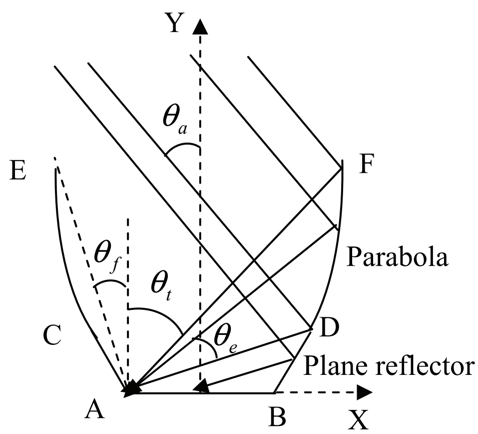

Figure1. Geometry of CPC-θ_e

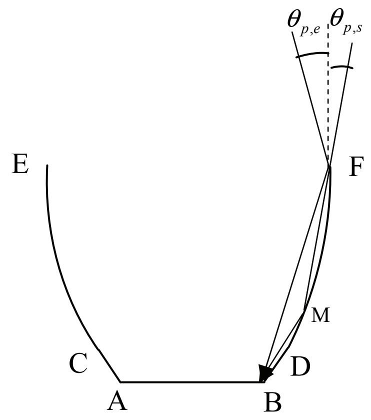

Figure 2. Angle range of solar rays that arrive on the absorber after more than two reflections.

2.2. Optical efficiency of CPC-θ_e

To calculate the collectible radiation of CPC-θ_e, it is essential to find the angular dependence of optical efficiency. For CPC-θ_e, all of radiation over its acceptance angle will arrive on the absorber at the incident angle (θ_{in}) less than θ_e as shown in figure 1; whereas for radiation incident at $\theta_p > \theta_a$, part of radiation incident on the plane reflector (see figure 2) and upper parabolic reflector (see figure 3) will arrive on the absorber at $\theta_{in} > \theta_e$. Thus the collectible radiation on the receiver includes the radiation incident at $\theta_{in} \le \theta_e$ (I_1), radiation reflecting from the plane reflector (I_2) at $\theta_{in} > \theta_e$ and that reflecting from the upper parabolic reflector to the opposite plane mirror first and then reflecting onto the absorber (I_3) at $\theta_{in} > \theta_e$. Therefore, the optical efficiency of CPC-θ_e is given by:

$$\eta(\theta_p) = \frac{I_1 + I_2 + I_3}{I_{ap}} = f_1 + f_2 + f_3 \tag{5}$$

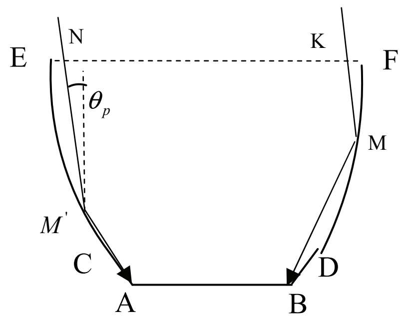

Figure 3. Fraction of radiation that arrive on the absorber after more than two reflections.

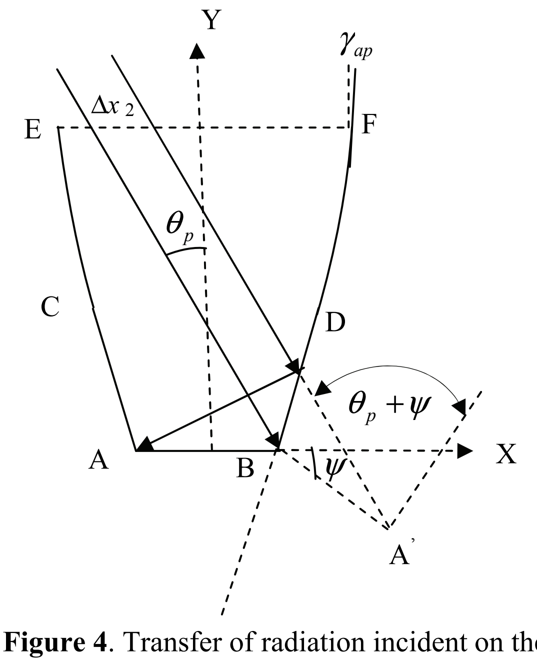

Figure 4. Transfer of radiation incident on the plane the mirror of CPC-θ_e at $\theta_a < \theta_p < \theta_t$

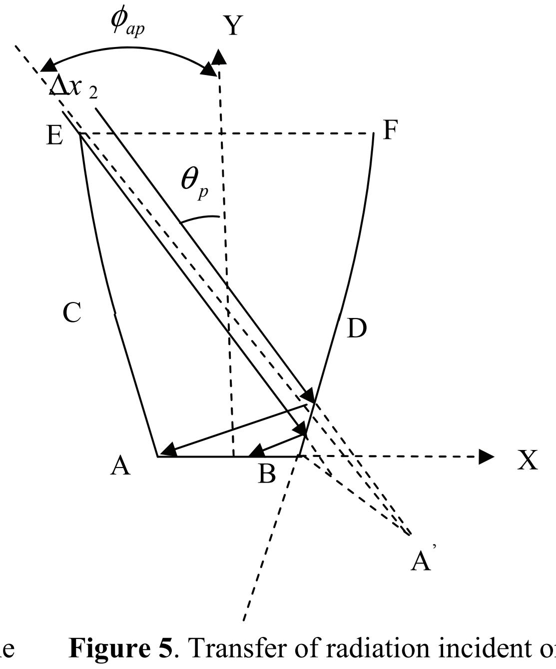

Figure 5. Transfer of radiation incident on mirror of CPC-θ_e at $\theta_t < \theta_p < \theta_{p,c1}$

In equation (5), the I_{ap} is the radiation incident on the aperture of CPC-θ_e, f_1 is the optical efficiency contributed by radiation arriving on the absorber at $\theta_{in} \le \theta_e$, f_2 is the efficiency contributed by radiation on the absorber after reflection from the plane reflectors at $\theta_{in} > \theta_e$, and f_3 is that contributed by radiation on the absorber after reflecting from the upper parabolic reflector first and then from its opposite plane reflector at $\theta_{in} > \theta_e$.

2.2.1. Calculation expressions of f_1. For radiation incident at small θ_p, partial radiation undergoes multi-reflection before arriving on the absorber [22]. To make calculations accurate, the two-reflection model, in which the radiation arriving on the absorber after more than two reflections is regarded as that arrives on the absorber after just two reflections, is employed, thus f_1 can be expressed by:

$$f_1 = f_0 + (f_{11} - f_{12})\rho + f_{12}\rho^2 \tag{6}$$

where f_0 is the fraction of radiation directly irradiating on the absorber; f_{11} and f_{12} stand for the fraction of radiation arriving on the absorber after more than one and two reflections, respectively. equation (6) also can be rewritten as:

$$f_1 = f_0 + f_{11}\rho - f_{12}\rho(1-\rho) = \eta_1 - f_{12}\rho(1-\rho) \tag{7}$$

where $\eta_1 = f_0 + f_{11}\rho = f_0 + (1-f_0)\rho$ is the f_1 of CPCs estimated based on the one-reflection model [20,22], and calculated by:

$$\eta_1 = \begin{cases} \rho + (1-\rho)/C_t & \theta_p \le \theta_f \\ \rho + 0.5(1-\rho)(1+C_t)(1-\tan\theta_p/\tan\theta_t)/C_t & \theta_f < \theta_p \le \theta_a \\ 0.5(1+C_t)(1-\tan\theta_p/\tan\theta_t)/C_t & \theta_a < \theta_p \le \theta_t \\ 0 & \theta_p > \theta_t \end{cases} \tag{8}$$

In the case of $\theta_f > \theta_a$, it is given by

$$\eta_1 = \begin{cases} \rho + (1-\rho)/C_t & \theta_p \le \theta_a \\ 1/C_t + 0.5\rho(1-1/C_t) - 0.5\rho(1+1/C_t)\tan\theta_p/\tan\theta_{t,2} & \theta_a < \theta_p \le \theta_f \\ 0.5(1+C_t)(1-\tan\theta_p/\tan\theta_t)/C_t & \theta_f < \theta_p \le \theta_t \\ 0 & \theta_p > \theta_t \end{cases} \tag{9}$$

where θ_f (see figure 1) is given by:

$$\tan\theta_f = \tan\theta_t (C_t - 1)/(C_t + 1) \tag{10}$$

As shown in figure 2, solar rays incident on the (right) reflector at $\theta_{p,e}$ and $-\theta_{p,s}$ just strike at the end (B) of the absorber after one reflection, thus solar rays incident at $-\theta_{p,s} < \theta_p < \theta_{p,e}$ will undergo more than two reflections before arriving on the absorber (see figure 3). Based on the reflection law of light, one has:

$$\theta_{p,e} = \theta_f - 2\gamma_{ap} \tag{11}$$

The γ_{ap} is the tilt-angle of the line tangent to the upper end of the parabolic reflector relative to y-axis (see figure 4), and can be found based on equation (1) as follow:

$$\tan\gamma_{ap} = \left.\frac{dx}{dy}\right|_{\theta=\theta_t} = \frac{\cos\theta_a - \cos\theta_t}{\sin\theta_a + \sin\theta_t} \tag{12}$$

The critical angle $\theta_{p,s}$ is subjected to following equation:

$$\gamma_M = 0.5(\theta_{p,s} + \gamma_{MB}) \tag{13}$$

where γ_M is the tilt-angle of the line tangent to M (see figure 3), γ_{MB} is the tilt-angle of line *MB*. For a given CPC, γ_M , γ_{MB} and $\theta_{p,s}$ as the function of θ_M can be respectively expressed by:

$$\tan\gamma_M = \frac{\cos\theta_a - \cos\theta_M}{\sin\theta_a + \sin\theta_M} \tag{14a}$$

$$\tan\gamma_{MB} = (x_m - 0.5) / y_M \tag{14b}$$

$$\tan\theta_{p,s} = (0.5C_t - x_M) / (h_t - y_M) \tag{14c}$$

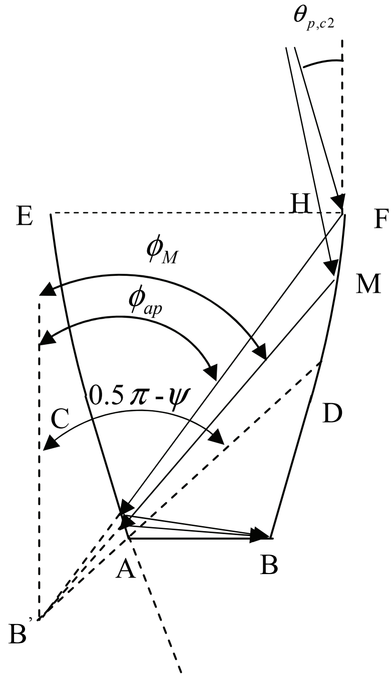

Figure 6. Transfer of radiation incident on the upper parabolic reflector of CPC-θ_e at $\theta_a < \theta_p < \theta_{p,c2}$

By substituting γ_M , γ_{MB} and $\theta_{p,s}$ obtained from equation (14) into equation (13) or by iterative calculations, one obtains θ_M , then $\theta_{p,s}$ is obtained from equation (14c). As shown in figure 3, the fraction of radiation arriving on the absorber after more than two reflections is given by $f_{12} = (KF + ED) / C_t$, and KF is given by:

$$KF = \begin{cases} 0.5C_t - x_M + (h_t - y_M)\tan\theta_p & (-\theta_{p,s} < \theta_p < \theta_{p,e}) \\ 0 & else \end{cases} \tag{15}$$

where x_M and y_M as the function of θ_M are x-, y-coordinates of critical point M, respectively, and θ_M can be calculated based on equation (13), (14a)-(14b) by setting $\theta_{p,s} = -\theta_P$. Similarly, *ED* can be calculated in the same way as finding *KF* by setting $\theta_p = -\theta_P$.

2.2.2. Calculation method of f_2. As shown in figure 4, BA' is the first image of the absorber formed by the plane mirror. According to the imaging principle of plane mirrors, rays pointing to the image of the absorber formed by the mirror must arrive on the absorber after reflecting from the mirror [12]. Therefore, when solar rays incident on the plane mirror at $\theta_p > \theta_a$, partial radiation will arrive on the absorber at $\theta_{in} = \theta_p + \psi > \theta_e$ (for radiation incident at $\theta_p = \theta_a$, the $\theta_{in} = \theta_e$, thus the opening angle of V-trough formed by two plane reflectors of the CPC-θ_e is $\psi = 2\gamma_D = \theta_e - \theta_a$), the remaining will redirect to the opposite mirror and then escape from CPC-θ_e or arrive on the absorber at $\theta_{in} = (\theta_p + 2\psi) > \theta_e$ [12]. Solar radiation arriving on the absorber after two reflections in between two plane mirrors must satisfy:

$$\begin{cases} \theta_p > \theta_a \\ \theta_p + 2\psi \le 0.5\pi \end{cases} \quad (16)$$

And this leads:

$$\theta_a > 2\theta_e - 0.5\pi \quad (17)$$

This means that solar rays entering the V-trough are possible to arrive on the absorber after two reflections for CPC-θ_e with $\theta_a > 2\theta_e - 0.5\pi$. Thus for CPC-$\theta_e$ with $\theta_e = 65^\circ$ and 60°, θ_a must be larger than 40° and 30°, respectively. In practical design of CPC-θ_e, θ_e is usually larger than 65° and $\theta_a < 35^\circ$ [17], and such CPC-θ_e is subjected to $\theta_a < 2\theta_e - 0.5\pi$. In this work, the CPCs subjected to $\theta_a < 2\theta_e - 0.5\pi$ are considered for simplifying analysis, thus no radiation arrives on the absorber of CPC-θ_e after more than two reflections in between two mirrors.

To ensure solar rays reflecting from mirrors arrive on the absorber, the θ_p should be subjected to $\psi + \theta_p \le 0.5\pi$, i.e. $\theta_p \le 0.5\pi - \psi$; but in the other hand, θ_p should be less than ϕ_{ap} due to the shade of reflectors on the opposite mirrors (see figure5-6). Therefore, the critical incident angle $\theta_{p,c1}$, solar rays incident at the angle less than which will arrive on the absorber at $\theta_{in} > \theta_e$, should take the smaller one of ϕ_{ap} and $0.5\pi - \psi$, namely:

$$\theta_{p,c1} = Min(\phi, 0.5\pi - \psi) \quad (18)$$

As shown in figure 5-6, ϕ_{ap} is the tilt-angle of the line linking the upper end *E* (*F*) of parabolic reflectors and the image of absorber end *A* (*B*), and is calculated by:

$$\tan\phi_{ap} = \frac{0.5 + \cos\psi + 0.5C_t}{h_t + \sin\psi} \quad (19)$$

Here $h_t = 0.5(C_t + 1)/\tan\theta_t$ is the height of CPCs. As shown in figure 4, the image (BA') of the absorber is fully irradiated by radiation incident at $\theta_p \le \theta_t$ and partially irradiated as $\theta_t < \theta_p < \theta_{p,c1}$

(see figure 5), therefore, the fraction of radiation on the absorber after reflection from the plane mirrors, Δx_2, is determined by:

$$\Delta x_2 / C_t = \begin{cases} (\cos\psi - \sin\psi \tan\theta_p) / C_t & (\theta_a < \theta_p \le \theta_t) \\ (h_t + \sin\psi)(\tan\varphi_{ap} - \tan\theta_p) / C_t & (\theta_t < \theta_p < \theta_{p,c1}) \\ 0 & \theta_p \ge \theta_{p,c1} or \theta_p \le \theta_a \end{cases} \quad (20)$$

$$f_2 = \rho \Delta x_2 / C_t \quad (21)$$

2.2.3. Calculation method of f_3. As shown in figure 6, solar rays incident on the upper end (F) of the parabolic reflector at $\theta_{p,c2}$ will just redirect to the image B' after reflection, thus, for solar rays entering CPC-θ_e at $\theta_a < \theta_p < \theta_{p,c2}$, the radiation incident on the upper parabola (FM) will redirect to the opposite mirror (AC) and then be reflected onto the absorber, and solar rays incident on lower part of parabola (MD) will finally escape from the cavity of the CPC-θ_e after multiple reflections within the CPC cavity. The polar angle θ_m of the critical point (M) on the parabola is subjected to following equation group:

$$\begin{cases} 2\gamma_m = \phi_m - \theta_p \\ \tan\phi_m = \dfrac{x_m + 0.5 + \cos\psi}{y_m + \sin\psi} \end{cases} \quad (22)$$

where x_m and y_m as the function of θ_m are *x* and *y* coordinates of point *M*, respectively; ϕ_m as the function of θ_m is the angle formed by line MB' and y-axis (see figure 6); γ_m, as the function of θ_m, is the slope of the line tangent to point *M* and determined by equation (12). Thus, given θ_p ($\theta_a < \theta_p < \theta_{p,c2}$), θ_m can be obtained by iterative calculations, then the fraction of solar radiation incident on the upper parabola (*FM*) that arrive on the absorber after reflection from the parabola first then from the opposite mirror, $\Delta x_3 / C_t$, can be calculated by:

$$\Delta x_3 / C_t \begin{cases} 0.5C_t - x_m + (h_t - y_m)\tan\theta_p & (\theta_a < \theta_p < \theta_{p,c2}) \\ 0 & else \end{cases} \quad (23)$$

$$f_3 = \rho^2 (\Delta x_3 / C_t) \quad (24)$$

According to the law of light reflection, the critical incident angle $\theta_{p,c2}$ (see figure 5) is given by:

$$\theta_{p,c2} = \phi_{ap} - 2\gamma_{ap} \tag{25}$$

It is noted that $\gamma_{ap} = 0$ and $\theta_{p,c2} = \theta_{p,c1}$ for full $CPC - \theta_e$. Analysis shows that, for truncated CPC-θ_e, $\theta_{p,c2}$ decreases with the increase of θ_t, and $\theta_{p,c1}$ is always larger than $\theta_{p,c2}$. It is also noted that no radiation will arrive on the absorber at $\theta_{in} > \theta_e$ after reflections from upper parabola first and then from the opposite mirror in the case of $\theta_t \geq 0.5\pi - \psi$ (see figure 6) because point *M* must be above the line $B^{'}A$ to ensure solar rays reflecting from *M* redirect to the image $B^{'}$ of absorber end *B*.

3. Mathematical method to calculate daily collectible radiation

It is assumed that CPC-θ_e is oriented in the east-west direction with the aperture being tilted at β from the horizon, and radiation reflected from the ground is not considered. Thus, the collectible radiation on absorber at any moment for isotropic sky diffuse radiation is given by:

$$I = C_t I_b g(\theta_{ap}) \eta(\theta_p) \cos\theta_{ap} + C_t \int_{-(0.5\pi-\beta)}^{0.5\pi} iu(\theta_p) \cos(\theta_p) d\theta_p \tag{26}$$

where I_b is the intensity of beam radiation; $i = 0.5 I_d$ is the directional intensity of sky diffuse radiation on the cross-section of CPC-troughs [12, 22]. Equation (26) can be rewritten as

$$I = C_t I_b g(\theta_{ap}) \eta(\theta_p) \cos\theta_{ap} + 0.5 I_d (C_{d,1} + C_{d2}) \tag{27}$$

where I_d is sky diffuse radiation on the horizon, and $C_{d,1}$ and $C_{d,2}$ are calculated as follows:

$$C_{d,1} = C_t \int_0^{0.5\pi} \eta(\theta_p) \cos\theta_p d\theta_p \tag{28}$$

$$C_{d,2} = C_t \int_0^{0.5\pi-\beta} \eta(\theta_p) \cos\theta_p d\theta_p \tag{29}$$

For a given CPC-θ_e, $C_{d,1}$ is a constant but $C_{d,2}$ is dependent on β, and both can be obtained by numerical calculations. Similarly, radiation on the absorber of CPC-θ_e at $\theta_{in} \leq \theta_e$ is expressed by:

$$I(\theta_{in} \leq \theta_e) = C_t I_b g(\theta_{ap}) f_1 \cos\theta_{ap} + 0.5 C_t I_d \int_{-(0.5\pi-\beta)}^{\theta_t} f_1 \cos\theta_p d\theta_p \tag{30}$$

The daily radiation on the absorber can be calculated by:

$$H_{day} = C_t \int_{-t_0}^{t_0} I_b g(\theta_{ap}) \eta(\theta_p) \cos\theta_p dt + 0.5 H_d (C_{d,1} + C_{d2}) \tag{31}$$

$$H_{day}(\theta_{in} \leq \theta_e) = C_t \int_{-t_0}^{t_0} I_b g(\theta_{ap}) f_1(\theta_p) \cos\theta_p dt + 0.5 C_t H_d \int_{-(0.5\pi-\beta)}^{\theta_t} f_1 \cos\theta_p d\theta_p \tag{32}$$

where H_d is the daily sky diffuse radiation on the horizon, $t_0 = \tau_{day}\omega_0 / 2\pi$ is the sunset time on the horizon, $\tau_{day}(24\times 3600s)$ is the day length, ω_0 is the hour angle of sunset on the horizon [5]. Given the geometry of CPC-θ_e and tilt-angle β, the last term of right hand in equation (32) can be numerically calculated. The daily radiation on the absorber with $\theta_{in} > \theta_e$, $H_{day}(\theta_{in} > \theta_e)$, can be simply calculated by subtracting H_{day} $(\theta_{in} \le \theta_e)$ from H_{day}. The incidence angle of solar rays on the aperture of CPC-θ_e (θ_{ap}) and projection incident angle (θ_p) at any moment of a day can be found based solar geometry [5, 22, 23]. The $g(\theta_{ap})$ in equation (26-27) is a control function, being 1 for $\cos\theta_{ap} > 0$ otherwise zero. On knowing θ_p and θ_{ap}, and f_1, f_2 and f_3 can be determined based mathematical procedure suggested in this work, then the H_{day} and $H_{day}(\theta_{in} > \theta_e)$ can be numerically calculated, finally summing H_{day} and $H_{day}(\theta_{in} > \theta_e)$ in all days of a year obtain the annual radiation on the absorber of CPC-θ_e ($S_{CPC-\theta_e}$) and annual radiation on the absorber with $\theta_{in} > \theta_e$ ($S(\theta_{in} > \theta_e)$). The annual radiation on the absorber of CPC-90 with $\theta_{in} > \theta_e$ can be obtained according to the method presented in the previous work of authors [20].

In this work, monthly horizontal radiation in Beijing ($\lambda = 39.95^\circ$), the capital of China, was used for the analysis [24], the daily sky diffuse radiation, H_d, and beam radiation (I_b) at any moment of a day are estimated based on the empirical correlations proposed by Collares-Pereira and Rabl [25]. The angle step for calculating $C_{d,1}$ and $C_{d,2}$ is take to be 0.005^o; the time step to calculate daily radiation on the absorber is taken to be 1 min; and the θ_a of CPCs is set to 26^o [26]. To investigate the optical performance of CPC-θ_e, two cases with the β being yearly fixed and yearly adjusted four times at three tilts are considered. For CPCs with β being yearly fixed (1T-CPC), the β is taken to be site latitude (λ) [26]; whereas for CPCs with β being yearly adjusted four times at three tilts (3T-CPC), the β is set to be λ during periods of 22 days around both equinoxes, and adjusted to $\lambda + 23^\circ$ and $\lambda - 23^\circ$ in winters and summers, respectively [12].

4. Results and discussions

4.1. *Optical efficiency comparison between CPC-θ_e and CPC-90*

As seen from figure 7-8, f_2 and f_3 is zero for radiation within the acceptance angle but not for radiation beyond the acceptance angle as expected. It is also seen that the f_3 is larger than f_2 for full CPC-65 and less than f_2 for truncated CPC-65. This means that for full CPC-θ_e, the contribution of upper parabola together with the opposite plane mirror to $S(\theta_{in} > \theta_e)$ is larger than that of plane mirror alone; whereas for truncated CPC-θ_e, the situation is reversed. Comparisons of optical efficiency between CPC-90 and CPC-65 are presented in figure 9 and 10. It is found that the optical efficiency of both CPCs is almost identical for radiation within the acceptance angle; whereas for radiation incident at $\theta_p > \theta_a$, the f of CPC-65 is always larger than that of CPC-90 for full CPCs, but for truncated CPCs with identical C_t and θ_a, the f of CPC-90 is larger than that of CPC-65 in the case of $\theta_p > 31^\circ$ due to the large edge-ray angle (θ_t) of CPC-90 and zero f_3 of CPC-65 (see figure 6)

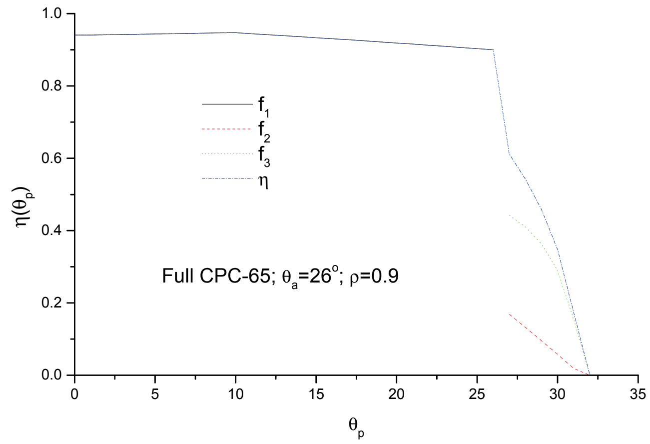

Figure 7. Angular variations of optical efficiency of full CPC-65

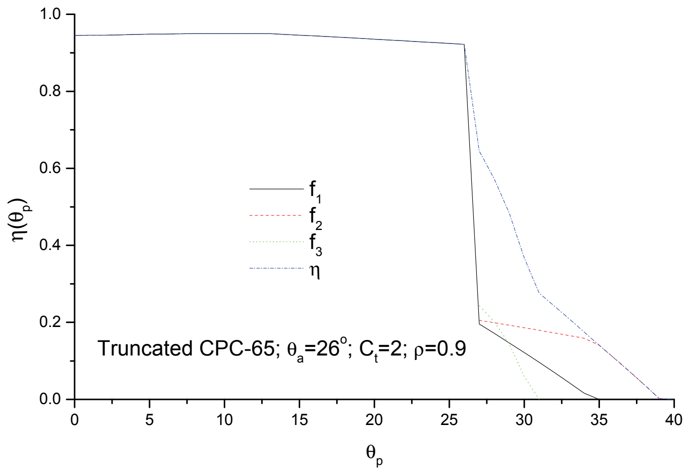

Figure 8. As in Fig.7 but for truncated CPC-65

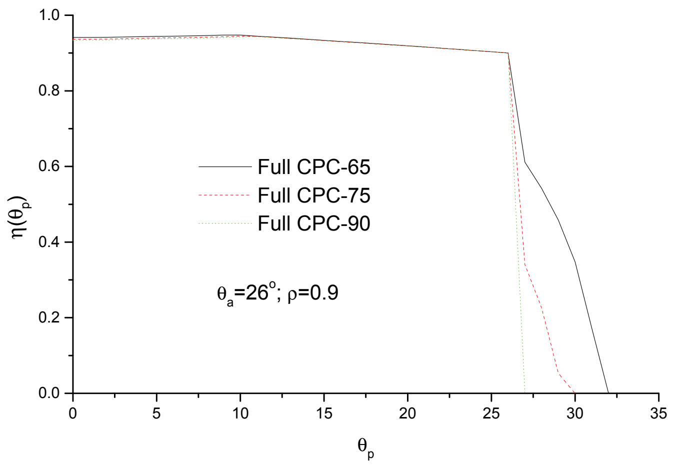

Figure 9. Angular variation of optical efficiency of full CPC- with different.

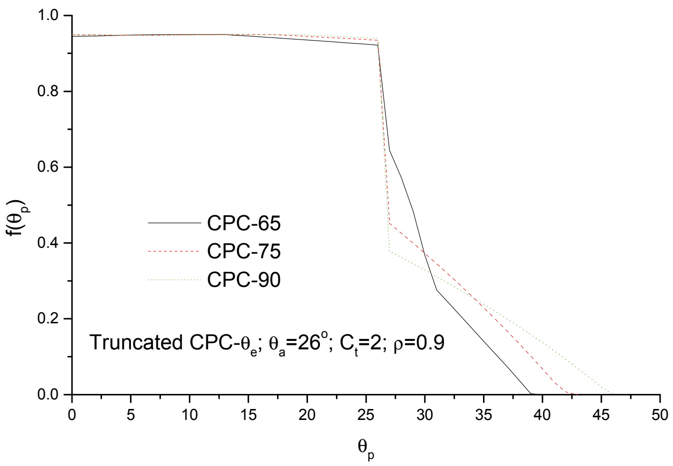

Figure 10. As in Fig.9 but truncated CPC-θ_e

4.2. Comparison of annual collectible radiation between CPC-θ_e and CPC-90

Figure 11 presents the ratio of annual radiation concentrated by CPC-θ_e to that by CPC-90 in the case of β being yearly fixed (1T-CPCs). It is seen that, for full CPCs with identical θ_a, the annual radiation collected by CPC-90 is always larger than that by CPC-θ_e due to large geometric concentration of CPC-90; whereas for truncated CPCs with identical θ_a and C_t, the annual radiation collected by CPC-θ_e are almost identical to that by CPC-90, and even slightly higher in the case of low C_t. The same results as in figure 11 are also found in figure12 in which β is yearly adjusted four times at three tilts (3T-CPCs).

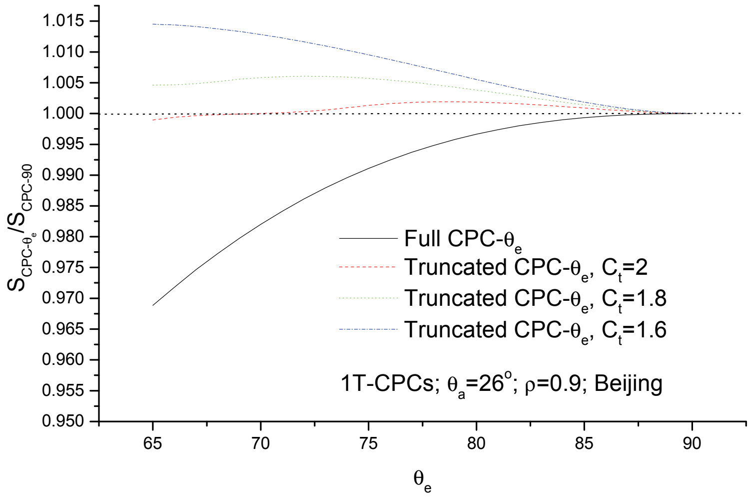

Figure 11. Comparison of annual collectibleradiation between 1T-CPC-θ_e and 1T-CPC-90.

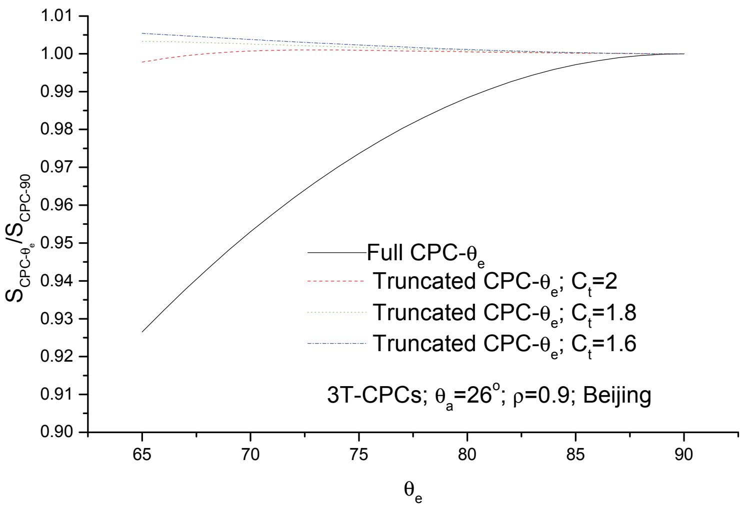

Figure12. Comparison of annual collectible radiation between 3T-CPC-θ_e and 3T-CPC-90.

4.3. Comparison of $S(\theta_{in} > \theta_e)$ between CPC-θ_e and CPC-90

The annual radiation on the absorber of CPCs with $\theta_{in} > \theta_e$ are presented in figure 13-14. It is seen that $S(\theta_{in} > \theta_e)$ of both CPC-θ_e and CPC-90 decreases with the increase of θ_e, and $S(\theta_{in} > \theta_e)$ collected by CPC-θ_e is much less than that by CPC-90 in the case of θ_e less than 70°, this implies that the CPC-θ_e based PV system should be more efficient than CPC-90 based PV system because the radiation on solar cells with $\theta_{in} > 65^\circ$ is poorly absorbed by solar cells.

It is noted that, given θ_e, the annual radiation on the absorber of CPC-90 is independent on its geometric concentration (C_t) and $S(\theta_{in} > \theta_e)$ keeps constant as a result of the fact that the $S(\theta_{in} > \theta_e)$ comes from the lower part of reflectors of CPC-90 [20], but $S(\theta_{in} > \theta_e)$ collected by CPC-θ_e is sensitive to C_t because critical angles $\theta_{p.c1}$ and $\theta_{p.c2}$ are sensitive to θ_t as shown in figure15. In fact, $S(\theta_{in} > \theta_e)$ collected by CPC-θ_e in a site depends on the angular dependence of ($f_2 + f_3$) and tilt-angle adjustment strategy, and in turn the angular dependence of ($f_2 + f_3$) is dependent on the edge-ray angle (θ_t) or C_t as seen from figure 16. As shown in figure 15, for 1T-CPC-65, $S(\theta_{in} > 65^o)$ increases with C_t due to high $f_2 + f_3$ for $\theta_p < 30^\circ$ (see figure15) and large C_t; whereas for 3T-CPC-65, $S(\theta_{in} > 65^o)$ decreases with C_t as a result of the fact that the sun is almost within the acceptance angle of CPCs (i.e. $\theta_p < \theta_a$) over the daytime in any day of a year, and $S(\theta_{in} > 65^o)$ mainly originates from the sky diffuse radiation, thus $S(\theta_{in} > 65^o)$ increases with θ_t or decreases with C_t. Figure14 also indicates that $S(\theta_{in} > 65^o)$ of 1T-CPC-65 is much larger than that of 3T-CPC-65, showing that the use of 3T-CPCs facilitates the improvement of photovoltaic performance CPC-θ_e based PV systems due to lower $S(\theta_{in} > \theta_e)$.

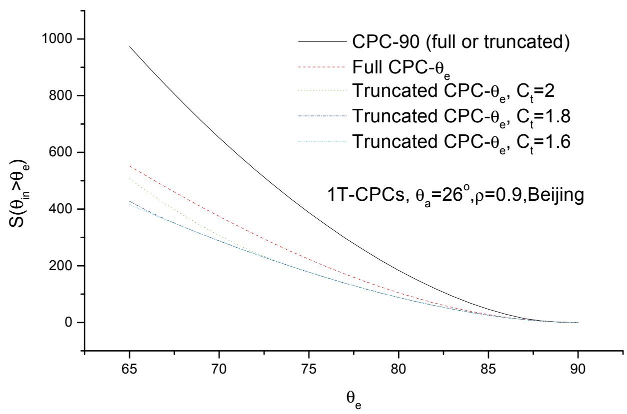

Figure 13. Annual radiation on the absorber of 1T-CPCs at $\theta_{in} > \theta_e$

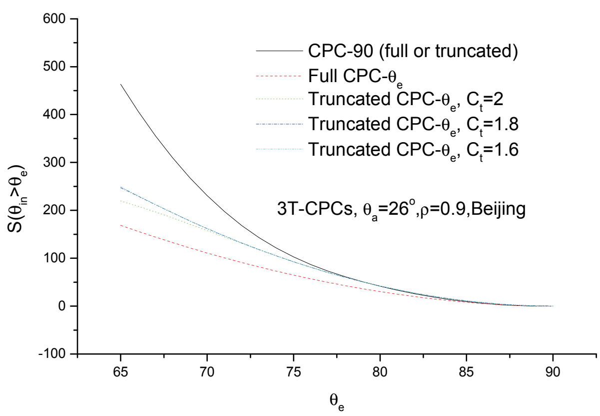

Figure 14. As in figure 13 but for 3T-CPCs

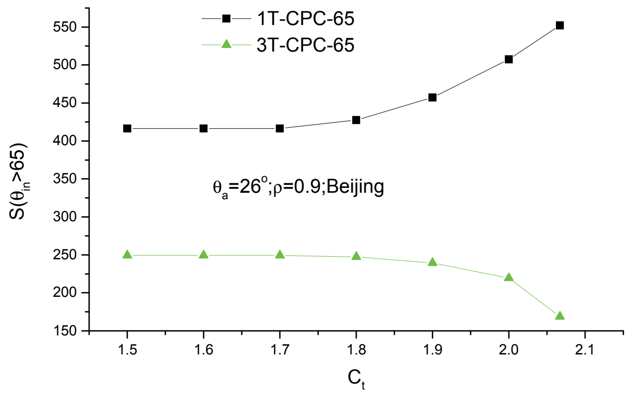

Figure 15. Effects of geometric concentration on S($\theta_{in} > 65$) of CPC-65

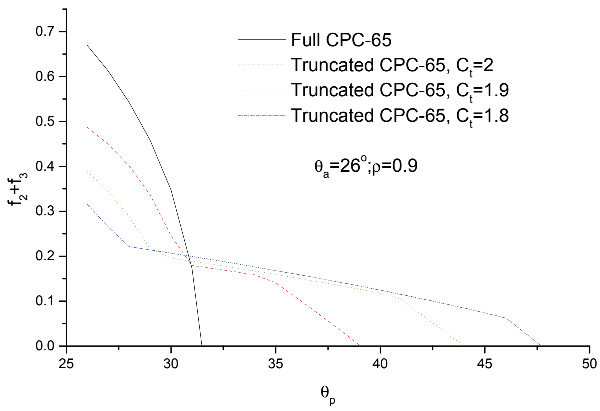

Figure 16. Optical efficiency for radiation arriving on the absorber at θ_{in}>65

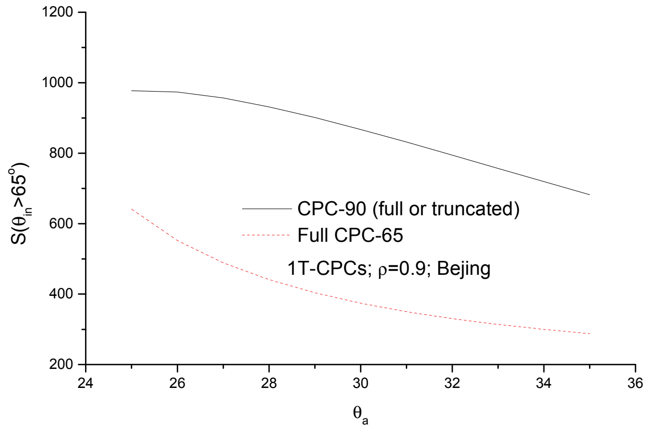

Figure 17. Effects of acceptance half-angle on S($\theta_{in} > 65$) of CPCs

Figure 17 shows the effect of acceptance angle (θ_a) on $S(\theta_{in} > \theta_e)$. It is seen that, given θ_e (such as 65°), $S(\theta_{in} > \theta_e)$ collected by CPC-90 and CPC- θ_e decreases with the increase of θ_a. This indicates that θ_a should be large as possible in the practical design of CPCs based on requirements of least daily operation hours, geometric concentration and strategy of tilt-angle adjustments [27].

5. Conclusions

The analysis in this work shows that part of radiation beyond the acceptance angle of CPC-θ_e would arrive on the absorber at $\theta_{in} > \theta_e$, the annual radiation on the absorber with $\theta_{in} > \theta_e$ depends on the acceptance angle, geometric concentration as well tilt-angle adjustment strategy of CPC-θ_e. Calculations indicate that, for full CPCs with identical θ_a, the annual radiation collected by CPC-θ_e is less than that by CPC-90 due to the large geometric concentration (C_t) of CPC-90; whereas for truncated CPCs with identical θ_a and C_t, the annual radiation collected by CPC-θ_e is almost identical to that by CPC-90, and even slightly higher. Calculations also show that the annual radiation on the absorber of CPC-θ_e with $\theta_{in} > \theta_e$ decrease with the increase of θ_e but always less than that of CPC-90, and this implies that the CPC-θ_e based PV system is more efficient than CPC-90 based PV system because the radiation on solar cells incident at large angle is poorly converted into electricity.

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