

Hydraulic fracturing model based on the discrete fracture model and the generalized J integral

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Abstract. The hydraulic fracturing technique is an effective stimulation for low permeability reservoirs. In fracturing models, one key point is to accurately calculate the flux across the fracture surface and the stress intensity factor. To achieve high precision, the discrete fracture model is recommended to calculate the flux. Using the generalized J integral, the present work obtains an accurate simulation of the stress intensity factor. Based on the above factors, an alternative hydraulic fracturing model is presented. Examples are included to demonstrate the reliability of the proposed model and its ability to model the fracture propagation. Subsequently, the model is used to describe the relationship between the geometry of the fracture and the fracturing equipment parameters. The numerical results indicate that the working pressure and the pump power will significantly influence the fracturing process.

1. Introduction

Unconventional resources reserves such as shale or tight gas are huge. However, their natural deliverability is insufficient to meet industrial needs due to their low permeability [1]. As hydraulic fractures can effectively increase the drainage area, hydraulic fracturing technique has become a widely-used stimulation for low-permeability reservoirs [2]. To ensure that the hydraulic fractures meet the stimulation requirement, it is necessary to simulate the hydraulic fracturing process and analyze the factors which affect the fracturing treatment.

Fracturing simulation focuses on rock deformation caused by a fracturing fluid and the fracture propagation criteria. Fluid leak-off was neglected by the early fracturing models [3-4], in which the ability of the fracturing fluid to deform the rock is overestimated. Subsequently, other models [5-8] employ the empirical “Fluid-loss coefficient” to account for fluid leak-off. Assuming that the calculation domain is infinite, the fracture geometry is calculated in the models mentioned above using the analytical solution of the displacement. In actuality, the deformation of the rock framework and the seepage interact with one another. Fluid-solid coupling theory is applied to describe this phenomenon in recent fracturing models [9-11], in which rock deformation and fluid flow are solved simultaneously. Thus, an accurate method of calculating the fluid flux across the fracture surface is necessary. To achieve high precision, the discrete fracture model (DFM) [12] is used to calculate the flux in the present work.

In linear elastic fracture mechanics, the failure criteria is met when the stress intensity factor (SIF) exceeds the fracture toughness. The commonly-used methods to calculate SIF are summarized as



follows. Advani, et al., [13] calculated the SIF by substituting the stress on the fracture surface into the analytic formulas of the SIF for infinite space. Similarly, Dean, et al., [9] obtained the SIF by using the analytic solution of the infinite solid. The J integral [14] is another method used to calculate the SIF. An integral path far from the fracture tip can be chosen in order to avoid the numerical error resulting from the stress singularity. Considering fluid-solid coupling, the generalized J integral is introduced into the present work.

Based on the discussion above, a hydraulic fracturing model based on the discrete fracture model and the generalized J integral is proposed.

2. Model description

2.1. Reservoir-flow model

The mass conservation equations of the fluid flow for the matrix and the fracture are given below, respectively [15]:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(-\rho \frac{K_m}{\mu} r \frac{\partial P_m}{\partial r} \right) + \frac{\partial}{\partial z} \left(-\rho \frac{K_m}{\mu} \frac{\partial P_m}{\partial z} \right) + \rho q_{fm} \delta(\mathbf{r} - \mathbf{r}_f) = 0 \quad (1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(-\rho w \frac{K_f}{\mu} r \frac{\partial P_f}{\partial r} \right) - \rho q_{fm} + \frac{\rho Q}{2\pi r_w} \delta(\mathbf{r} - \mathbf{r}_0) = 0 \quad (2)$$

where P_m is the fluid pressure in the matrix, P_f is the fluid pressure in the fracture, μ and ρ are the viscosity and the density for the fluid, respectively, $\mathbf{r}_f$ is the fracture strike, w is the fracture width, $\mathbf{r}_0$ and r_w are the location and the radius for the well, Q is the well injection, r and z are the radial coordinate and the axial coordinate, respectively, $\delta(\mathbf{r})$ is the Dirac function, K_m is the matrix permeability, K_f is the fracture permeability equal to $w^2/12$, and q_{fm} is the fluid flux across the fracture surface.

In the present work, the embedded discrete fracture model is introduced. The fluid flux across the fracture surface is calculated according to the method proposed by Zhou, et al., [12] as follows:

$$q_{fm} = -\frac{K_{fm}}{\mu} \frac{\partial P}{\partial n_f^+} - \frac{K_{fm}}{\mu} \frac{\partial P}{\partial n_f^-} = -\frac{K_{fm}}{\mu} \frac{P_f - (P_m)_{M^+}}{d^+} - \frac{K_{fm}}{\mu} \frac{P_f - (P_m)_{M^-}}{d^-} \quad (3)$$

where $d^\pm$ is the distance from the fracture grid to the reference point $M^\pm$ in the matrix, and K_{fm} is the permeability between the fracture and the matrix which is the harmonic mean of K_m and K_f .

2.2. Geo-mechanical model

The system of equations representing the solid mechanics includes the equilibrium equation, the constitutive equation and the geometric equation. Considering the effect of in-situ stress, variations, such as the displacement, are decomposed into the initial portion caused by in-situ stress and the effective portion caused by other external forces. For instance, the decomposition of ϕ is $\phi = \phi' + \phi''$, where ϕ' is the initial portion and ϕ'' is the effective portion.

The equations for the initial portion are given as follows:

$$\frac{\partial \sigma'_{ji}}{\partial x_j} = 0 \quad (4)$$

$$\sigma'_{ij} = 2G\varepsilon'_{ij} + \lambda\theta'\delta_{ij} \quad (5)$$

$$\varepsilon'_{ij} = \frac{1}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) \quad (6)$$

The effective portion satisfies the following equations:

$$\frac{\partial \sigma''_{ji}}{\partial x_j} + f_i'' = 0 \quad (7)$$

$$\sigma''_{ij} = 2G\varepsilon''_{ij} + \lambda\theta''\delta_{ij} + \sigma_{0,ij}\delta_{ij} \quad (8)$$

$$\varepsilon''_{ij} = \frac{1}{2} \left(\frac{\partial u''_i}{\partial x_j} + \frac{\partial u''_j}{\partial x_i} \right) \quad (9)$$

where σ'_{ij} is the initial stress, σ''_{ij} is the effective stress, ε'_{ij} is the initial strain, ε''_{ij} is the effective strain, u'_i is the initial displacement in the i-direction, u''_i is the effective displacement in the i-direction, θ' is the initial volume strain, and θ'' is the effective volume strain, $\sigma_{0,ij}$ is the in-situ stress, G is the shear modulus, λ is the Lamé coefficient, δ_{ij} is the Kronecker delta, and f_i'' is the body force of the rock or the impact of the fracturing fluid on the rock. Based on the equilibrium equation of the rock and the fluid-solid coupling theory [16], the expression of f_i'' is described as follows:

$$f_i'' = -\frac{\partial(\alpha P_m \delta_{ij})}{\partial x_j} \quad (10)$$

where α represents Biot's coefficient.

2.3. Fracture propagation model

From a linear elastic fracture mechanics perspective, the fracture will extend when the SIF exceeds the fracture toughness [17]. In this work, the generalized J integral is used to calculate the SIF as follows [14, 18]:

$$K_I = \left(\frac{E}{1-\mu^2} J \right)^{1/2} \quad (11)$$

$$J = \frac{1}{a} \left[\int_{\Gamma} w r dz - \int_{\Gamma+l^++l^-} \sigma_{ij} n_j \frac{\partial u_i}{\partial r} r d\Gamma - \iint_D f_i \frac{\partial u_i}{\partial r} r dr dz \right] \quad (12)$$

where K_I is the SIF; E is the elastic modulus; μ is Poisson's ratio; w is the strain energy density; a is the distance from the fracture tip to the axis of symmetry; Γ , l^+ and l^- are the integral contours; D is the integral area; n_j is the unit vector in the j-direction; u_i is the actual displacement in the i-direction, which equals $u'_i + u''_i$; f_i is the body force of the rock, which equals f_i'' ; and σ_{ij} is the actual stress.

3. Model validation

In this section, the reliability of the model is verified by comparing the numerical predictions with analytic results of specific examples. The calculation domain is the cylindrical body shown in figure 1.

There is a concentric horizontal well with a radius of r_w in the domain. The radial fracture extends from the middle of the inner surface. The specific parameters are listed in table 1.

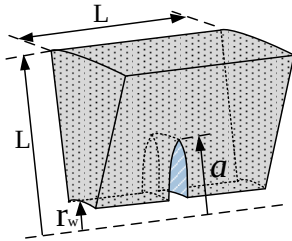


Figure 1. Schematic diagram of the calculation domain.

Table 1. Specific parameters of the model.

Fluid density (kg/m ³)	982.6
Elastic modulus (GPa)	35.0
Fluid viscosity (mPa·s)	0.456
Poisson's ratio	0.22
Well radius (m)	0.07
Injection (m ³ /min)	0.5
Fracture toughness (MPa·m ^{1/2})	1.5

3.1. Comparison with the radial fracture model

The simulation results of the radial fracture propagation are compared with the analytic solution. It is assumed that: (a) the rock deformation is quasi-static; (b) the lag of the fluid is neglected; (c) the fluid leak-off is neglected.

Since the fracture width tends to zero near the fracture tip, the pressure drop primarily occurs in this area. Therefore, the assumption of a uniform pressure P along the fracture is reasonable. When the fracture is in a critical state, the SIF is equal to the fracture toughness. Using the fracture volume and the SIF analytic forms in infinite space [19], the relationship between the fracture radius and time is obtained as follows:

$$\frac{8\sqrt{\pi}K_{IC}(1-\mu^2)}{3EQ}a^{1/2}(a^2-r_w^2)=t \quad (13)$$

where K_{IC} is the fracture toughness, a is the fracture radius, r_w is the well radius, and t is time.

The simulation results are shown in figure 2. Results indicate that the numerical results will tend to the analytic solution as the calculation domain increases. However, when the size of the calculating domain is insufficiently large, there is significant difference between the numerical result and the analytic solution. This indicates that the influence of the outer boundary condition must be taken into consideration.

3.2. Influence of the solid boundary conditions

The influence of the boundary conditions on the SIF are studied in this example. The fracture radius is 20m. The stress applied on the fracture surface is 15MPa. Other parameters are listed in table 1. The outer boundary conditions are set as zero stress and zero displacement, respectively.

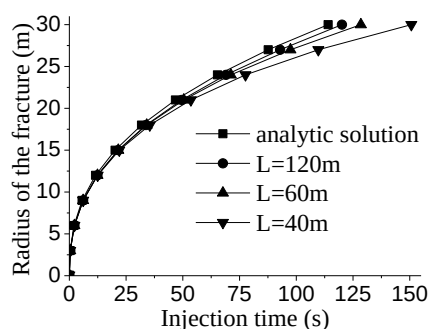


Figure 2. Comparison of the proposed model with the radial fracture model.

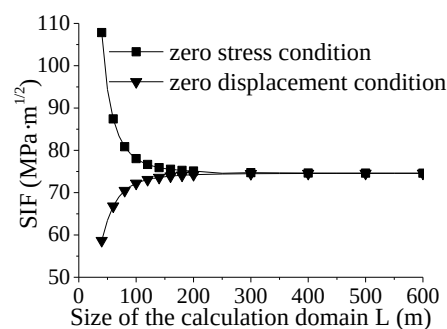


Figure 3. Influence of the solid boundary conditions on the SIF.

The results are presented in figure 3, which indicates that the SIF under the two different conditions tends to the same value as the calculation domain increases. However, when the size of the calculating domain is insufficiently large, the SIF under the zero stress condition is always greater than that under the zero displacement condition. Moreover, the difference will increase significantly when the size of the calculation domain decreases.

4. Application

The influence of the fracturing equipment parameters on the fracturing process is analyzed in this section. The calculation domain is shown in figure 1. The radius of the outer boundary is 300m and the axial thickness is 200m. The matrix permeability is 2 mD, the Biot's constant is 0.89, and the other property parameters are listed in table 1. The fluid pressure exerted on the axial outer boundary is 14.06MPa. The radial outer boundary is assumed to be impermeable. The in-situ stresses in the axial and the radial directions are assumed to be -12 and -22 MPa, respectively. The simulation results are shown in figure 4.

Figure 4(a) and figure 4(b) show the variations of the SIF and the pump power with the fracture radius, respectively, when the well pressure is fixed. This indicates that the SIF increases monotonically as the fracture radius increases. However, to maintain a fixed well pressure, the necessary pump power will increase rapidly as the fracture radius increases. Under the limitation of the maximum output power of the fracturing equipment, the fracture will cease to extend in some radius.

Figure 4(c) and figure 4(d) show the variations in the fracture radius and the fracture width at the wellbore with the well pressure, respectively, when the pump power is fixed. Results indicate that low well pressure corresponds to long and narrow fractures, while high well pressure corresponds to short and wide fractures.

This numerical example indicates that the working pressure and the pump power will influence the fracturing process significantly.

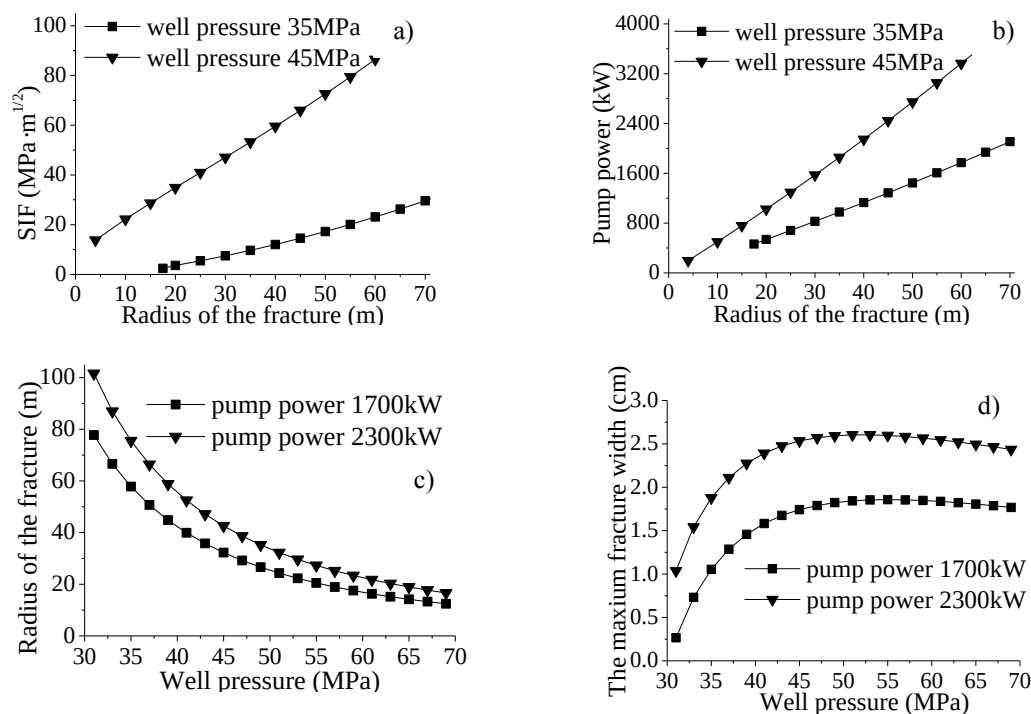


Figure 4. Influence of fracturing equipment parameters on the fracturing process.

5. Conclusions

A hydraulic fracturing model which combines the discrete fracture model and the generalized J integral has been presented. Compared to traditional models, the proposed model calculates the fluid flux across the fracture surface by avoiding the use of the empirical “fluid leak-off coefficient”. In addition, by using the generalized J integral to calculate the SIF, both the assumption of infinite space and the numerical error resulting from the stress singularity are avoided. These ideas can improve the accuracy of the hydraulic fracturing model effectively, and are thus practically important for the optimization of fracturing design.

Several examples have been provided in this paper. The primary conclusions are as follows: (a) the simulation result agrees well with the analytic solution. Results indicate the reliability of the proposed model. (b) The model can be used to describe the relationship between the geometry of the fracture and the fracturing equipment parameters. The numerical results indicate that the working pressure and the pump power will significantly influence the fracturing process.

The proposed model has the ability to accurately and effectively simulate the model I fracture propagation, and has practical value in fracturing design. In further research, the model can be extended to simulate mixed model fracture propagation by introducing mixed model stress intensity factors, thus providing a much wider range of applications.

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