

Dynamics of Agent-Based Growing Network

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Abstract. In this work we describe the dynamics of growing network using an agent-based model. We restrict our analysis for the case of a closed spatial boundary and apply certain rules that must be followed by each agent. We show the emergence of an outbreak in the number of agents and connectivity degree before reaching a saturated state. Using the concept of Gibbs-Network entropy we show that the saturated connectivity entropy always converge to the same constant independent of the initial agent number and boundary dimension.

1. Introduction

The dynamics of growing network has been an attracting topic for investigation due to its importance for explaining many phenomena from natural to social complex systems. It is a well known fact that any object/entity interaction and relation phenomena can be described as graph or network. In reality, some network tends to change time, either in number of vertices or number of vertices edge respectively. Their changes tend to continually grow, such as the World Wide Web and social network. This kind of network called growing network. Some research has been conducted on growing network, such as its topology and properties, as growing network more likely evolve to complex network [1]. Many complex networks have a degree distribution $P(k)$ that follows a power law, whereas some other network exhibit exponential distribution [2]. Moreover, in order to connect two indirect connected nodes, typically a few nodes needs to be passed (small-world) and their shortest path length is small [3].

The dynamics of growing network has been an attracting topic for investigation due to its importance for explaining many phenomena from natural to social complex systems [4]. In this paper, we created growing network simulation based on agent interaction which obeys certain rules. Later, we investigated our simulated network by Gibbs-Network-Entropy which adopted from the concept of Gibbs's Entropy in statistical physics [5], as follows:

$$S = -\sum_k p_k \log p_k$$

which p_k denotes the probability of an agent to be in a degree of connectivity k .

Our simulation rules govern the interaction between agents. Initially some agents are placed randomly on an element in the matrix. Later, each agents evolve simultaneously. If there are two agents that are not connected to each other, both agents will be connected each other and new



agents will be spawned between them and connected to both agents respectively. Also there is restriction which limits each agent only interacts with one agent each iteration.

2. Algorithm

In our model, we used three main matrices, which are Identity (abbreviated as IDEN), Display (DISP), and Edge (EDG). IDEN is $n \times n$ sized square matrix which any elements in the matrix contains integer i , which $i > 0$ defined that the element is occupied by an agent i and it should be unique, whereas $i = 0$ defined that the element is unoccupied. DISP is representation of degree of each agent, which its element position references degree of agent with same position. Moreover, EDG is $n^2 \times n^2$ sized adjacency matrix which defined network of agent interaction.

Simulation runs with integer t as iteration counter (as simulation time). Initially some agent will be placed randomly in IDEN elements. Within one iteration, to simulate agents' random motion, elements value of IDEN will be swapped with its adjacent elements randomly or stay not swapped. Agent interaction scanning performed after all agents' motion processed, which illustrated in Figure 1. First in Figure 1(a), scanning focused around IDEN(2,2). There are 2 unconnected agents in shaded area. Based on interaction rule, a randomly placed new agent will appear in these shaded area, named Agent 4 (Figure 1(b)). We continue the scan to the next cell (Figure 1(c)). In Figure 1(d) and Figure 1(e), any interacted agent (Agent 4) cannot interact with her un-interacted agent (Agent 3) since within one iteration, Agent 2 and Agent 4 has interacted with other agent (Agent 1). Later, in Figure 1(f) each agent's degree of connectivity will be described as DISP matrix, which each element in DISP with same coordinate as element in IDEN represent its degree of connectivity.

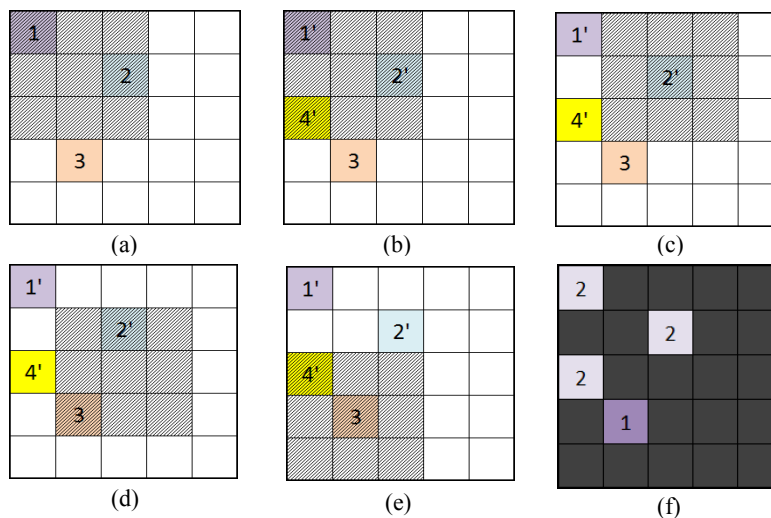


Figure 1. Agent interaction scheme in IDEN matrix as depicted in Figure 1(a) to Figure 1(e), and DISP matrix in Figure 1(f).

3. Gibbs-Network-Entropy of The Simulated Network

We consider an initial condition with 15 agents at random location inside cells as shown in Fig. 2(a). Each agent walks randomly until two of them, according to above mentioned generate a newborn agent. It is observed that at certain critical condition occur emergent outbreak explosion by rapidly increasing amount of agent as depicted in Fig. 2(b) and Fig. 3(c). The growth is becoming saturated at longer time as demonstrated in Fig. 2(d).

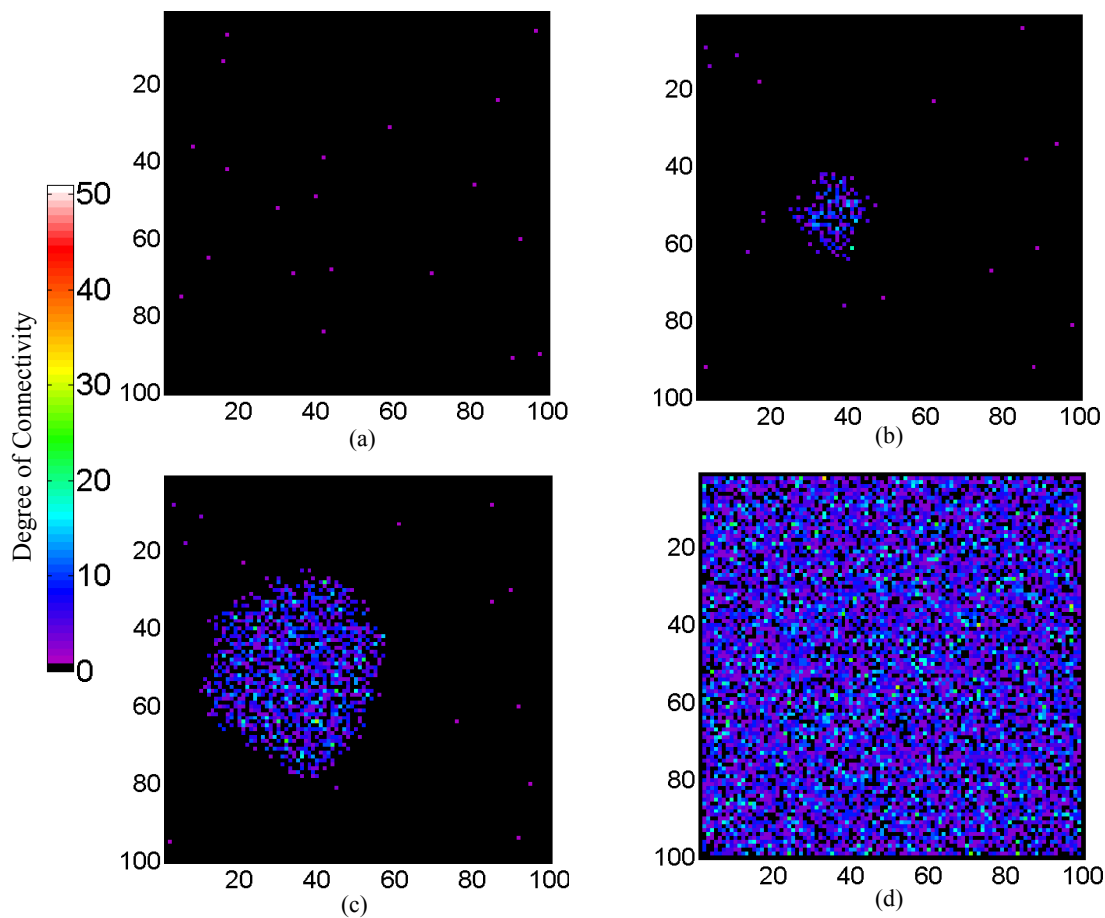


Figure 2. (a) Initial condition ($t = 0$), (b) emergent of “outbreak” ($t = 168$), (c) outbreak of agents ($t = 198$) (d) saturated condition ($t = 420$)

To explain this phenomenon, we plot in Fig. 3(a) and 3(c) the number of growing agents (N) and the associated Gibbs-Network-Entropy, respectively. It is shown in the Fig. 3(a), as expected that the amount of agents grows exponentially shortly after emergence of “outbreak” and become saturated at longer time. The corresponding Gibbs-Network-Entropy exhibits fluctuating characteristic whilst at the saturated state it become relatively steady. This phenomenon indicates that the network tends to be self-organized although visually it looks like a random pattern.

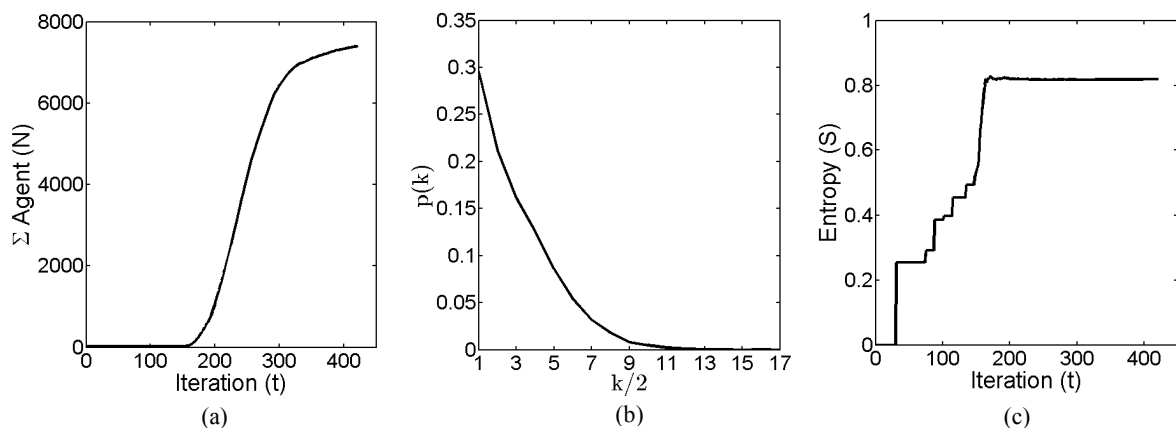


Figure 3. (a) Number of agent (b) degree of connectivity probability and (c) entropy

An interesting feature that occurs from the simulation can be seen in Fig. 3c which is the emergence of a stable total entropy at around 0.8. This number always occur from the same sum of individual entropies related to each degree of connectivity regardless of the choice of initial agent position and amount of initial agent as long as the computational area is large and given enough time to reach equilibrium. Although it is not fully clear, we believe at present that this feature might arise from the interaction rule and topology of the mesh. Further work is required (e.g. choosing different mesh such as triangular or hexagonal) to investigate this emergence.

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