The effects of the coordination on the fragmentation of a single grain

Salami Y., Dano C., Hicher P.-Y., Colombo G. and Denain P.

Research Institute in Civil and Mechanical Engineering (GeM, UMR CNRS 6183), École Centrale Nantes, Nantes, France.

Email: younes.salami@ec-nantes.fr

Abstract. The main objective of this study is to understand the effects of the coordination number on the behavior of a single grain, before and during fracture. A new apparatus is designed, with the purpose of conducting multipoint crushing tests, and to investigate the effects of the contacts number, type and position on the failure of a particle. Some of the tests were monitored using digital image correlation (DIC), in order to get an in-depth view into the mechanics of fracture. The fragmentation of the grains was studied, and the cracks were shown to follow specific paths. The failure forces of multiple contact configurations were compared to show the effect of the number and position of contact forces on the primary and secondary cracks. It was shown that the position and magnitude of the contact forces plays a significant role in the fragmentation of the grain.

1. Introduction

Investigating particle crushing in soils is crucial in understanding its behavior, because, soil behavior could be explained from a grain level analysis, where the grain interactions aren't limited to sliding and rotating, but also to fragmenting. It is widely accepted that the probability of grain breakage is a combination of the distribution of local forces on the grains as well as the distribution of the particle strength. The larger particles are usually weaker, but can see their survival probability significantly increased if their coordination number is high. Smaller particles however, have a higher resistance to crushing, but their coordination number is limited by their size. In addition, the strength of particles, generally represented by a Weibull distribution as done by McDowell & Amon [2] is usually identified through point load tests or splitting tests between stiff plates, inducing a diametral compression of the tested particle [1]. This methodology is not necessarily representative of the map of contacts into a stressed granular packing. Finally, the confining effect provided by the contacts on the particle crushing force is only seldom taken into account whereas many authors have assumed its existence (Auvinet & Marsal, [2]; Tsoungui, [3]). Surprisingly, no experimental systematic investigation has been conducted to precise this.

Numerically, the multi-axial compression of round particles was investigated from a fracture mechanics point of view by Tsoungui *et al.* [3]–[5], Sukumaran *et al.* [6] and Ben-nun & Einav [7]. They proposed two failure criteria for a two dimensional grain subjected to a random distribution of contact forces. In Tsoungui's criterion, the complex distribution of forces is replaced by an equivalent distribution (see Figure 1-(a)) of orthogonal forces F_{max} and F_{min}, orientated by the tilt angle Φ, in the form of an inclined cross (Figure 1 –(b)). This model considers that fracture occurs according to a tensile splitting mode (mode I), in a manner similar to that of a grain between two diametrically opposed forces. A series of finite element simulations were then performed on disks of radius R under diametrical compression, and disks subjected to multiple contact forces. These simulations confirmed the idea that the crack originates from the center of the disk where the tensile stresses are concentrated, before propagating through the continuum. The stresses perpendicular to the loading axis near the grain center σ_{xx}^0 for the two simulations (for pure diametral compression and for the equivalent set of contact forces F_{min} and F_{max}) can become positive (tension), while the other stresses are always in compression for both cases, and cannot possibly produce a mode I fracture. σ_{xx}^0 (positive for tension and negative for compression) is expressed by (1) for the diametric compression, and (2) for the equivalent set of contact forces:

$$\sigma_{xx}^0 = \sigma_{xx}(x \to 0, y \to 0) \approx \frac{F}{\pi R} \tag{1}$$

$$\sigma_{xx}^0 = \frac{F_{min} - 3F_{max}}{\pi R} \tag{2}$$

The stresses near the center of the grain appear to be independent from the contact and loading conditions. When the tensile stresses inside the disk σ_{xx}^0 exceed a critical value σ_c, assumed to be the same for every contact configuration, a central crack that moves along the direction of F_{max} causes the specimen to break. The failure criterion is expressed in terms of the hydrostatic stress p and the shearing stress τ in Eq. (3):

$$\sigma_{xx}^0 = 2\tau - p \geq \sigma_c \tag{3}$$

This formula suggests that a grain subjected to a random distribution of contact forces has the highest probability of breakage when $2\tau \gg p$, meaning a force configuration close to a diametral compression. In the case when $2\tau \ll p$, the grain has a low probability of breakage since its stress state tends to a hydrostatic pressure.

Ben-Nun & Einav [7] proposed an alternative criterion, supposed to overcome the principal limitation of Tsoungui's model: the inability of a grain to crush under isotropic compression, or when the set of forces applied is symmetrical. This criterion supposes that the cracks result from an in-plane shear fracture mode (mode II) (Figure 1–(c)). The average of the normal components of the contact forces is compared to a critical threshold force. This critical force is based on Sukumaran's DEM simulations [6], and the resulting formula for the failure force is (4):

$$F_c = d\sigma_c f_w f_D f_{CN} \tag{4}$$

σ_c being the tensile stress at failure of a grain of size d, f_w, f_D and f_{CN} are reduction factors taking account respectively the variability in strength, curvature and coordination number. The first factor f_w is deduced from a statistical law (Weibull [8]) describing the distribution of the flaws or imperfections directly affecting the strength. The two other factors are defined empirically, and the one accounting for the effect of the coordination number C is expressed as a function of the crushed grain size d and the average size of the contacting grains D (5):

$$f_{CN} = (C-1)\exp\left(\left(\frac{D}{d}\right)\frac{(C-2)(C-3)}{4C}\right) \tag{5}$$

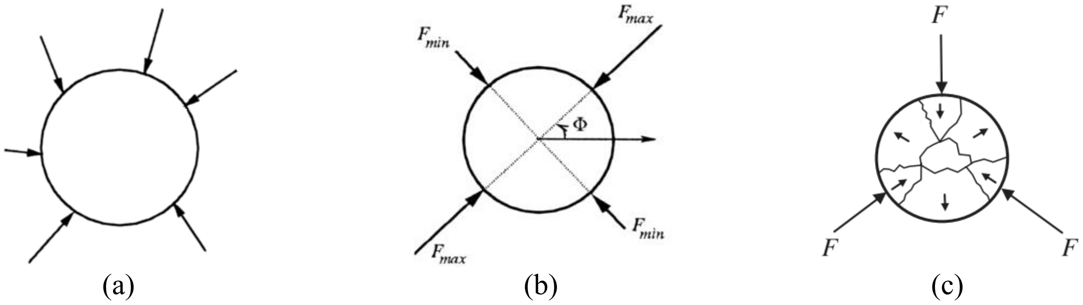


Figure 1. The two failure criteria for a grain under an arbitrary set of contact forces (a), according to Tsoungui *et al.* (b), and to Ben-Nun & Einav (c). (after[3], [7]).

2. The experimental procedure

A new device, used to simulate the effects of the surrounding particles on the fragmentation behavior of a specimen, was designed. It is worth noting that the results presented here are only preliminary since further improvements are planned on the device. The apparatus consists of a round metal mounting frame, equipped with multiple clamping bars intended to lock the displacements of a cylindrical specimen from various points on its lateral surface. The bars can be moved freely around the frame, and then be locked in a desired position. This apparatus is then placed on a loading frame, and the specimen is crushed under a particular configuration of contacts. The contacts imposed on the specimen can either be linear or along a surface. The loading frame has a capacity of 50 kN, and the force and displacements are measured using a load cell and a displacement transducer, with a resolution of 0.06 N and 0.01 mm respectively. The results presented here correspond to a displacement velocity of 0.6 mm/min.

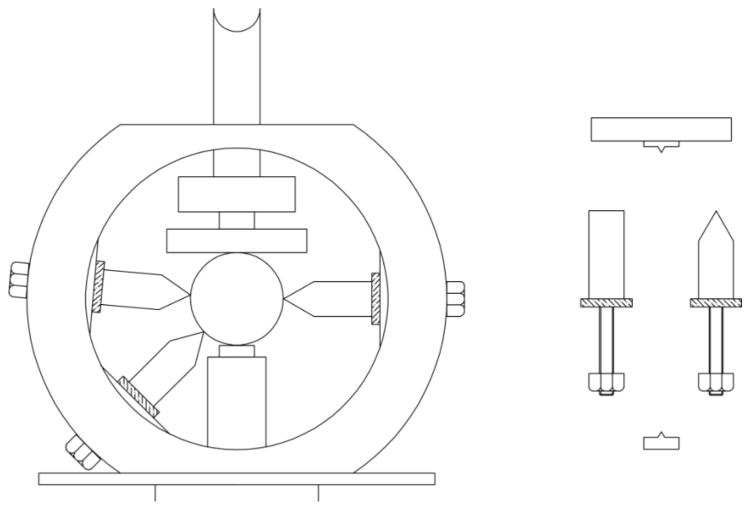


Figure 2. Schematic view of the apparatus and the clamping bars.

The tests are conducted on cylindrical disks of standardized mortar with a diameter of 52 mm and a width of 22 mm, prepared in accordance with the French standard NF EN 196 –1 in batches of 15 units. A CEM1 52.5R cement and a Hostun sand HN 0.6/1.6 are used for this formulation. The water/cement ratio is 0.5. The disks are then cured for 7 days in a humidity chamber (100% relative humidity and a temperature of 18°C). The preparation mode was perfected during the first weeks, as we tried to minimize all sources of discrepancy by controlling every step of the process. From the mixture of the paste, to the vibration of the specimens, to the storage conditions, everything was handled with extreme care, to ensure a standard deviation less than 0.5 for the diametric compression tests, and less than 0.6 for the other configurations.

The cylindrical (disk-like) shape was chosen to represent a two dimensional system, and to eventually compare the results with the theoretical 2D crushing models. Besides, this configuration provides us with an unobstructed view into the specimen's base as the contacts are along the lateral surface, allowing direct access to the surface strains through Digital Image Correlation.

3. Digital image correlation

Digital image correlation (DIC) is an optical experimental technique allowing the measurement of surface displacements in the case of a 2 dimensional analysis. This technique has been widely validated theoretically and experimentally, and has recently appeared in many research papers in material science as a robust and accurate method. The technique is quite easy to carry out as it doesn't require any specific technology other than a digital camera and DIC processing software. The strains are calculated from the computed displacements using the continuum mechanics theory.

The digital camera used has a resolution of 1040x1392 pixels and delivers up to 256 levels of gray output. The camera is programmed to take one picture each second (maximum capability), and is placed at 50 cm from the specimen (see Figure 3). The data is stored on a computer, and the image correlation is performed later using the software VIC2D. The configurations tested are: 5 Brazilian tests (diametral compression), 5 tests in the 4-90 configuration, 5 in the 6-45 configuration, and 2 tests in a random configuration (See Figure 4).

Figure 3. The camera setting for the DIC

4. Results

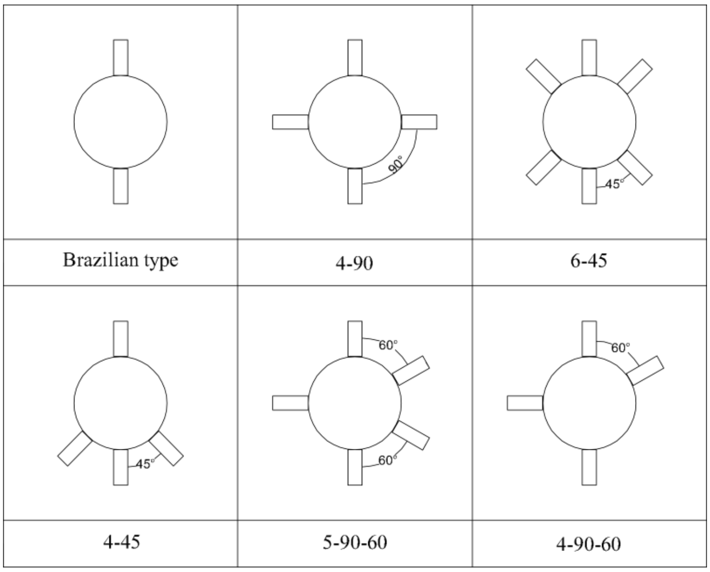
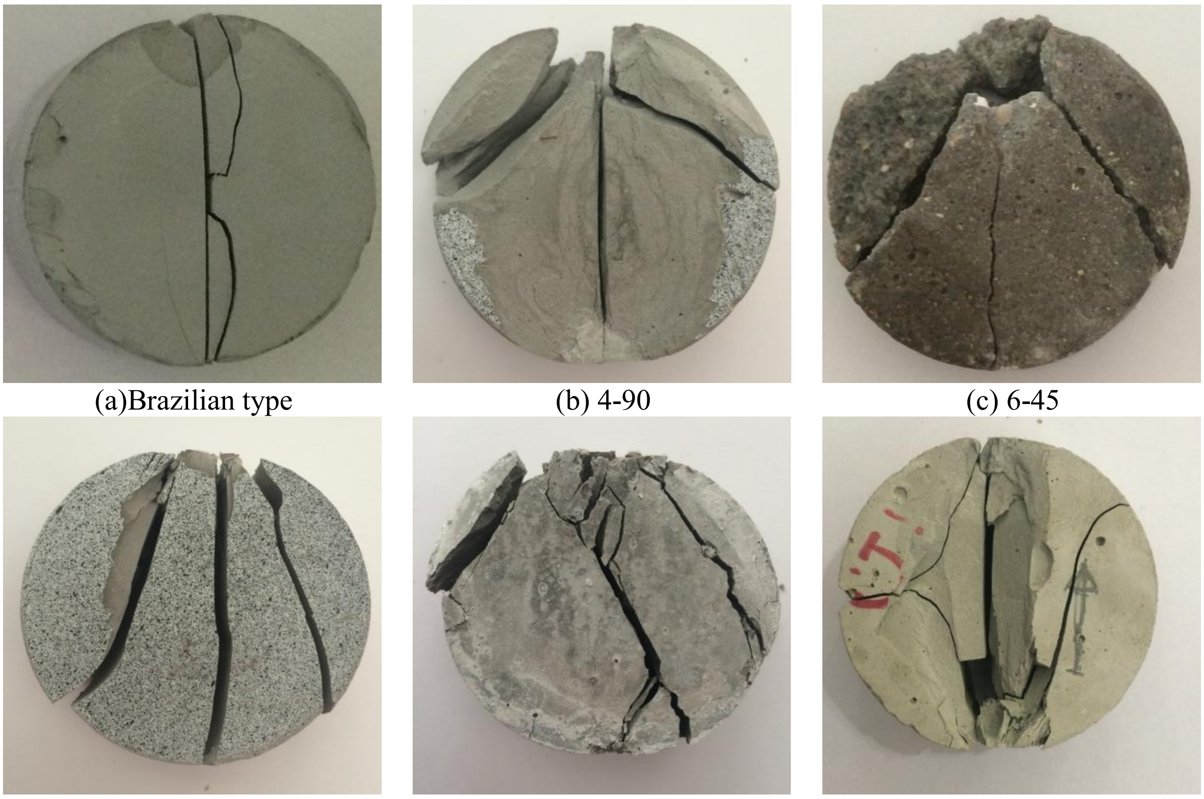


Figure 4. The different loading configurations tested

4.1. The fragmentation of the disk

For all of the tests conducted, a central crack appears along the loading axis. Other cracks develop later, following specific paths corresponding to each configuration, and apparently originating from the contact area. Since the displacement imposed is along the diametric axis, a volume fracture model [5] should explain the appearance of the central crack, although the digital image correlation suggests otherwise. The secondary cracks could be explained using a contact fracture model.

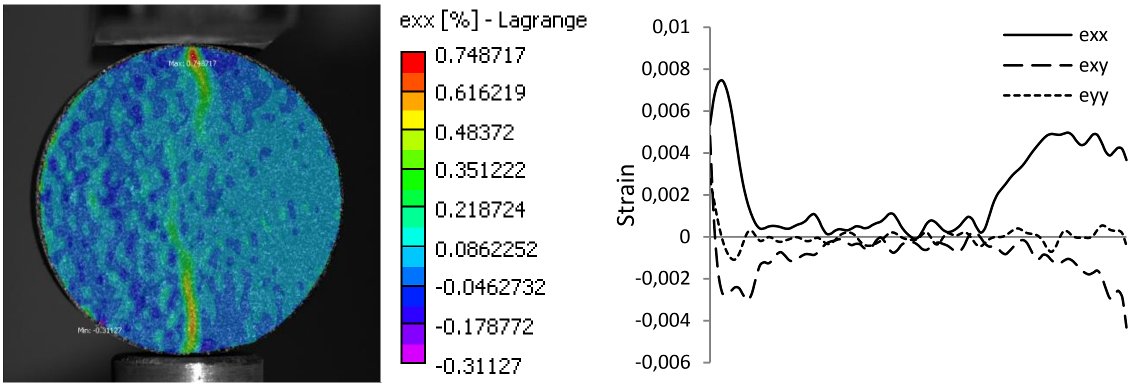


(a)Brazilian type (b) 4-90 (c) 6-45

(d) 4-45 (e) 5-90-60 (f) 4-90-60

Figure 5. Typical fragments resulting from the crushing of cement disks in the configurations studied Figure 4

The predominance of the effect of the two contacts along the vertical axis, since they are the ones carrying the load, leads to the appearance of the loading axis crack first. This was explained in the literature by the formation of a traction zone far away from the contacts (generally near the center of the disk) and ultimately splitting the specimen in half. The digital image correlation of a diametral compression test shows that the extension strain is highly concentrated near the contacts (Figure 6), and this traction zone advances from the top and the bottom, before merging in the form of a first diametral crack. Because of the small area of contact, both the horizontal and vertical compressive strains are highly concentrated near the contacts. Wang & Xing [9] determined a minimum loading angle for the crack initiation to be at the center of the cylindrical specimen, and proposed using a specimen with flattened edges to better reproduce the theoretical behavior. The contact area is then large enough to spread the load along a greater area, and the flat contact geometry is stable and prevents the specimen from moving during the test. It was also suggested that a longer duration of the test could result in a flattening of the contact area, and might lead to a tensile zone near the center of the disk.



(a) (b)

Figure 6. (a) Distribution of the strains in the x direction prior to fracture (b) the strains along a vertical line

In multipoint configurations, secondary cracks result from another fracture mechanism. The cracks appear to start from one of the secondary contacts before reaching one of the two contacts along the loading axis. These cracks appear to be caused by a shear fracture mode II, as the shear strains are at their lowest negative value along this second crack (see Figure 7). The high shear strains along the first diametric crack are simply caused by the friction of the two initial fragments

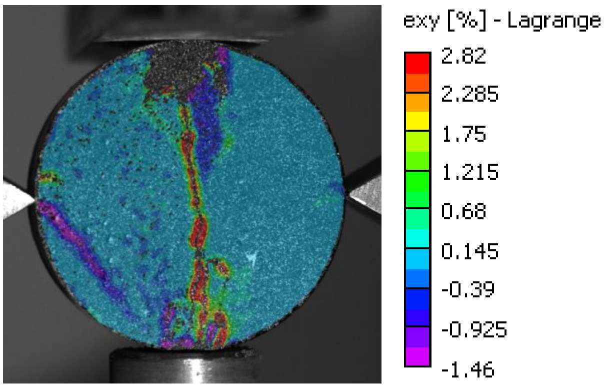


Figure 7. Distribution of the shear strains along the surface of the disk

To sum up, in almost all of the tests conducted in the different configurations in Figure 4, two initial cracks appear near the contacts, and propagate along the diametral loading axis before joining and splitting the specimen in half. Secondly, one or more cracks start from near the lateral contacts, and reach the top or bottom contact. These cracks seem to follow a shearing fracture mode. In addition, a crack that starts from one of the lateral contacts will most likely join the farthest loading contact.

4.2. The effects of the contacts number

For all the tests performed, the failure forces corresponding to the first and second crack appearances, marked by a sudden drop in the force, were recorded. The mean value for each configuration is presented in Table 1. We only considered the tests with surface contacts. The first column F_1 corresponds to the force for the first crack (the one along the loading axis), and F_2 to the force for a second crack. We chose not to take into account other cracks that appear later in the loading, as the solid is highly damaged. It should be noted that we were able to keep the standard deviation below 0.6 kN for all of the configurations. From a first look into the table, we can immediately see that a higher coordination number means a higher resistance to crushing. Another important consideration is that whenever a set of two contact forces are diametrically opposed, the resistance is slightly higher. This is apparent in the comparison between the 4-90 and the 4-90-60 configurations. This raises the issue of the importance of the contact position on the crushing behavior of a grain. Until now, only the number of contacts was thought to affect the critical stress of a grain.

The lateral contacts allow the grain to sustain greater forces until they finally fail.

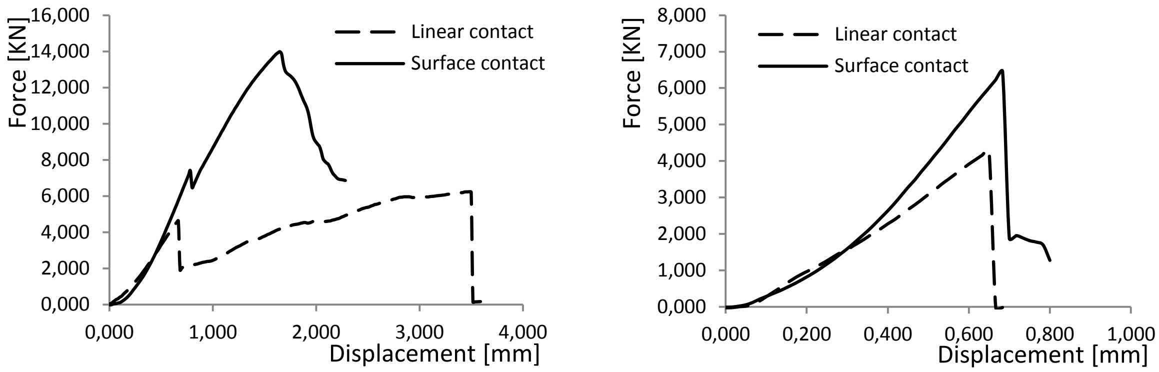
Table 1. Mean value of the critical force for the first two cracks.

Contact configuration	**F_1(kN)**	**F_2(kN)**
Brazilian	5.95	-
4-90-60	6.45	9.70
4-90	7.12	13.70
4-45	7.28	9.88
5-90-60	8.84	13.75
Random (5contacts)	8.44	12.76

The secondary cracks appear to be equally affected by the contacts positions.

4.3. The effects of the contact type

The effect of the contact type is investigated through a series of tests on two sample groups composed of 15 identical mortar disks each. The tests on the first group consist of 3 diametral compression tests (Brazilian type) and 12 tests in the 4-90 configuration (Figure 4), with a surface contact type for both the clamping bars and the compression axis contacts. The same tests were performed on the second group, the only difference being the contact type, which is now linear. Typical force-displacement curves are presented in Figure 8:



(a) (b)

Figure 8. Typical force-displacement curves for (a) 4x90 configuration and for (b) Brazilian type crushing tests

For each of the diametral compression type tests, a mean value of the critical force for the linear contacts is 4.28kN, and 6.25kN for the surface contacts. This force is associated with a diametral crack originating near the contacts along the loading axis, and propagating diametrically through the specimen, breaking it in half. Both the contact types are non-conforming, so the stresses are highly concentrated near the contacts independently from the shape of the body, with an even higher concentration in the case of the linear contacts than the surface ones. This generates localized strains around the contacts favoring the crushing of the grain at a lower stresses.

In the 4-90 configurations, two force peaks are observed for both contact types, the first corresponding to the diametral crack, and the second to the secondary cracks. We can see that the first peaks in the 4-90 configurations are slightly higher than the ones in the Brazilian configuration. The failure post second peak is sudden in the case of the linear contact, but more ductile like in the case of the surface contact. The surface contact provides a higher adherence area blocking the specimen from moving after failure, explaining the post peak behavior of the surface contact: the two fragments are held together by the surface contacts, and the specimen continues to carry the load applied, causing multiple little cracks to appear, and leading to an increasing damage, especially near the contacts. In the case of a linear contact, the tip of the contact penetrates in the crack after failure, leading to a quick crack opening, and a sudden failure.

4.4. The effects of the contact force
A preliminary investigation on the effect of the contact force was conducted through 12 compression tests in the 4-90 configuration. The lateral contacts were tightened to simulate the effect of a pre-stress. No force was measured, as this experiment goal was just to get an oversight of the problem, but a specific tightening of the clamping bolts ensured reasonably comparable conditions for the tests.

The first crack along the loading diameter appears, but no other crack followed, instead, many little cracks developed near the contacts, causing the specimen to break into a multitude of small fragments. A ductile-like behavior like the one in Figure 8 (a) is then observed in the force displacement curves. The mean critical forces associated with the crack in this case is 11.36 kN, which is higher than the 7.12 kN at which a 4-90 specimen without any pre-stress breaks.

5. Conclusion
The importance of the coordination number was investigated through a series of crushing tests on mortar cylinders, with varying contacts numbers and position. 6 configurations of contacts positions were tested, and monitored using DIC analysis. It was found that each configuration results in a specific set of fragments. Digital image correlation allowed us to track each crack and determine its point of initiation, and its defined path. It was confirmed that the number of contacts plays a determining role in the crushing of a single grain, as the grains with the higher coordination number are harder to break. Our new device allowed us to control the contact's position, and to show the importance of the contact's position on the failure of the grain.

References

[1] J. C. Jaeger, "Failure of Rocks Under Tensile Conditions," *Int. J. Rock Mech. Min. Sci.*, vol. 4, pp. 219–227, 1967.

[2] G. Auvinet and R. Marsal, "Statistical model of grain breakage," *Proceedings, Fifth Pan-American Conf. Soil Found. Eng. Buenos Aires*, vol. 1, pp. 193–204, 1975.

[3] O. Tsoungui, D. Vallet, and J. C. Charmet, "Numerical model of crushing of grains inside two-dimensional granular materials," *Powder Technol.*, vol. 105, pp. 190–198, 1999.

[4] O. Tsoungui, D. Vallet, and J.-C. Charmet, "Use of contact area trace to study the force distributions inside 2D granular systems," *Granul. Matter*, vol. 1, pp. 65–69, 1998.

[5] O. Tsoungui, D. Vallet, J.-C. Charmet, and S. Roux, "Size effects in single grain fragmentation," *Granul. Matter*, vol. 2, pp. 19–27, 1999.

[6] B. Sukumaran, I. Einav, and Arcady Dyskin, “Qualitative Assessment of the Influence of Coordination Number on Crushing Strength using DEM,” *5th World Congr. Part. Technol.*, pp. 1–8, 2006.

[7] O. Ben-Nun and I. Einav, “The role of self-organization during confined comminution of granular materials.,” *Philos. Trans. A. Math. Phys. Eng. Sci.*, vol. 368, pp. 231–247, 2010.

[8] W. Weibull, “A statistical distribution function of wide applicability,” *Journal of applied mechanics*, vol. 18. pp. 293–297, 1951.

[9] Q.-Z. Wang and L. Xing, “Determination of fracture toughness KIC by using the flattened Brazilian disk specimen for rocks,” *Eng. Fract. Mech.*, vol. 64, no. 2, pp. 193–201, Sep. 1999.