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System reliability analysis of granular filter for protection against piping in dams

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Abstract. Granular filters are provided for the safety of water retaining structure for protection against piping failure. The phenomenon of piping triggers when the base soil to be protected starts migrating in the direction of seepage flow under the influence of seepage force. To protect base soil from migration, the voids in the filter media should be small enough but it should not also be too small to block smooth passage of seeping water. Fulfilling these two contradictory design requirements at the same time is a major concern for the successful performance of granular filter media. Since Terzaghi era, conventionally, particle size distribution (PSD) of granular filters is designed based on particle size distribution characteristics of the base soil to be protected. The design approach provides a range of D_{15f} value in which the PSD of granular filter media should fall and there exist infinite possibilities. Further, safety against the two critical design requirements cannot be ensured. Although used successfully for many decades, the existing filter design guidelines are purely empirical in nature accompanied with experience and good engineering judgment. In the present study, analytical solutions for obtaining the factor of safety with respect to base soil particle migration and soil permeability consideration as proposed by the authors are first discussed. The solution takes into consideration the basic geotechnical properties of base soil and filter media as well as existing hydraulic conditions and provides a comprehensive solution to the granular filter design with ability to assess the stability in terms of factor of safety. Considering the fact that geotechnical properties are variable in nature, probabilistic analysis is further suggested to evaluate the system reliability of the filter media that may help in risk assessment and risk management for decision making.

1. Introduction

Seepage and piping is one of the major sources of concern for the safety of the dams [1, 2, and 3]. Piping is a phenomenon in which the hydraulic gradient at the exit point exceeds the critical hydraulic gradient value. The critical exit point is normally at the toe of the structure where seeping water flowing through foundation exits. Under the influence of seepage flow, soil at the exit point has the tendency to get eroded. Once eroded, the erosion continues and moves in the backward direction, ultimately, leading to the failure of the whole structure [4]. Conventionally, granular filters are provided to protect the dam from failure against piping. Figure 1(a) depicts typical piping failure in a concrete dam and Figure 1(b) shows provision of granular filter provided at the downstream end for protection against piping. Similarly, in earth dams, granular filters are provided as toe filter, blanket drains and (or) chimney drains as per design requirements as shown in Figures 1(c) & 1(d), respectively. Detailed discussions on the topic "granular filters for dams" are available in FEMA [5] document. It is well understood that the critical location for the occurrence of piping in any water retaining structure is at the toe of the structure where the seeping water exits and the flow is in the vertically upward direction resulting in the reduction of vertical effective stress due to upward seepage and a condition very much favourable for development of piping.

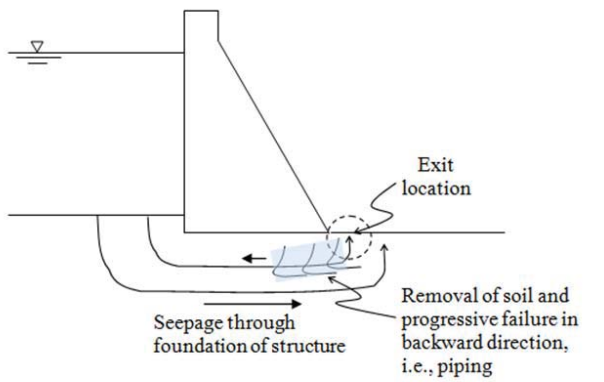

Figure 1(a) Seepage through the foundation of a water retaining structure and phenomenon of piping in dams

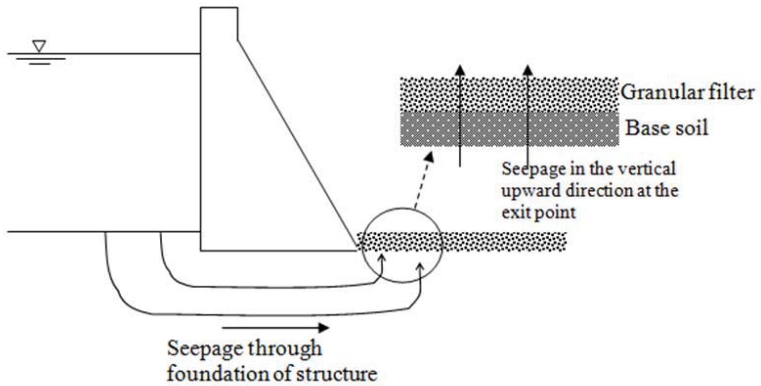

Figure 1(b) Provision of granular filters for protection of dam against piping

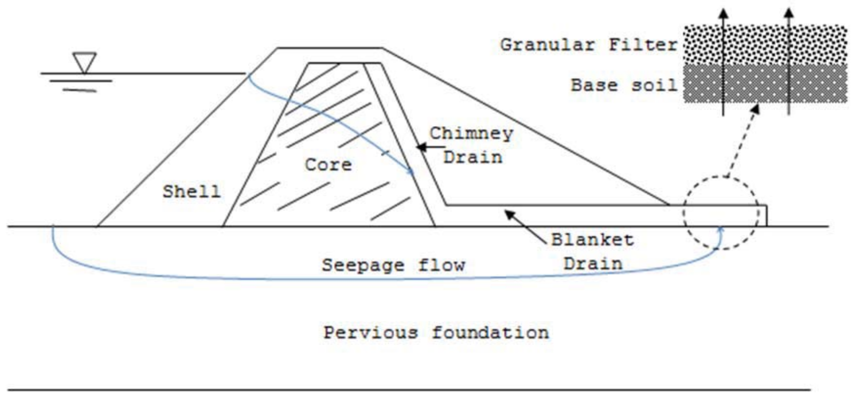

Figure 1(c) Provision of granular filters (chimney drain and blanket drain) for protection of earth dam

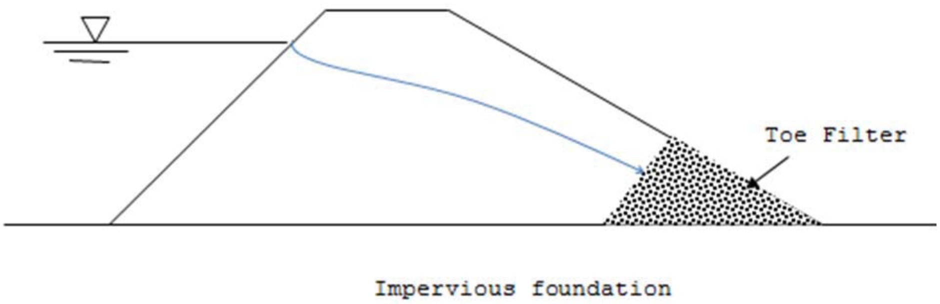

Figure 1(d) Provision of toe (granular) filters for protection of earth dam

Classical examples of seepage and piping failures in water retaining structures are reported in literature [2, 6]. Few examples are failure of right abutment of Fontenelle Dam (1965), Teton dam failure (1976) and Cowlitz County dam failure (2002).

2. Granular filter Design criteria

Granular filters protect the base soil from migration or being washed away under the influence of seepage forces and at the same time it allows the smooth passage of seeping water to avoid development of excess pore water pressure and loss of shear strength in the base soil., The size of voids in the filter media should be small enough to fulfil the first requirement, and, at the same time it should be large enough to fulfil the second requirement. Satisfying these two conflicting requirements at the same time is a challenging task for the design of granular filter. Terzaghi [7] suggested design criteria for obtaining the particle size distribution (PSD) characteristics of the granular filter media based on the PSD characteristics of the base soil. Later, several researchers [8, 9, 10, 11, 12, 13, 14, 15, and 16] established filter design guidelines on the same line of thoughts depending on soil types, experimental procedure, experience and specific design requirements, such as, self healing and internal stability. In the existing filter design guidelines, consideration of geotechnical properties of the base soil as well as filter media and existing hydraulic conditions is lacking. A few theoretical studies on the soil transport phenomenon [10, 17] help in understanding the physical mechanisms of the soil particle transport and retention. Recent works into piping phenomenon focused on the development of predictive mathematical models for particle transport and filter clogging [18, 19, 20, 21, 22 and 23] conducted study on the estimation of the time of development of piping for prediction purposes. Researchers like [24, 25] provided excellent literature review on critical appraisal of granular filter design. Terzaghi [7] in his original work provided the following criteria for the PSD design of granular filters:

Soil retention criterion

$$\frac{D_{15f}}{D_{85b}} < 4.0 \tag{1}$$

Permeability criterion

$$\frac{D_{15f}}{D_{15b}} > 4.0 \tag{2}$$

where, D_{15f} and D_{15b} are the diameters of particles at 15% passing for filter material and base soil, respectively; D_{85b} is the diameter of particles at 85% passing for base soil. The design criteria provide a range of D_{15f} values and there exists an infinite number of options for the choice of appropriate particle size distribution characteristics of the filter media. Although, these guidelines are well established and have been successfully used through decades of experiences, the safety of the filter media with respect to the two design criteria cannot be quantified. Authors [26, 27, and 28] proposed the following analytical solutions for quantifying the factor of safety with respect to soil particle migration and permeability criteria in granular filter design:

Soil retention criterion

$$FS_m = \frac{\left[\frac{d_s^3}{6} + \frac{1}{2\sqrt{2}} d_s^3 \tan\phi'\right](G_s - 1)}{\left[\frac{3}{32} d_s n_f D_{of}^2 + \frac{1}{4} d_s^3\right] i} \tag{3}$$

Permeability criterion

$$FS_p = \frac{(G_s - 1)}{mpH_t i}\left[\left(\frac{D_{15f}}{D_{15b}}\right)^2 \frac{(mp-1)^3}{m(p-1)^3} H_b + H_f\right] \tag{4}$$

Where,

$$m = \frac{(\gamma_d)_b}{(\gamma_d)_f} \tag{5}$$

$$p = \frac{G_s \gamma_w}{(\gamma_d)_b} \tag{6}$$

In the above equations, ϕ' is the effective angle of internal friction of the base soil, n_f is the porosity of filter media, D_{of} is the size of pore channels in the filter media with the assumption that the filter media can be represented as a bundle of tubes, d_s is the weighted average particles size of the base soil, H_t is the total thickness of base soil (H_b) and filter media (H_f), G_s is the specific gravity of the soil particles, $(\gamma_d)_b$ and $(\gamma_d)_f$ are the dry unit weights of the base soil and filter material, respectively, and i is the existing hydraulic gradient. It can be observed that the proposed analytical equations takes into consideration some of the basic geotechnical properties like effective angle of internal friction, unit weight, representative particle sizes, pore size, and existing hydraulic conditions and enable carrying out a stability assessment as well as for the design of granular filter media in terms of factor of safety.

3. Uncertainties Issues: Factor of safety vs. probabilistic approach

The input parameters required in equations (3) & (4) are obtained through various methods, such as, field or laboratory testing, correlation, or mathematical modelling. Soil being a natural material, it has inherent variation in its properties. These factors contribute to the uncertainty in the estimation of the soil input parameters. Conventionally, uncertainties are taken care by use of *factors of safety* in the design philosophy. In geotechnical design and practices, a factor of safety in the range of 1.5 to 3.0 is *generally* accepted [28]. Selection of appropriate values of *factor of safety* comes from past experience, good engineering judgment and the confidence level of the designer. It lacks rationality and there is no mathematical basis. Hence, a question that often arises in practice is to know "how safe is safe?" or to what extent the question of safety can be addressed adequately.

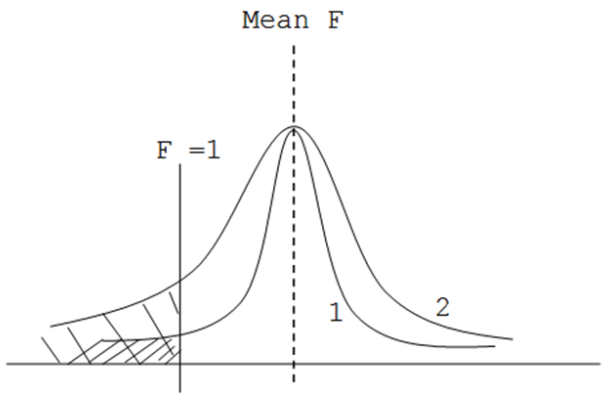

Figure 2 Comparison of two cases with same system response (mean factor of safety) but different failure probabilities or associated risks

Considering the fact that variation in the input parameters introduces variation in the output response, in the present study, the output response, i.e., stability of the system, is measured in terms *of factor of safety (F) and its variations*. The system fails when $F < 1.0$. Figure 2 compares the two cases in which the mean system response (i.e., mean F) is same but the extent of variation in F is different due to the extent of variation in the input parameters being different. The area below the *frequency curve* for F < 1.0 defines the probability of failure or the risk associated with the system. It can be observed that although, the mean system response is although same in the two cases, a higher risk (or probability of failure) is associated with case 2. Such rationality is never achieved in conventional factor of safety approaches and there is no means of identifying the factors for risk reduction and making the system more reliable. The extent of variation and its influence on the system response can be captured in a probabilistic framework. In the probabilistic approach, input parameters are, generally, assumed to be normally or log-normally distributed continuous random variables and parameters of distribution are related to unbiased estimate of mean and variance of the measured data set [29, 30].

Sample Mean (μ)

$$\mu_{FS} = \frac{1}{N}\sum_{i=1}^{N} FS_i \tag{7}$$

Variance (σ^2)

It is a measure of dispersion of data about the mean value. The square root of variance is defined as standard deviation (σ).

$$\sigma_{FS}^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(FS_i - \overline{FS}\right)^2 \tag{8}$$

The coefficient of variation (δ), which is defined as the ratio of sample standard deviation (σ) and the sample mean (μ), is commonly used to quantify geotechnical uncertainties. The advantage of using the coefficient of variation is that it is dimensionless and provides a better estimate of relative dispersion of data around mean. Where site-specific data are not available, typical values of coefficients of variation measured in soil properties are taken from published literature [31, 32, and 33]. In the probabilistic analysis, performance of the system is assessed in terms of an index called *reliability index* (β).USACE [34] made specific recommendations on target reliability indices (β) in geo-technical and infrastructure projects.

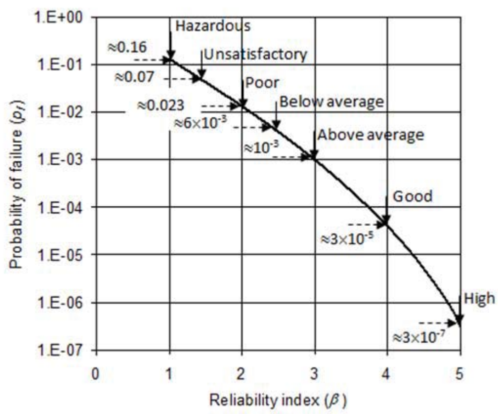

Figure 3. USACE (1997) guidelines for reliability index (β) and corresponding probability of failure (p_f) (adapted from Table B-1) (Phoon, 2004)

The suggested guidelines state that a reliability index (β) value of at least 3.0 is expected for an average performance of the system and 5.0 for excellent performance. There are many methods of estimation of reliability index (β) available in literature [30, 35, and 36].

3.1 System Reliability

The two conflicting design requirements for granular filters introduce multiple failure modes in the base soil- filter media system. Hence, occurrence of anyone of the failure mode will involve failure or non-performance of the granular filter media system. In the estimation of system reliability, evaluation of reliability of system (in terms of reliability indices, β) with respect to individual failure modes, i.e., component reliability is first obtained. A detailed discussion on system reliability approach is provided in [36, 37]. It can be noted that the two failure modes in granular filter are not statically independent as the analytical equations (3) and (4) share several input parameters. If the correlation coefficient between two failure modes is taken as (ρ), the system reliability can be evaluated. For correlated normal variates, Ditlevsen [38] suggested following bounds (equation 9) for joint probability of failure Pr[MP], which is an approximation as the exact solution is cumbersome to determine. Here, *M* (m) is used for particle migration and *P* (p) is used for permeability consideration.

$$max\,(p_m, p_p) \;\leq\; pr[MP] \;\leq p_m + p_p \tag{9}$$

Where

$$p_m = \Phi\,(-\beta_m)\Phi\left(-\frac{\beta_m - \rho\beta_p}{\sqrt{1-\rho^2}}\right) \tag{10}$$

$$p_p = \Phi\,(-\beta_p)\Phi\left(-\frac{\beta_p - \rho\beta_m}{\sqrt{1-\rho^2}}\right) \tag{11}$$

Where p_m and p_p are the probabilities of failure with respect to particle migration and permeability consideration, respectively. β_m and β_p are the reliability index values from particle migration and permeability consideration and these individual components of reliability indices are first evaluated.

3.2 Reliability analysis of individual component (β_i)

If the performance function g(x) is defined as g(x) = FS-1, where FS is factor of safety with respect to particle migration and permeability consideration (as the case may be), the reliability index (β) through First Order Reliability Method (FORM) for log-normally distributed FS is obtained from the following expression [30]

$$\beta = \frac{\ln\left[\left(\overline{FS}\right)\sqrt{\left(1+\delta_{FS}^2\right)}\right]}{\sqrt{\ln\left[\left(1+\delta_{FS}^2\right)\right]}} \tag{12}$$

where, $\overline{FS}$ is the mean factor of safety and δ_{FS} is the coefficient of variation in *FS* whose values are obtained through Monte Carlo Simulation (*MCS*). For MCS, N numbers of random data (sample) are generated for each set of input parameters that follows the characteristics of probability density function (*pdf*) defined for that input parameters. Factor of safety with respect to soil particle migration $(FS_m)_i$ and permeability $(FS_p)_i$ are evaluated with the help of equations (3) and (4), respectively for each set of the input parameters. It provides *N* number of data set for FS_m and FS_p and the mean and variance of factor of safety are then evaluated using equations (7) and (8). Reliability index values with respect to particle migration (β_m) and permeability criteria (β_p) are then estimated using equation (10). For system reliability evaluation, the sample correlation coefficient (r) between FS_m and FS_p is to be obtained from the following expression:

$$r = \frac{\sum_{i=1}^{N}(FS_i - \overline{FS}_m)\left(FS_i - \overline{FS}_p\right)}{\sqrt{\sum_{i=1}^{N}(FS_i - \overline{FS}_m)^2\left(FS_i - \overline{FS}_p\right)^2}} \tag{13}$$

4. Illustrative Example

In the present section, evaluation of system reliability of a base soil-filter media system is presented by means of an example application. From the particle size distribution (*PSD*) of the base soil, the weighted average particle size (d_s) is assumed to be obtained as 0.716 mm. To protect this base soil from particle migration and the onset of piping, granular filter layer is provided. The *PSD* of granular filter layer is such that the D_{of} (pore size of filter media) value is obtained as 0.519mm. Other geotechnical properties of base soil and filter media are provided in Table 1.

Table 1 (a). Geotechnical properties of base soil

Properties of base soil	Numerical values
Dry unit weight [$(\gamma_d)_b$]	13.0kN/m^3
Angle of internal friction (ϕ')	32°
D_{15b}	0.750 mm
Weighted average particle size (d_s)	0.716 mm
Parameter "*p*"	2.00
Parameter "*m*"	0.93

Table 1 (b). Geotechnical properties of filter media

Properties of filter media	Numerical values
Dry unit weight [$(\gamma_d)_f$]	14.0 kN/m^3
D_{15f}	2.8 mm
Void ratio (e_f)	0.857
Porosity (n_f)	0.461

For the reliability analysis, coefficients of variation in the input soil parameters are taken from the range of values suggested in the literature (Table 2). It is worth mentioning here that the applied hydraulic gradient (i) is assumed as a deterministic parameter.

Table 2. Coefficient of variation (δ%) considered in the input soil parameters and representative particle sizes

Input soil parameters	Range CoV%	Selected CoV%
d_s, D_{of}, D_{15b}, D_{15f}	10-15% [39]	15%
Dry unit weight (γ_d)	3-7% [33, 34]	7%
Effective friction angle (ϕ')	2-13% [33, 34]	13%

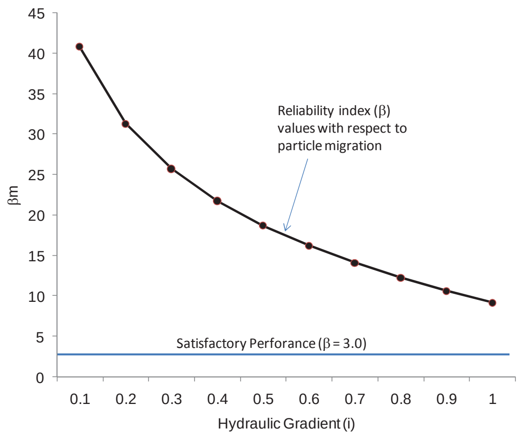

Figure 4(a) Reliability index values with respect to particle migration

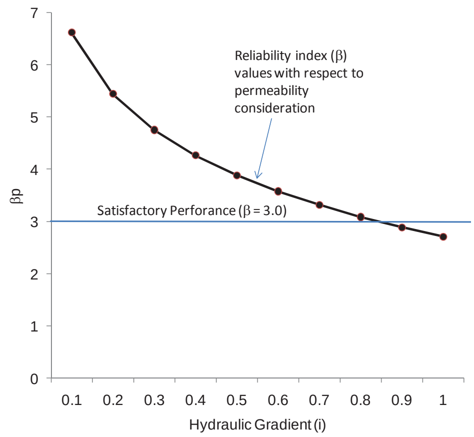

Figure 4(b) reliability index values with respect to permeability

Evaluation of reliability index with respect to two failure modes, i.e., particle migration (β_m) and permeability criteria (β_p) require information on mean and variance of the factor of safety with respect to particle migration (FS_m) and permeability (FS_p), respectively. The mean ($\overline{FS}_m$ or $\overline{FS}_p$) and coefficient of variation of the factor of safety [$(\delta_{FS})_m$ or $(\delta_{FS})_p$] with respect to these two criteria can be evaluated through Monte Carlo Simulations (sample size N) with the help of the proposed analytical solutions. The reliability index values from the two failure modes (β_m & β_p) were estimated by utilizing commercially available software tool STRUREL [COMREL 8 -TI (Time invariant analysis)] [40] and results are compared in the following Figures 4a & 4b.

It can be noted that the granular base soil filter media system indicates acceptable values of reliability indices with respect to particle migration for all the ranges of hydraulic gradient values considered. On the other hand, with respect to permeability, filter performance is satisfactory up to a hydraulic gradient value of 0.8.

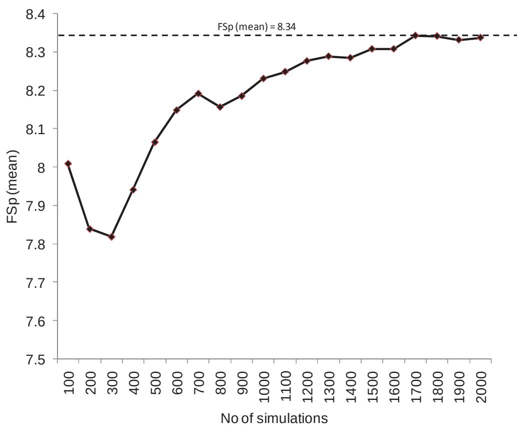

Figure 5 (a) Estimate mean FS_m with sample size

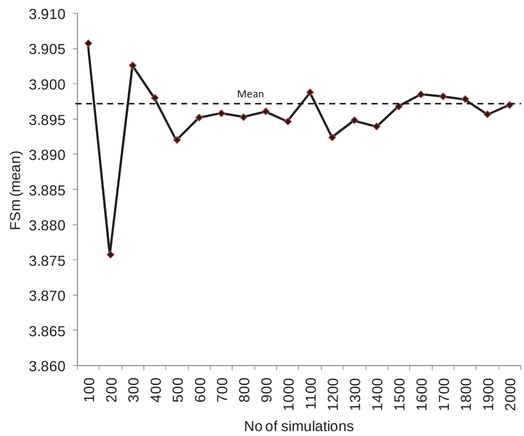

Figure 5 (b) Estimate mean FS_p with sample size

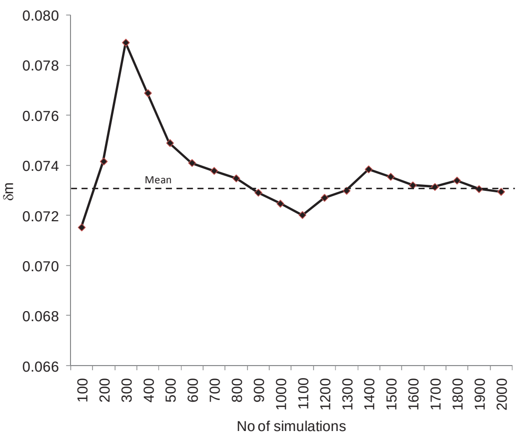

Figure 5 (c) Estimate δ_m with sample size

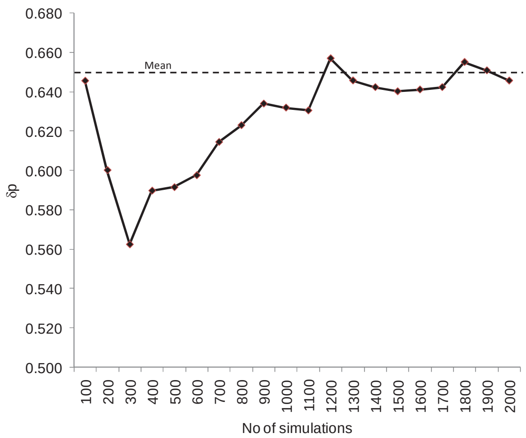

Figure 5 (d) Estimate δ_p with sample size

5.2 Sample size (N) for Monte Carlo simulations

Sample size (N) for Monte Carlo simulation determines both accuracy of the results and time for the analysis. A higher sample size would provide more accurate results but it is time consuming. On the other hand, a smaller sample size although would provide faster results but would compromise the accuracy of the results. To obtain the sample size (N) for Monte Carlo simulations, mean and coefficient of variation of output response, i.e., FS_m and FS_p were obtained for increasing numbers of simulations as shown in the Figure 5. It can be noted that after 1600-1800 simulations, the estimated mean and coefficient of variation stabilize. Hence, in the present study, a sample size of 2000 is considered to be sufficient for the Monte Carlo Simulation.

5.3 System reliability evaluation

For system reliability evaluation of base soil – filter media system, the (sample) correlation coefficient (r) between FS_m and FS_p is obtained using equation (13). FS_i is the factor of safety value obtained for each set of input parameters through Monte Carlo simulations and the value of r is obtained as +0.83. It can be noted that both the factor of safety with respect to particle migration (FS_m) and the one with respect to permeability (FS_p) are positively correlated. It is noted that reliability index values at lower hydraulic gradients are quite high and it can be safely assumed that the system reliability is high for low values of hydraulic gradient. In Table 3, the component reliability indices (β_m , β_p) at high hydraulic gradients are reported.

Table 3. Component reliability indices at higher hydraulic gradients

Hydraulic gradient (i)	Reliability index (β_m)	Reliability index (β_p)
1.1	7.563	2.513
1.2	5.726	2.286
1.3	3.604	2.024
1.4	1.092	1.714

Figure 6 shows the system reliability indices (lower bound) values for the base soil – filter media system at higher hydraulic gradients. It is noted that the base soil- filter media system is likely to have better system performance level up to a hydraulic gradient of 1.3, beyond which, it decreases.

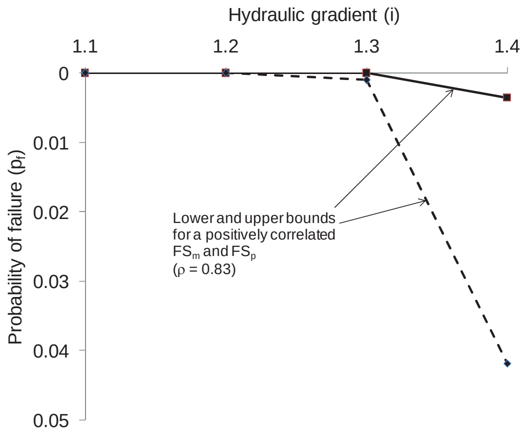

Figure 6. Lower and upper bound for positively correlated FS_m and FS_p

5. Conclusion

- The proposed analytical solutions with respect to soil particles migration and permeability criteria takes into consideration some of the basic geotechnical properties of the base soil and filter media as well as the existing hydraulic conditions.
- The variability in the geotechnical parameters and its influence on the system response can be captured through reliability analysis using presented analytical solutions.
- Reliability analysis when used in conjunction with the conventional factor of safety approach provides a sound mathematical basis for the decision making process.
- Mutually dependent failure modes in a base soil-filter media system, FS_m and FS_p are positively correlated with correlation coefficient +0.83.
- System reliability analysis provides an overall assessment of the system response where there exist multiple failure modes and the failure of one component affects the performance of the other ones.

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