Reliability-based Assessment of Stability of Slopes

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Abstract. Multiple sources of uncertainties often exist in the evaluation of slope stability. When assessing stability of slopes in the face of uncertainties, it is desirable, and sometimes necessary, to adopt reliability-based approaches that consider these uncertainties explicitly. This paper focuses on the practical procedures developed recently for the reliability-based assessment of slope stability. The statistical characterization of model uncertainty and parameter uncertainty are first described, followed by an evaluation of the failure probability of a slope corresponding to a single slip surface, and the system failure probability. The availability of site-specific information then makes it possible to update the reliability of the slope through the Bayes' theorem. Furthermore, how to perform reliability-based design when the statistics of random variables cannot be determined accurately is also discussed. Finally, case studies are presented to illustrate the benefit of performing reliability-based design and the procedure for conducting reliability-based robust design when the statistics of the random variables are incomplete.

1. Introduction

The uncertainties in both the solution models and the input parameters make it impossible to capture the true performance (or stability) of a slope with a single factor of safety (FOS), which is the outcome from a deterministic analysis. For example, slopes with the same FOS value may exhibit different risk levels depending on the variability of the solution models and soil parameters (Li and Lumb 1987). Subsequent to the pioneering researches such as Wu and Kraft (1970), Matsuo and Kuroda (1974), Alonso (1976) and Tang et al. (1976), extensive studies have been conducted on the probabilistic assessment of slope stability, including reliability analysis of slope failure along a given slip surface (e.g., Liang et al. 1999; El-Ramly et al. 2002), searching for the most critical slip surface (e.g., Bhattacharya et al. 2003; Xue and Gavin 2007), system reliability analysis (e.g., Oka and Wu 1990; Ching et al. 2009; Zhang et al. 2011&2013), effect of spatial variability of soil properties on slope stability (e.g., Griffiths and Fenton 2004; Cho 2007; Griffiths et al. 2009; Li et al. 2015), effect of environment action on slope reliability (e.g., Babu and Murthy 2005; Zhang et al. 2005; Frattini et al 2009; Santoso and Phoon 2011), and reliability-based design of slopes (Christian et al. 1994; Roh and Hong 2009; Salgado and Kim 2014). The outcome from a reliability analysis is regarded as an important input for the qualitative risk assessment and management of landslide hazards (e.g., Mostyn and Li 1993; Nadim et al. 2005; Cassidy et al. 2008; Zhang et al. 2012; Huang et al. 2013).

The probabilistic framework is generally considered effective for uncertainty quantification, incorporating information from various sources, and expressing the degree of belief about slope stability. Over the past decades, reliability methods have gradually evolved into a useful and practical tool that is complementary to the existing deterministic slope stability analysis. Nevertheless, the development in the field of probabilistic assessment of slope stability is still not well known in the general community of slope engineering. For many engineers, the probabilistic methods are sometimes considered as attractive but difficult to implement, which limits its application in practice. In an effort to bridge such a gap, the goal of this paper is to introduce practical methods and examples for addressing the following important questions in slope reliability analysis:

- How uncertainties in slope stability analysis can be characterized?
- How the failure probability of a slope can be calculated efficiently both for an individual slip surface and for the slope as a system?
- How the performance information can be used to update the reliability of a slope?
- How to implement reliability-based design with incomplete probabilistic information?

The structure of this paper is as follows. First, methods for estimating the uncertainties of solution models and soil parameters are presented. Second, how to evaluate the reliability of a single slip surface and multiple slip surfaces are introduced. Third, the update of the reliability-based slope stability analysis result using the past performance information is outlined. Finally, two application examples are studied to illustrate the use of probabilistic methods in the design of slopes in practice.

2. Uncertainties Characterization in Slope Stability Analysis

2.1. Model uncertainty

In that the solution models adopted in slope engineering are only abstractions of the real world, model uncertainties always exist (Ang and Tang 1984; Cheung and Tang 2005; Zhang et al. 2009). If the model uncertainty is not considered, the prediction from a solution model might be biased. In principle, the model uncertainty (i.e., uncertainty of solution models) can be determined by a systematic comparison between model predictions and observed performances (i.e., field observations or model tests) (Gilbert and Tang 1995; Zhang et al. 2009). Such a comparison is, however, difficult, as in slope engineering the model input parameters can hardly be determined with certainty. Thus, the model uncertainty must be analyzed considering uncertainties in the model input parameters.

Let $g(\boldsymbol{\theta})$ denote a general slope stability model, where $\boldsymbol{\theta}$ denote the uncertain input parameters in this model. Assume the observed performance d can be related to the model prediction $g(\boldsymbol{\theta})$ and model error ε as follows:

$$d = g(\boldsymbol{\theta}) + \varepsilon \tag{1}$$

Assume further that ε is a normal variable with a mean of μ_ε and a standard deviation of σ_ε. The task of model uncertainty characterization is then reduced to the estimation of μ_ε and σ_ε. Let $f(\mu_\varepsilon, \sigma_\varepsilon)$ denote the prior probability density function (PDF) of μ_ε and σ_ε. Based on Bayes' theorem, the posterior PDF of μ_ε and σ_ε can be evaluated as follows (Zhang et al. 2009):

$$f(\mu_\varepsilon, \sigma_\varepsilon \mid \mathbf{d}) \approx k f(\mu_\varepsilon, \sigma_\varepsilon) \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi(\sigma_\varepsilon^2 + \sigma_{g(\boldsymbol{\theta}_i)}^2)}} \exp\left[-\frac{1}{2} \frac{(d_i - \mu_{g(\boldsymbol{\theta}_i)} - \mu_\varepsilon)^2}{\sigma_\varepsilon^2 + \sigma_{g(\boldsymbol{\theta}_i)}^2} \right] \tag{2}$$

where k is a normalization constant to make the derived posterior PDF valid; d_i is the ith observation; and $\mu_{g(\boldsymbol{\theta}_i)}$ and $\sigma_{g(\boldsymbol{\theta}_i)}$ are the mean and standard deviation of $g(\boldsymbol{\theta}_i)$ caused by uncertainties in $\boldsymbol{\theta}_i$.

The above formulation was adopted in Zhang et al (2009) to study the model uncertainty associated with the Morgenstern-Price method (Morgenstern and Price 1965) for slope stability analysis, in which the test results of 20 model slopes examined in centrifuge by Kim (1980) were analyzed. The prior

distribution of μ_ε is assumed to be a normal variable with a mean of 0 and a standard deviation of 0.5, while the prior distribution of σ_ε is assumed as a lognormal variable with a mean of 0.5 and a standard deviation of 1.0. After the values of $\mu_{g(\theta)}$ and $\sigma_{g(\theta)}$ are determined, the posterior statistics of μ_ε and σ_ε are then calculated based on the Markov Chain Monte Carlo (MCMC) simulation, and the results are summarized in Table 1. A comparison of the prior and posterior statistics of these variables shows a significant reduction in the uncertainties in μ_ε and σ_ε. Table 1 shows that the mean of the model error of the Morgenstern-price method is close to zero, indicating this method is almost unbiased. The standard deviation of the model error is about 4.4%. For comparison, Duncan and Wright (1980) determined that any limit equilibrium method satisfying all conditions of equilibrium could predict the FOS within an accuracy of 5% from what is considered the correct answer.

Table 1. Comparison of statistics of model error calibrated based on centrifuge tests

	μ_ε		σ_ε		ε	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Prior	0	0.5	0.5	1	0	1.225
Posterior	-0.002	0.010	0.042	0.008	-0.002	0.044

The data in Table 1 show the model error associated with the Morgenstern-Price method used to analyze the model slopes tested in Kim (1980). When the Morgenstern-Price method is used to study slopes in the field, the statistics of the model error might be different. Nevertheless, when the observed performance data of similar slopes in a region are available, Equation (2) can also be used to calibrate the model error associated with the slope stability model for analysing the slopes in this region.

2.2. Parameter uncertainty
The uncertainty associated with the parameters of the slope stability model can often be assessed through laboratory and in-situ tests (e.g., Vanmarcke 1977; Einstein and Baecher 1983; Lacasse and Nadim 1996; Duncan 2000). When slope failure is observed, back analysis is also useful in deriving the uncertainties of the model input parameters (e.g., Gilbert et al. 1998; Tang et al. 1999). At the moment of slope failure, the FOS of the slope is equal to unity. Based on the normal assumption of the model error in Equation (1) as mentioned previously, if μ_ε and σ_ε are known, the chance to observe the slope failure event can be written as follows:

$$L(\boldsymbol{\theta}\,|\,F) = \phi\left(\frac{g(\boldsymbol{\theta}) + \mu_\varepsilon - 1}{\sigma_\varepsilon}\right) \tag{3}$$

where F denotes the event of observing slope failure. Let $f(\boldsymbol{\theta})$ denote a PDF that represents the knowledge about $\boldsymbol{\theta}$ prior to the back analysis. According to Bayes' theorem, the posterior PDF of $\boldsymbol{\theta}$ is

$$f(\boldsymbol{\theta}\,|\,F) = \frac{\phi\left(\dfrac{g(\boldsymbol{\theta}) + \mu_\varepsilon - 1}{\sigma_\varepsilon}\right) f(\boldsymbol{\theta})}{\displaystyle\int \cdots \iint \phi\left(\dfrac{g(\boldsymbol{\theta}) + \mu_\varepsilon - 1}{\sigma_\varepsilon}\right) f(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}} \tag{4}$$

The posterior PDF given by Equation (4) generally does not have an analytical expression. Therefore, for an arbitrary prior distribution, samples of the posterior distribution can be conveniently obtained using MCMC simulation (Juang et al. 2012a; Wang et al. 2013a). When $\boldsymbol{\theta}$ is multivariate normal, its posterior distribution can be approximated by a multivariate normal distribution based on the system identification theory (Zhang et al. 2010), as described below. Let $\boldsymbol{\mu_\theta}$ and $\mathbf{C_\theta}$ denote the prior mean and prior covariance of $\boldsymbol{\theta}$, respectively. Let $\boldsymbol{\mu_{\theta|d}}$ and $\mathbf{C_{\theta|d}}$ denote the posterior mean and covariance matrix of $\boldsymbol{\theta}$, respectively. In that the slope stability model $g(\boldsymbol{\theta})$ is often quite linear along a

given slip surface, $\mu_{\theta|d}$ and $C_{\theta|d}$ can be determined approximately using the system identification (SI) for linear systems as follows (Zhang et al. 2010; Wang et al. 2013a):

$$\mu_{\theta|d} \approx \mu_{\theta} + C_{\theta}H^T\left(HC_{\theta}H^T + \sigma_{\varepsilon}^2\right)^{-1}\left(1 - g(\mu_{\theta}) - \mu_{\varepsilon}\right) \tag{5}$$

$$C_{\theta|d} \approx \left(\frac{H^TH}{\sigma_{\varepsilon}^2} + C_{\theta}^{-1}\right)^{-1} \tag{6}$$

$$H = \left.\frac{\partial g(\theta)}{\partial \theta}\right|_{\theta=\mu_{\theta}} \tag{7}$$

where $g(\mu_{\theta})$ is the predicted FOS calculated at point μ_{θ}; and H is a row vector representing the sensitivity of $g(\theta)$ with respect to θ at point μ_{θ}.
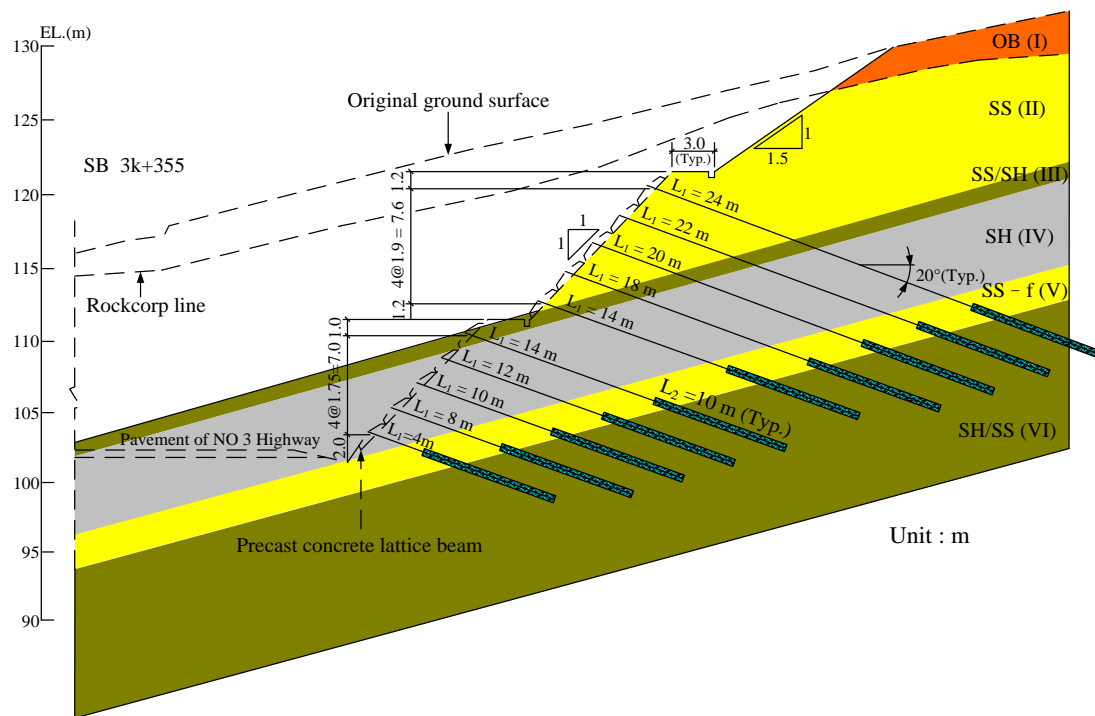

Figure 1. Geometry and geological profile of a cross-section of the slope studied in Wang et al. (2013a) [OB (I): Overburden soil; SS (II): Sandstone with vertical joints; SS/SH(III): Alternations of thin sandstone and shale; SH(IV): Dark gray shale; SS-f (V): Sandstone with trace fossils; and SS/SH(VI): Alternations of thin sandstone and shale]. (Wang et al. 2013a)

A probabilistic back analysis example adapted from Wang et al. (2013a) is illustrated in Figure 1. The cut slope, formed by highway construction, failed suddenly approximately 12 years after completion. The weather was sunny at the time of the landslide and there was no occurrence of tremors, thus excluding both heavy rainfall and earthquake as the trigger. The location of the slip surface was determined from a site investigation, which can be simplified as a single plane failure surface. The FOS of the rock slope can be analyzed using the method developed by Hoek and Bray (1981). For this slope, the friction angle of sliding surface φ (°) and the design forces of all rock anchors T (ton) are the major uncertain variables to be back analyzed, i.e., $\theta = \{\varphi, T\}$. For

demonstration purposes, the prior distribution of φ and T is assumed as a bivariate normal distribution with the statistics listed in Table 2. The model error is assumed to follow a normal distribution with mean $\mu_\varepsilon = 0.05$ and standard deviation $\sigma_\varepsilon = 0.07$. This input information is then used to derive the back analysis results using the SI method, as listed in Table 2. Comparing the prior and posterior mean values, both the mean values of φ and T are reduced. The updated outcomes also show a reduction in the uncertainties of φ and T.

Table 2. Prior and posterior statistics of φ and T in the back analysis of slope failure

	φ (°)		T (ton)	
	Mean	Std. Dev.	Mean	Std. Dev.
Prior	21	3.15	60	22.8
Posterior	12.88	1.49	33.05	20.86

3. Reliability Analysis of Slopes

A consideration of the uncertainties involved in the slope stability analysis results in an uncertainty in the FOS of the slope. In such a case, either failure probability or reliability index can be used to measure the stability of the slope. The traditional approach for computing the reliability index of a slope involves minimizing the FOS over a range of possible failure surfaces and then calculating the reliability index related to this surface (Chowdhury et al. 1987; Honjo and Kuroda 1991; Christian et al. 1994; Wolff et al. 1995). However, the slip surface that yields the minimum FOS (termed deterministic most critical slip surface) does not always coincide with the slip surface that yields the minimum reliability index (known as the probabilistic most critical slip surface) (e.g., Hassan and Wolff 1999; El-Ramly et al. 2002; Zhang et al. 2013). Indeed, a soil slope may have many potential slip surfaces, which means that the failure probability of the slope is often larger than that corresponding to any single slip surface (Cornell 1971; Oka and Wu 1990; Chowdhury and Xu 1995). Therefore, system reliability is more appropriate for analyzing the reliability of a slope (Chowdhury and Xu 1995; Ching et al. 2009; Low et al. 2011; Li et al. 2003; Zhang et al. 2013; Zeng and Jimenez 2014).

3.1. Reliability analysis along a single slip surface

Determining the probabilistic most critical slip surface is of significant importance in slope reliability analysis, which is possible through a rigorous constrained optimization algorithm. Although the Hassan and Wolff (1999) procedure is empirical, Zhang et al. (2013) found that it was quite accurate for many applications. Let θ_i denote the ith element of $\boldsymbol{\theta}$ with $\mu_{\theta i}$ and $\sigma_{\theta i}$ respectively being the mean and standard deviation of θ_i. Let n denote the dimension of $\boldsymbol{\theta}$ and let $\boldsymbol{\theta}^{(i)} = \{\mu_{\theta 1}, \mu_{\theta 2}, \ldots, \mu_{\theta i} - \sigma_{\theta i}, \ldots, \mu_{\theta n}\}$. Hassan and Wolff (1999) suggested the following procedure to search for the probabilistic most critical slip surface:

- For $i = 1, 2, \ldots, n$, search the slip surface with minimum FOS using $\boldsymbol{\theta}^{(i)}$ as input parameters. As such, n FOS values as well as n slip surfaces can be obtained;
- Among the n slip surfaces obtained, the slip surface with the minimum FOS is the one with the maximum failure probability, i.e., the probabilistic most critical slip surface. The argument for this is that when the random variable contributing the most to the uncertainty in the computed FOS takes a low value (but of reasonable likelihood) in its possible range, the location of the critical surface is dominated by this random variable.

After the probabilistic most critical slip surface is identified, its reliability index can be evaluated using existing reliability methods such as the first order reliability method (FORM) (e.g., Ang and Tang 1984). However, as deterministic computer programs are widely used for slope stability analysis, it is advantageous to be able to use the existing stand-alone deterministic computer programs for the reliability analysis. To this end, the response surface method is adopted. It is noted that along a given

slip surface, the relationship between FOS and shear strength parameters is often quite linear, which can be approximated by a second order polynomial as follows (Zhang et al. 2013):

$$g'(\mathbf{\theta}) = b_0 + \sum_{i=1}^{n} b_i \theta_i + \sum_{i=1}^{n} b_{n+i} \theta_i^2 \qquad (8)$$

where b_i ($i = 0, 1, \ldots, 2n$) is the unknown coefficient. There are ($2n + 1$) unknown coefficients in Equation (8). To calibrate these coefficients, the FOS of a slip surface is first evaluated at the following ($2n + 1$) points: $\{\mu_{\theta1}, \mu_{\theta2}, \ldots, \mu_{\theta n}\}$, $\{\mu_{\theta1} \pm m\sigma_{\theta1}, \mu_{\theta2}, \ldots, \mu_{\theta n}\}$, $\{\mu_{\theta1}, \mu_{\theta2} \pm m\sigma_{\theta2}, \ldots, \mu_{\theta n}\}$, $\ldots$, and $\{\mu_{\theta1}, \mu_{\theta2}, \ldots, \mu_{\theta n} \pm m\sigma_{\theta n}\}$, where m is a parameter selected by the user. By equating the FOSs calculated with Equation (8) to those calculated using the numerical slope stability model at the selected points, the unknown coefficients can be solved. The calibrated second order polynomial can then be used as the performance function for the subsequent slope reliability analysis using the existing reliability methods.

3.2. System reliability
The number of potential slip surfaces of a slope can be large. The failure probability of the most critical slip surface only provides a lower bound value for the system failure probability of the slope. Nevertheless, due to the high correlation among FOS of various slip surfaces, the system reliability of a soil slope is often governed by a few "representative" slip surfaces. Several methods have been suggested for identifying the representative slip surfaces (e.g., Ji and Low 2012; Zhang et al., 2013). In the Hassan and Wolff method, each uncertain variable is decreased from its mean value to locate a slip surface where the location of the slip surface is governed by that random variable, resulting in n slip surfaces. In the recent study by Zhang et al. (2013), it has been shown that these n slip surfaces can indeed be used as representative slip surfaces for the system reliability analysis. Thus, the system reliability of a slope can then be analyzed using the following procedure:
- Identify the representative slip surfaces using the Hassan and Wolff procedure;
- Approximate the relationship between FOS and uncertain variables using Equation (8) for each representative slip surface;
- Calculate the failure probability based on the fitted second-order polynomials using Monte Carlo simulation (MCS).

As an example, a slope adapted from Feng and Fredlund (2011) as shown in Figure 2 is studied. Through analysing 9,261 potential slip surfaces, the failure probability of the most critical slip surface is 0.120 (COV = 2.7%) and the system failure probability of the slope is 0.187 (COV = 2.1%) based on MCS with 1×10^4 samples. Using the Hassan and Wolff procedure, the identified representative slip surfaces are shown in Figure 2. As slip surfaces 1 and 2 are the same, there are two representative slip surfaces in this example. Slip surface 3 is the same as the most critical slip surface. Based on the two identified representative slip surfaces as shown in Figure 2, the failure probability of the slope is 0.184 with a COV of 2.1% using Monte Carlo simulation with 1×10^4 samples, consistent with the system failure probability calculated based on a 9261 slip surfaces.
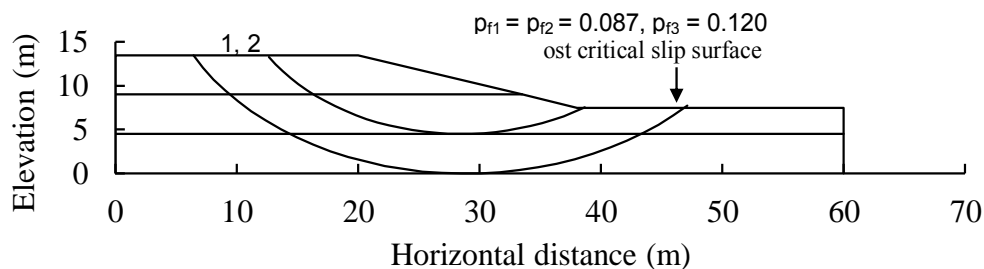

Figure 2. Representative slip surfaces identified by the Hassan and Wolff method (Zhang et al. 2013)

4. Updating Slope Stability with Observed Performance

Common information inherent in many slope reliability analyses includes knowledge learned from geotechnical explorations and tests, engineering judgment, published data, and regional experiences. In addition to the above knowledge, another important source of information is the past performance of the slope of concern: the slope survived or failed at a certain state previously. Such performance information is the outcome of a full-scale test directly performed on the slope under investigation, and could provide valuable information for the safety assessment, upgrading analysis, and remedial design of the slope. This section intends to illustrate how the site-specific past performance information can be used to update the reliability of slope.

4.1. Consideration of survival information

Let FOS_1 denote the FOS of the slope with known performance at a previous moment. If the slope did not fail, it means $FOS_1 > 1$. Let $p_{f2|FOS1>1}$ denote the conditional failure probability of the slope at a future moment considering its past survival information. Mathematically, it can be written as:

$$p_{f2|y_1>1} = P\left(FOS_2 < 1 \,|\, FOS_1 > 1 \right) \tag{9}$$

Using the rule of conditional probability, $p_{f2|FOS1>1}$ becomes:

$$p_{f2|y_1>1} = \frac{p_{f2} - p_{f12}}{1 - p_{f1}} \tag{10}$$

In Equation (10), p_{f1} and p_{f2} are failure probabilities at the previous moment and at a future moment without considering the past performance information, which can be calculated with the conventional reliability methods such as first order reliability method (FORM) (e.g., Low et al. 2011); and p_{f12} is the probability indicating a simultaneous violation of the limit-state functions of $FOS_1 - 1 = 0$ and $FOS_2 - 1 = 0$, which can be computed as follows (Zhang et al. 2011):

$$p_{f12} \approx \left(a + b \right)\left(\frac{\pi - v}{\pi} \right) \tag{11}$$

$$a = \Phi\left(-\beta_1\right)\Phi\left(-\frac{\beta_2 - \rho_{12}\beta_1}{\sqrt{1 - \rho_{12}^2}} \right) \tag{12}$$

$$b = \Phi\left(-\beta_2\right)\Phi\left(-\frac{\beta_1 - \rho_{12}\beta_2}{\sqrt{1 - \rho_{12}^2}} \right) \tag{13}$$

$$v = \cos^{-1}\rho_{12} \tag{14}$$

$$\rho_{12} = \frac{\mathbf{Y}_1^* \mathbf{R_Y}^{-1} \left(\mathbf{Y}_2^*\right)^{\mathrm{T}}}{\beta_1 \beta_2} \tag{15}$$

where β_1 and β_2 are the FORM reliability indexes of the slope at the previous moment and at the future moment, respectively; $\mathbf{Y}$ = reduced variables of $\mathbf{\theta}$; $\mathbf{R_Y}$ = correlation matrix of $\mathbf{Y}$; and $\mathbf{Y}_1^*$ and $\mathbf{Y}_2^*$ = the design points of the slope at the state with known performance and at the future state, found with FORM, respectively.

To illustrate the effect of survival information on reliability updating, an infinite slope example is adapted from Zhang et al. (2011), as shown in Figure 3. The FOS of the slope can be calculated using the infinite slope model (Wu and Abdel-Latif 2000). Let $m = h_w/h$ denotes the normalized water table. Let m_1 denote the level of groundwater at which the slope has survived and let m_2 denote the maximum groundwater table within a certain design period. Due to the presence of observation error,

statistical uncertainty, and modelling uncertainty, the values of m_1 and m_2 may not be exactly known. Consider a general case where both m_1 and m_2 are uncertain. Suppose that m_1 is normally distributed with a mean of μ_{m1} and a standard deviation of 0.05 and that m_2 is normally distributed with a mean of 0.5 and a standard deviation of 0.05.
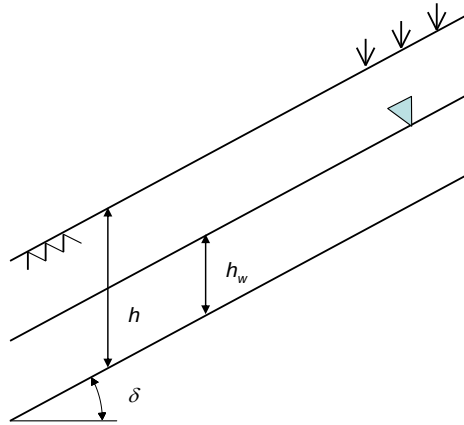
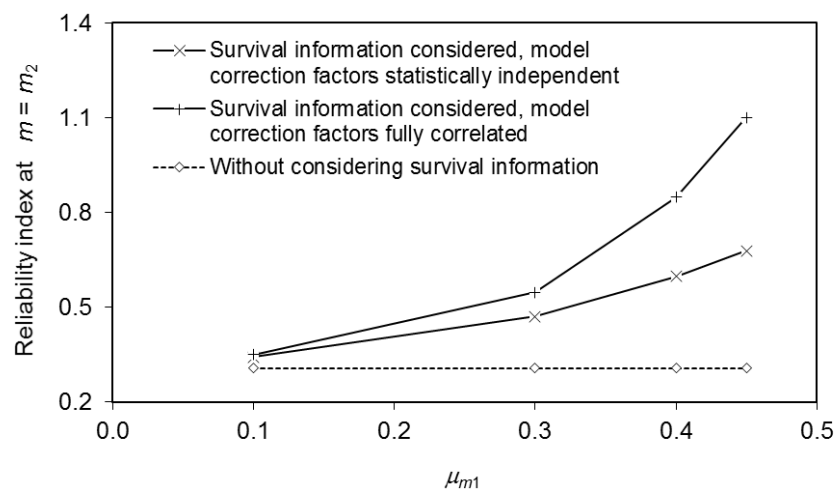**Figure 3.** An existing slope for reliability assessment (Zhang et al. 2011)

Figure 4. Reliability index of the infinite slope at $m = m_2$ when μ_{m1} takes various values (Zhang et al. 2011)

Let ε_1 and ε_2 denote the model correction factors in computing FOS_1 and FOS_2, respectively. Consider first the case where ε_1 and ε_2 are statistically independent. Figure 4 shows the reliability indexes of the slope at $m = m_2$ considering the survival information of $FOS_1 > 1$. After considering the survival information, the reliability index of the slope at $m = m_2$ increases, indicating that we are more confident in the safety of the slope. Figure 4 also shows that the amount of improvement in reliability index increases with μ_{m1}, implying that the updating effects will be more obvious when the slope survives a more critical state. Hence, ignoring the survival information would underestimate the reliability index of a slope and may result in an uneconomical design. Assuming the model correction factors (i.e., ε_1 and ε_2) are fully correlated, the reliability index of the slope at $m = m_2$ is re-evaluated and the results are also plotted in Figure 4. With this latter assumption, the assessed reliability index is larger than that obtained by assuming ε_1 and ε_2 being statistically independent given the same μ_{m1}.

This is because the assumption of model correction factors being fully correlated implies a stronger association between the performances of the slope when the groundwater is at various levels; hence, the past survival information will be more useful in updating the reliability of the slope at $m = m_2$.

4.2. Consideration of failure information

Suppose it is known that a slope has failed at a previous state, i.e., $FOS_1 = 1$. Let $p_{f2|FOS1=1}$ denote the failure probability of the slope at a future moment considering its past failure information. Based on the conditional probability theorem, $p_{f2|FOS1=1}$ can be written as:

$$p_{f2|y_1=1} = \frac{P\left(FOS_2 < 1 \cap FOS_1 = 1\right)}{P\left(FOS_1 = 1\right)} \qquad (16)$$

Let β_1 and β_2 denote the reliability indexes of the slope at the previous moment and at the future moment without considering the information of past performance. Let ρ_{12} denote the correlation coefficient between the FOSs of the slope at these two moments. The conditional failure probability shown in Equation (16) can be estimated based on β_1, β_2, and ρ_{12}, which has been given in Madsen et al. (1986), as follows:

$$p_{f2|FOS_1=1} = \Phi\left(-\frac{\beta_2 - \rho_{12}\beta_1}{\sqrt{1-\rho_{12}^2}}\right) \qquad (17)$$
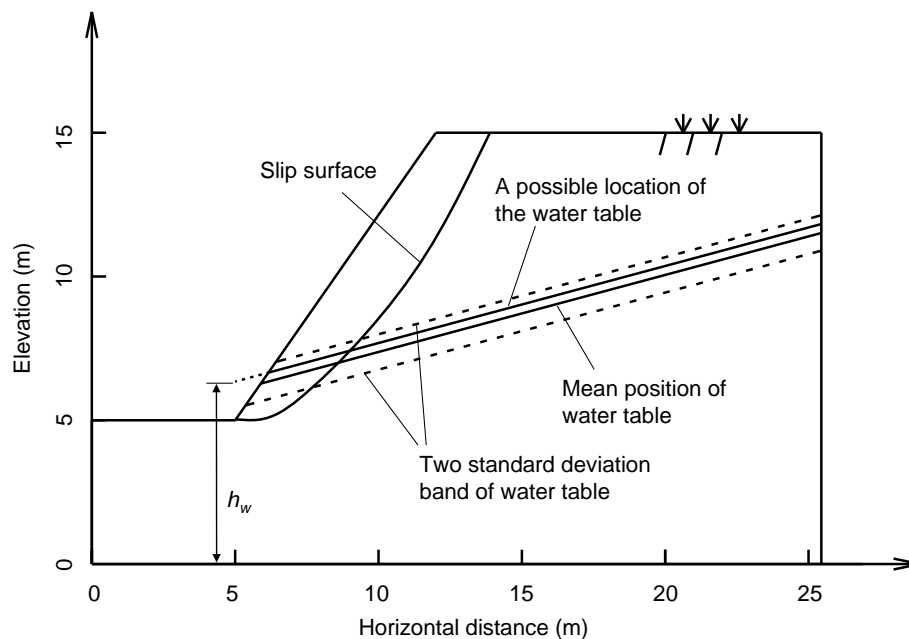

Figure 5. A failed cut slope for repair design (Zhang et al. 2011)

To show the impact of past failure information on reliability evaluation, Figure 5 shows the geometry of a failed cut slope that has a slope angle of 55°. The exact position of groundwater table at the moment of slope failure is unknown, and is assumed as normally distributed around its mean position with a standard deviation of 0.3 m, as shown in Figure 5. The location of the groundwater table is characterized by the elevation of the projection line of the groundwater table at the slope toe, which is denoted as h_w (Figure 5). Based on the previous assumption about the groundwater table, h_w is normally distributed with a mean of 6 m and a standard deviation of 0.3 m. Suppose the lowering of the slope angle is used to repair the slope. Based on Equation (15), the correlation coefficients between

the FOSs of the slope at the angle of 55$^{\text{o}}$ and at a new slope angle are all around 0.74 when the angle of the new slope varies within the range of 35$^{\text{o}}$ to 50$^{\text{o}}$. Based on Equation (17), the reliability indices of the slope at various slope angles, considering the past failure information, are calculated and plotted in Figure 6.

The reliability index of the new slope at various slope angles with and without considering the past failure information is shown in Figure 6. Here the past failure information can significantly affect the reliability indexes of the slope at other slope angles. When the slope angle is 55$^{\text{o}}$, the reliability index of the slope, considering the failure information, is lower than that obtained without considering the previous failure information, which is more consistent with failure of the slope at this particular angle. As the slope angle decreases, the reliability index of the slope considering the failure information increases more rapidly than without considering the failure information. This inclusion rather than omission of failure information thus reduces the uncertainty in the slope stability analysis, making the reliability of the slope more sensitive to the change in slope angle. To reach a target reliability index of 3.0, we only need to flatten the slope to approximately 40$^{\text{o}}$ when the previous failure information is considered. For comparison, if the previous failure information is not considered, the slope would need to be flattened to about 35$^{\text{o}}$ to meet the same target reliability index.
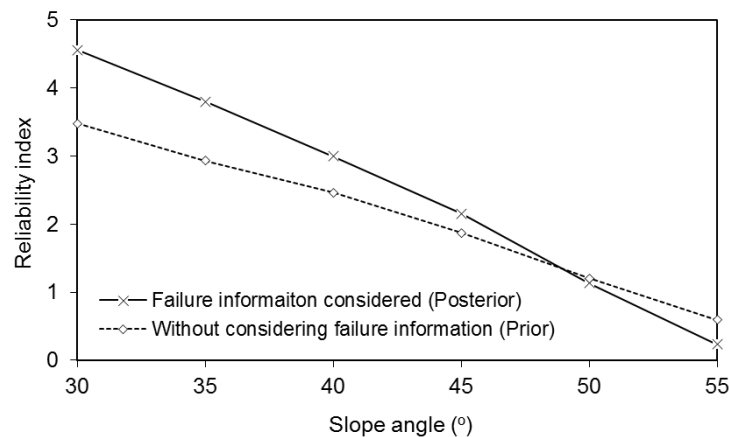

Figure 6. Reliability index of the slope at various slope angles with and without considering past failure information (Zhang et al. 2011)

5. Reliability-Based Design of Slopes with Incomplete Probabilistic Information

Most probabilistic methods require detailed statistics of uncertain variables, which are sometimes difficult to obtain due to budgetary constraints for site investigation and field and laboratory tests. The results from a probabilistic analysis, however, may depend on the input statistics of the uncertain variables. As such, while an initially acceptable design may not satisfy the target reliability index or failure probability if the variation of the geotechnical properties is underestimated; an initially cost-efficient design may not be cost-efficient if the variation of the geotechnical properties is overestimated (Juang et al. 2012b&2013). To address such a dilemma, the robust design concept initiated by Taguchi (1986) for improving product quality and reliability in industrial engineering is recently explored for design of slopes (Juang et al. 2012b&2103; Wang et al. 2013b; Gong et al. 2015), in which the easy-to-control parameters can be systematically adjusted such that the performance of a slope becomes insensitive to the statistics of variables that are hard to determine accurately. Naturally, higher design robustness is achieved at a higher cost. Thus, a multi-objective optimization considering both cost and robustness is needed to select the optimal designs among those in the acceptable design space. The robust reliability-based design methodology will be illustrated later in this paper with a slope in Hong Kong.

6. Application Example 1: Reliability-Based Design of Slopes

6.1. Engineering background

The #6 landslide of Shang-Shan Highway lies in Zhejiang Province, China. From May to June in 2000, the slope deformation accelerated due to continuous rainfall, with an obvious overall landslide occurring on June 10 of that year. The typical geological cross section A2-A2′ of #6 landslide, shown in Figure 7, is adopted for the following reliability analysis. The post-failure investigation indicates that the slip surface of the landslide occurred along the surface between the overlying soil and the bedrock as marked in Figure 7. The landslide mass is composed of gravel and gravelly clay. After the failure event, a series of engineering measures including drainage holes and reinforced piles were adopted to stabilize the slope in December, 2001. This landslide is reanalyzed here to illustrate the benefit of preforming reliability-based design.
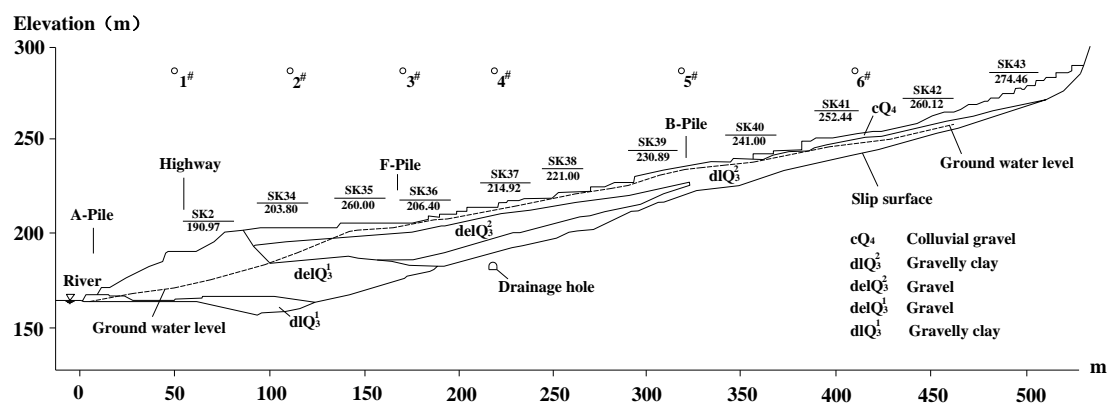

Figure 7. Cross section A2-A2′ of #6 landslide

6.2. Quantification of uncertainties

An analysis of the slope stability requires determining the variances in cohesion (c), the friction angle (φ) of the slip surface materials as well as the pore pressure ratio (r_u) at the moment of the slope failure. Extensive laboratory tests determined mean values of c and φ were 21.6 kPa and 7.3°, respectively, and standard deviations of c and φ were 9.0 kPa and 3.0°, respectively. The determination of the pore pressure ratio r_u at the moment of slope failure seems difficult because the ground water level was not measured then. On the basis of the monitoring data of the ground water level after the slope failure, the mean value of r_u was estimated as 0.35, and the standard deviation of r_u was assumed as 0.1. Let $\theta = \{r_u, c, \varphi\}$. Based on the above analysis, the mean and standard deviation of θ are $\mu_\theta = \{0.35, 21.6, 7.3\}$ and $\sigma_\theta = \{0.1, 9, 3\}$, respectively. Assuming that the elements of θ are statistically independent, the Morgenstern-Price method is then used for the slope stability analysis. For this particular slope, the mean and standard deviation of the model error is assumed as 0.02 and 0.07, respectively.

6.3. Back analysis of slope failure

The slope failure can provide valuable information about the parameters of the slope. Assuming that θ follows the multivariate normal distribution, the probabilistic back analysis method derived by Equations (5)-(7) is applicable. Figure 8 shows the spreadsheet template for implementing the above probabilistic back analysis method. The **H** vector is obtained based on sensitivity analysis of the slope stability model around the mean point. Based on Figure 8, the posterior values of mean and standard deviation of θ are $\mu_{\theta|d} = \{0.323, 26.126, 9.356\}$ and $\sigma_{\theta|d} = \{0.096, 7.574, 2.03\}$, respectively. Although the elements of θ are uncorrelated in the prior distribution, they are correlated in the posterior

distribution, which is a reflection of the constraint that the FOS of the slope at the moment of failure should be equal to unity.

	A	B	C	D	E	F	G	H	I	G	K
1		μ_θ	C_θ						r_u	c	ϕ
2	r_u	0.35	0.01	0	0		H	-0.712	0.0147	0.06	
3	c	21.6	0	81	0		$g(\mu_\theta)$	0.752			
4	ϕ	7.3	0	0	9		d_{obs}	1			
5											
6		μ_ε	σ_ε				$HC_\theta H^T$				
7		0.02	0.07				0.055084				
8											
9		$H^T H$					$\mu_{\theta\|d}$	$C_{\theta\|d}$			
10		0.5072	-0.01	-0.043			0.322929	0.0092	0.1414	0.064	
11		-0.01	0.0002	9E-04			26.12589	0.1414	57.364	-10.74	
12		-0.043	0.0009	0.004			9.35598	0.0642	-10.74	4.122	
13											

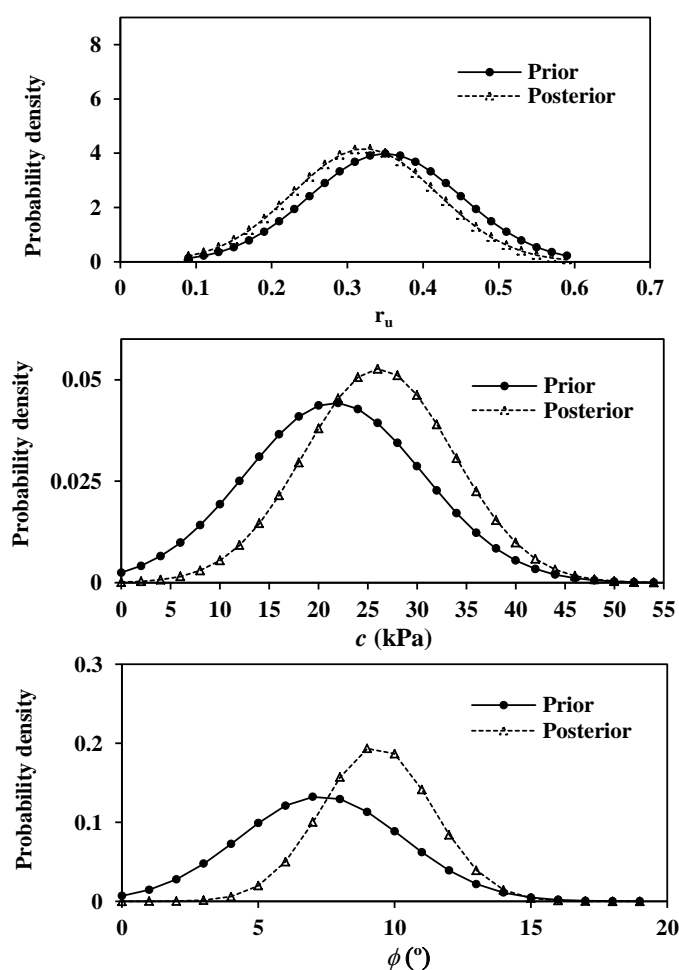**Figure 8.** Spreadsheet of probabilistic back-analysis of slope failure

Figure 9. Prior and posterior probability density of r_u, c and ϕ

As θ is assumed as multivariate normal, the posterior distribution of θ is also the multivariate normal. Figures 9(a), 9(b), and 9(c) compare the prior and posterior PDF of for r_u, c and ϕ, respectively. Note that after a back analysis, the obvious reduction in the spread of the posterior distributions indicates a corresponding reduction in the uncertainties in these input parameters. The degree of updating for different variables, however, differs. The probabilistic back analysis algorithm can automatically allocate the observed information to different variables considering their relative importance and contribution to the uncertainty of the overall performance of the slope.

6.4. Design of reinforcing piles
To restore the stability of the landslide, we provide as an example the use of reinforcing piles for slope stabilization. First designed using the traditional deterministic analysis, the adopted values of c, ϕ, and r_u are 26 kPa, 9.1°, and 0.25, respectively. Based on Chinese Specifications for Design of Highway Subgrades (JTGD30-2004), the target FOS is 1.3. This deterministic analysis in turn determined that a lateral resisting force of 14,334 kN is required to meet the target FOS.

Reliability methods are then used for design of the reinforced piles. To show the effect of back analysis, both the prior distribution of θ and the posterior distribution of θ are used to calculate the relationship between resisting force of the piles and the reliability index of the landslide, as shown in Figure 10. As the total resisting force increases from 5,000 kN to 15,000 kN, the reliability index changes from about 0.4 to 1.7 based on the prior distribution, and changes from 1.2 to about 3.8 based on the posterior distribution. If a resisting force of 14,334 is applied, the reliability index of the landslide will be only 1.65 based on the prior distribution of θ, which may be considered as inadequate. However, if the posterior distribution of θ is employed, the reliability index of the slope is approximately 3.75 when the resisting force is 14,343, which can often be considered as adequate. The reliability method can not only be used to assess the reliability of a design, but can also be used to determine the required resistance to achieve a certain target reliability level. For instance, to meet a target reliability index of 3.0, the required resisting force is 12,500 kN.
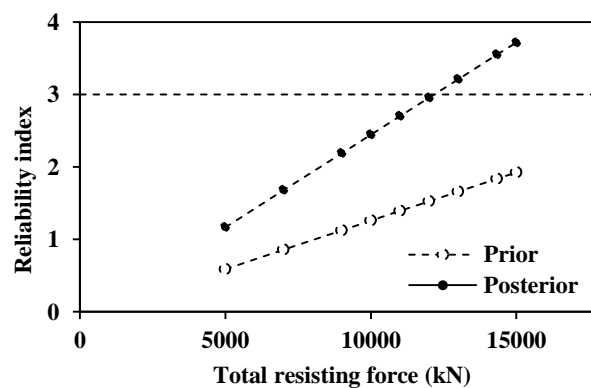

Figure 10. Reliability index at various lateral resisting forces

7. Application Example 2: Reliability-Based Robust Design of Slopes

7.1. Engineering background
The second case history in this article details the use of the robust design approach on the Sau Mau Ping rock slope in Hong Kong. The rock mass of the Sau Mau Ping slope is un-weathered granite with sheet joints, which is formed by the exfoliation processes during the cooling of granite. An initial and elemental analysis by Hoek (2006) determined that the Sau Mau Ping slope was composed of a single unstable block with a water-filled tension crack involving only a single failure mode. The geometry of

the Sau Mau Ping slope is illustrated in Figure 11. Following Hoek (2006), the slope before remediation has a height H of 60 m and a slope angle Θ of 50°. The potential failure plane is inclined at 35° ($\psi = 35°$) and the unit weight of rock (γ) is 2.6 ton/m^3, which are assumed to be fixed values. A deterministic model with a single failure mode developed by Hoek and Bray (1981) is adopted herein for the case study of the Sau Mau Ping slope, the detailed formulation for which can be found in Hoek (2006) and Wang et al. (2013b&2015).
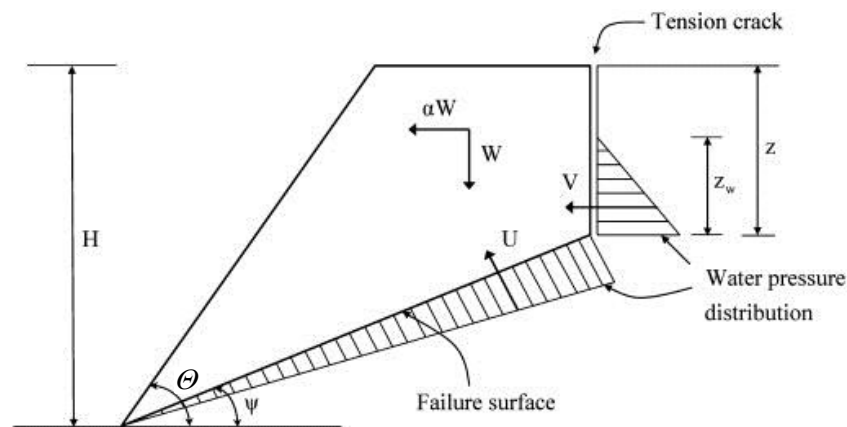

Figure 11. Illustration of geometry for plane sliding of Sau Mau Ping rock slope (Wang et al. 2013b)

There are two approaches for the remedial design of a rock slope: (i) geometry design, in which the slope height and slope face angle are reduced, and (ii) reinforcing of the rock slope using rock bolts and cables. Stabilizing rock slopes by means of geometry design is generally more cost-efficient than the use of cables and bolts (Hoek 2006). For ease of illustration, we will focus on the geometrical design of the Sau Mau Ping slope, with slope height H and slope angle Θ as design parameters. Robust reliability-based design of rock bolts for the Sau Mau Ping slope can be found in Wang et al. (2015).

7.2. Quantification of uncertainties
As summarized by Hoek (2006), five uncertain variables are considered in the reliability analysis of the Sau Mau Ping slope: the cohesion of rock discontinuities (c), friction angle of rock discontinuities (ϕ), tension crack depth (z), percentage of the tension crack depth filled with water (i_w), and gravitational acceleration coefficient (α). Following the previous studies in Hoek (2006) and Low (2007), c, ϕ and z are assumed to follow the normal distribution, and i_w and α are assumed to follow the truncated exponential distributions with mean and upper and lower bounds listed in Table 3. The mean values of these uncertain parameters can usually be adequately estimated even with a small sample size (Wu et al. 1989); however, the coefficient of variation (COV) and the correlation of coefficient (ρ) of these uncertain parameters are difficult to ascertain (Gong et al. 2014). In past studies (Hoek 2006; Low 2007), COV and ρ of these uncertain parameters were primarily estimated by engineering judgment or based on published literature.

According to a survey of literature by Lee et al. (2012), the COV of cohesion c, denoted as COV[c], typically ranges from 10% to 30%; the COV of friction angle ϕ, denoted as COV[ϕ], typically varies in the range from 10% to 20%. Furthermore, the coefficient of correlation for c and ϕ, denoted as $\rho_{c,\phi}$, is likely to be negative varying from -0.2 to -0.8; and the coefficient of correlation for z and i_w, denoted as $\rho_{z,iw}$, may vary from -0.2 to -0.8. Then, the mean and the COV values of these statistics are estimated. The mean of COV[c], denoted as $\mu_{COV[c]}$, is 0.20 and the COV of COV[c], denoted as $\delta_{COV[c]}$ is 17%; the mean of COV[ϕ], denoted as $\mu_{COV[\phi]}$, is 0.14 and the COV of COV[ϕ], denoted as $\delta_{COV[\phi]}$, is 12%; the mean of $\rho_{c,\phi}$, denoted as $\mu_{\rho c,\phi}$, is -0.50 and the COV of $\rho_{c,\phi}$, denoted as

$\delta_{\rho c, \phi}$, is 25%; and, the mean of $\rho_{z, iw}$, denoted as $\mu_{\rho z, iw}$, is -0.50 and the COV of $\rho_{z, iw}$, denoted as $\delta_{\rho z, iw}$, is 25%.

Table 3. Statistics of uncertain variables for the Sau Mau Ping rock slope (Wang et al. 2013b)

Random variables	Probability distribution	Mean	Std. dev.
c	Normal	10 ton/m^2	–
ϕ	Normal	35°	–
z	Normal	14 m	3 m
i_w	Exponential with mean 0.5, truncated to [0, 1]		
α	Exponential with mean 0.08, truncated to [0, 0.16]		

7.3. Robust design of slope geometry for slope stabilization

For the two design parameters of the Sau Mau Ping slope, the slope height *H* may typically range from 50 m to 60 m, and slope angle *Θ* may typically range from 44° to 50°. For convenience of construction, slope height *H* may be rounded to the nearest 0.2 m and slope angle *Θ* may be rounded to the nearest 0.2°. Thus, a discrete design space is adopted herein: *H* may assume 51 discrete values and *Θ* may assume 31 discrete values in their possible ranges, which yields a total of 1,581 candidate designs.

   When the statistics of the random variables are uncertain, the calculated failure probability of the slope is also uncertain. Within the framework of the robust reliability-based design, a target failure probability (i.e., p_T = 0.0062) can be set as the safety constraint and used to screen the unsatisfactory designs based on the mean value of the failure probability (μ_p). The standard deviation of the failure probability (σ_p) can be used to gauge the robustness, with a smaller standard deviation of the failure probability signaling higher design robustness. In this example, the cost of a rock slope design is approximated as the volume of rock mass that is to be excavated (Duzgun et al. 1995). The design is considered as most competitive when it is optimal to both cost efficiency and design robustness while satisfying the safety constraint. Mathematically, the robust reliability-based design can be formulated as a multi-objective optimization problem as follows

Find   **d** = [*H*, *Θ*]

Subject to: $H \in$ {50m, 50.2m, 50.4m,…, 60m } and $\Theta \in$ {44°, 44.2°, 44.6°,…, 50°}

           $\mu_p < p_T$ = 0.0062

Objectives: Minimizing the standard deviation failure probability (σ_p),

           Minimizing the cost for rock slope design

   NSGA-II (Deb et al. 2002) can be used to solve the above multi-objective optimization, in which the designs optimal to both objectives (cost and robustness) are searched iteratively in the discrete design space. With this algorithm, 89 non-dominated optimal designs are selected into a Pareto Front, as shown in Figure 12. As can be seen from this figure, an apparent trade-off relationship exists between design robustness and cost efficiency, and greater design robustness can only be attained at the expense of a higher cost. The Pareto Front is useful as a design aid for the final decision making especially when an acceptable cost or robustness level is specified. For instance, when the maximum acceptable cost is set at 200 units, 38 designs on the Pareto Front are deemed acceptable (Figure 12). The design with the lowest σ_p value in this cost range is the best design based on the robustness criterion, and in this scenario, it is a design with *H* =50 m and *Θ* = 50°. The optimal design of *H* =50 m and *Θ* = 50° has a cost of 195.1 units and a standard deviation of failure probability of 1.25×10^{-3}.

   Furthermore, the feasibility robustness index β_β, defined as follows, may be computed and used as a decision making aid.

$$P[p_f - p_T \leq 0] = \Phi(\beta_\beta) \geq P_0 \tag{18}$$

The feasibility robustness index β_β is readily computed for each of the 89 non-dominated optimal designs on the Pareto Front. As expected, the design with higher feasibility robustness costs more. When a target feasibility robustness level (β_β^T) is selected, the least-cost design that satisfies the feasibility robustness level can easily be identified, which renders a final design using the proposed robust design method. As an example, Table 4 shows the final designs identified using the robust design method for different levels of target feasibility robustness.
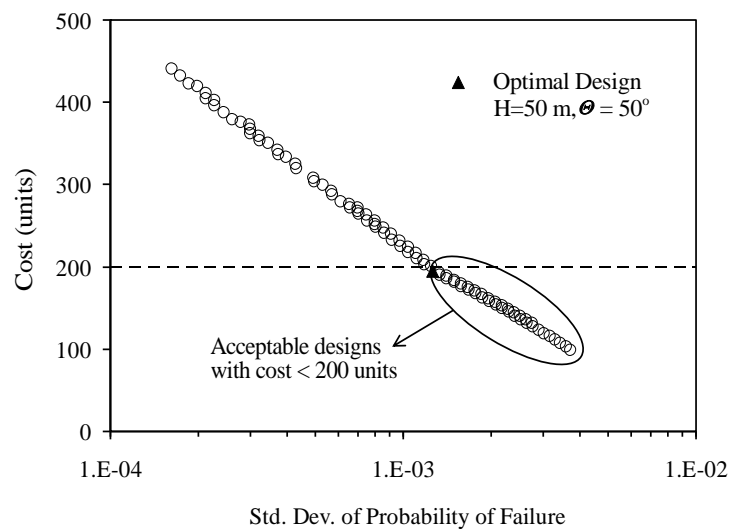

Figure 12. Pareto Front obtained by NSGA-II and selection of final optimal design based on the acceptable cost level (Wang et al. 2013b)

Table 4. Selected final designs at various target feasibility robustness levels (Wang et al. 2013b)

β_β^T	P_0	H (m)	Θ (°)	Cost (units)
0.5	69.15%	54.0	50.0	107.0
1.5	93.32%	51.4	49.8	170.6
2.5	99.38%	50.0	45.4	378.9

8. Concluding Remarks

The characterization and assessment of the effect of uncertainties on slope stability evaluation are important in the analysis and design of slopes. In recent years, the probabilistic methods have gradually evolved into a practical tool for stability analysis of slopes considering uncertainties, and simple yet sound methods are available for addressing challenging problems that are hard to answer within a deterministic framework. In this paper, we have introduced and illustrated practical methods developed recently to address important problems related to reliability-based assessment of slope stability. The following conclusions can be drawn from this paper.

- Probabilistic methods are now available for characterizing the uncertainty associated with the slope stability model considering the presence of parameter uncertainties. The model error associated with a slope stability model can be determined if the field performances of similar slopes in a region are available. For a specific slope, simple probabilistic back analysis methods can be used to update the probabilistic distribution of uncertain variables based on the observed failure. These methods provide practicing engineers tools for better uncertainty characterization as the basis for slope reliability analysis.

- The most critical slip surface of a slope can be found with the empirical procedure suggested by Hassan and Wolff quite accurately. After the most critical slip surface is found, the failure probability can be calculated based on a second-order polynomial-based response surface method. The Hassan Wolff procedure can also be used to locate representative slip surfaces that govern the system failure probability of a slope. Combined with response surface method, the Hassan and Wolff method can be extended into a practical tool for evaluating the system reliability of a slope efficiently.

- The past performance information of a slope can be viewed as the outcome of a full-scale test directly performed on the slope under investigation, and can be directly used for reliability updating. When the past performance information is considered, the uncertainty in the slope stability analysis can be reduced, and the reliability of the slope can be evaluated more realistically.

- When the probabilistic distribution of random variables cannot be determined accurately, the reliability-based design approach can be implemented as a multi-objective optimization problem by considering design robustness as one additional design objective. In such a case, a set of non-dominated optimal designs that satisfy the safety (reliability) reθuirement can be found, which show the trade-off relationship between design robustness and cost efficiency. The robust design method can be a useful design aid in the environment of high uncertainties. It complements the reliability-based methods for the analysis and design of a slope.

Acknowledgements

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