

Experimental analysis of the dynamic behavior of a rotating disk submerged in water

A Presas¹, D Valentín¹, E Egusquiza¹, C Valero¹ and U Seidel²

¹Center for Industrial Diagnostics and Fluid dynamics, University Polytechnic of Catalonia, Av. Diagonal, 647, Barcelona, 08028, Spain.

²Voith Hydro Holding GmbH & Co. KG; Alexanderstraße 11, 89522 Heidenheim, (Germany)

E-mail: alex.presas@mf.upc.edu, david.valentin@mf.upc.edu, egusquiza@mf.upc.edu, valero@mf.upc.edu, Ulrich.Seidel@voith.com

Abstract. To study the dynamic behavior of turbine runners (natural frequencies and mode shapes) not only the added mass effect of still water has to be considered. Also the effect of rotation may not be neglected in the dynamic response. In the present study, the dynamic behavior of a rotating disk submerged in water is studied. For this purpose an experimental test rig has been developed. It consists of a rotating disk submerged in water that can be excited and its response can be measured from the rotating system by a slip ring system. For the excitation an impact device installed on the casing has been used. The response is measured with miniature accelerometers screwed on the disk. The influence of rotation on the dynamic response has been determined experimentally.

1. Introduction

The dynamic behavior of turbine runners in operation is a difficult topic to be studied experimentally since the runner is a complex structure confined and submerged, that during its operating condition rotates. The rotation of this part with small gaps between the static guide vanes and the rotating blades leads to a complex flow that excites the runner. This excitation is known as Rotor Stator Interaction excitation or RSI. RSI may produce some damages on the structure as reported in some cases [1]. Therefore it is of paramount importance to determine the effects of the confinement (small gaps to the casing) and the effects of the rotation on the natural frequencies of this part of the machine. In order to study these effects in a systematic way, simplified models such as rotating and submerged disks are necessary.

Some works have been carried out regarding the dynamic behavior of submerged structures and its added mass effect. Kwak in [2] studied the hydroelastic vibrations of circular plates. Nevertheless, the effect of nearby rigid walls was not considered. This problem was studied firstly by Lamb [3]. He considered the contact plate-water in one side. Rodriguez [4] and Harrison [5] studied the influence of only one nearby rigid wall in the natural frequencies of a rectangular plate. Their results concluded that the distance plate-wall has a great influence on the added mass effect; i.e. the natural frequencies of the plate are generally reduced when the plate is closer to the rigid wall. The case of confined plates, which makes the problem much more complex, has been studied recently by Askari in [6]. He provided a complete formulation for the flow above and under the disk. The influence of the radial gap



and the influence of the free surface in the natural frequencies was also determined. Although the influence of the surrounding fluid, the nearby rigid walls and the free surface in the hydroelastic vibration of plates has been considered in the mentioned cases, no experimental results have been found where the effect of the rotation of the surrounding water is considered.

This effect has to be considered to describe the real operating condition of hydraulic turbomachinery. Kubota in [7] investigated this problem. He proposed an analytical model to study the effect of rotation in the natural frequencies of a rotating disk in water. He used a simplified model for the disk structure and for the flow in the tank. With these assumptions, an analytical expression was deduced in that paper to calculate the natural frequencies of the disk. The solution was provided for the case that only one surface of the disk is in contact with a high density fluid (water) and all the fluid is moving at the same rotating speed with respect to the disk. The influence of the viscosity of the fluid was not considered and the case of a confined disk in rotation was not studied. Furthermore, no experimental results were given in that study.

In this paper, the effect of the rotation on the natural frequencies and mode shapes of a submerged disk is studied experimentally. For this purpose a rotating disk test rig has been made. It consists on a disk that can be excited and its dynamic response measured from the rotating system. For the excitation an impact device fixed on the casing has been used. To measure the disk vibration miniature accelerometers have been attached on the disk. The first several natural frequencies and mode shape of the disk have been determined for the disk rotating in water and in air. Results are contrasted with the analytical model proposed in [7]

2. Analytical model

2.1. Natural Frequencies and mode shapes

The model presented in this paper can be found in [7]. The simplified case of study is shown in figure 1.

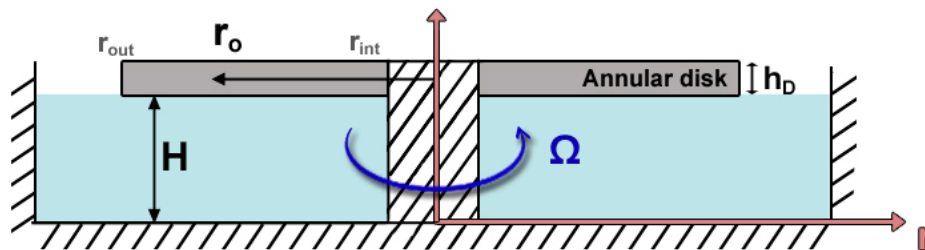


Figure 1: Fluid rotating with respect to the disk

The basic assumptions for this model is that there is a potential flow that rotates uniformly with a velocity Ω with respect to the disk. Furthermore it is considered that the differential coefficients of the motion of the fluid and the disk in the radial direction are negligibly small [7] and the disk and flow motion is studied in the averaged radius $r_o = \sqrt{r_{int} \cdot r_{out}}$. In this case, the equation that has to be accomplished for this flow in cylindrical coordinates is:

$$\frac{1}{r_o^2} \frac{\partial^2 \phi}{\partial \varphi^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (1)$$

The boundary conditions that have to be accomplished in this case is that the axial velocity of the flow on the rigid wall is equal zero, and on the disk surface is equal to the vibration of the disk, i.e.:

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=0} = 0 \quad (2)$$

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=(H)} = \frac{\partial w}{\partial t} \quad (3)$$

The vibration of the disk, particularized in the averaged radius r_o , for each mode n can be expressed as following (equation (4)):

$$w = \sum_{n=\pm 1}^{\infty} A_n e^{jn\theta} e^{j\omega_n t} \quad (4)$$

In this case, only the modes with no nodal circles and n diametrical modes are considered. ω_n are the natural frequencies of the disk. The characteristic equation given in [7] for this model is:

$$\left(\left[\coth \left(\frac{nH}{r_o} \right) \right] \frac{\rho_F r_o}{n} + \rho_D h_D \right) \omega_n^2 + \left(\left[\coth \left(\frac{nH}{r_o} \right) 2n\Omega \right] \frac{\rho_F r_o}{n} \right) \omega_n + \left(-\frac{D^*}{r_o^4} n^4 + \left[\coth \left(\frac{nH}{r_o} \right) n^2 \Omega^2 \right] \frac{\rho_F r_o}{n} \right) = 0 \quad (5)$$

ρ_D, ρ_F are the densities of the disk and surrounding fluid respectively. D^* is a parameter that has units of stiffness and can be calibrated with the natural frequencies of the disk in air (see equation (6)). This analytical equation leads us to calculate the natural frequencies for each mode n . The solutions obtained in equation (5) have to be substituted in equation (4) in order to determine the mode shapes of the disk.

2.1.1. Surrounding fluid air

When the surrounding fluid is air, since the density ρ_F is small, equation (5) can be simplified as:

$$(\rho_D h_D) \omega_n^2 + \left(-\frac{D^*}{r_o^4} n^4 \right) = 0 \quad (6)$$

The parameter D^* can be calibrated comparing the natural frequencies obtained with equation (6) and exact analytical methods presented in some references [8, 9].

Equation (6) gives the same solutions ω_n for each pair of n -positive and n -negative. If this pair of natural frequencies is substituted in equation (4), the motion of the disk can be rewritten as:

$$w = \sum_{n=\pm 1}^{\infty} 2A_n \cos(n\theta) \cos(\omega_n t) \quad (7)$$

This motion can be understood as a standing wave (all the points of the disk moving in phase or in counterphase) viewed from the disk with $2 \cdot n$ nodal points in the circle.

2.1.2. Surrounding fluid water

In this case the pair of n -positive and n -negative has two different solutions in equation (5) for ω_n . The motion of the disk can be expressed as:

$$w = \sum_{n=\pm 1}^{\infty} A_n (\omega_n t + n\theta) \quad (8)$$

For each n (positive or negative), there is a different natural frequency. Corresponding to each natural frequency, there is a mode shape with frequency in time $\omega_n/2\pi$ that is a travelling wave (all the points moving with a different phase to each other). In this case the absolute value of n indicates the number of nodal diameters. The sign of n indicates the travelling direction of the travelling wave.

If it is positive, the wave travels in the same direction as the disk and if it is negative it travels in the opposite direction.

3. Experimental tests

3.1. Test rig

To study the dynamic behavior of a rotating disk submerged in water, a test rig setup has been developed. It consists of a disk connected to a variable speed motor. The disk rotates up to 8 Hz . When the disk is rotating, it is excited with an impact device installed on the casing.

The disk vibration is measured with miniature and submersible accelerometers Dytran 3006-A, that are screwed on the disk. The added mass of the sensors and actuators is less than 1% of the mass of the disk, so it is checked that after installation of them, the dynamic behavior of the disk doesn't change. Due to the limited number of channels only 4 sensors can be used simultaneously.

All the excitation and response signals of the rotating frame are transmitted to the stationary frame through a slip ring system Michigan S10. This system is attached at the tip of the shaft.

The rotating speed of the motor Mavilor MLV-072 is digitally controlled. The rotation of the motor is transmitted to the shaft through a cog belt. The vibrations of the motor are isolated from the rest of the test rig through a silent block. A view of the test rig used is shown in figure 2 (without showing the transmission motor-shaft).

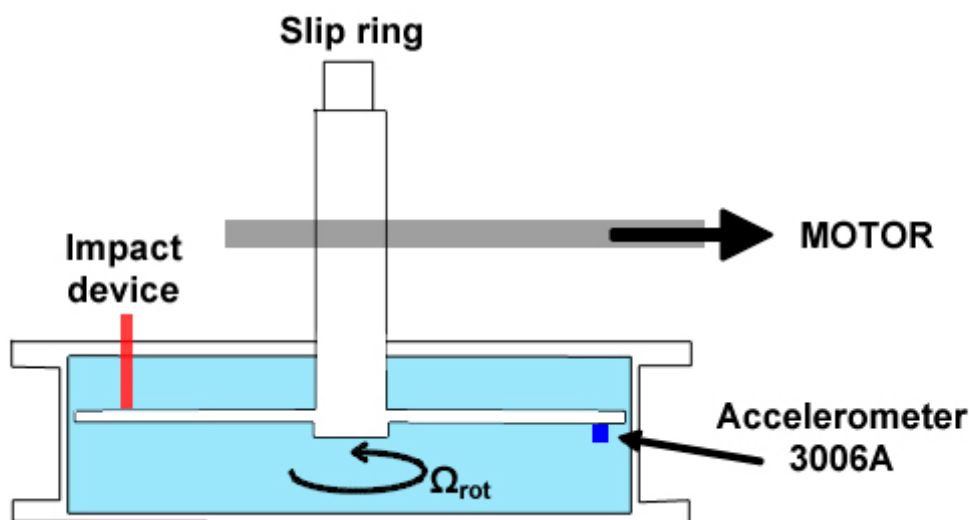


Figure 2: Overview of the test rig

3.2. Accelerometers disk

The disk is made of stainless steel and has a hole on its center in order to attach the shaft. The nomenclature used for the accelerometers is A-X, where X is the angle with respect to the 0° direction in counterclockwise direction, when the disk is attached to the shaft and viewing the test rig from the top. Four accelerometers are screwed on the disk (A-0, A-90, A-180 and A-210).

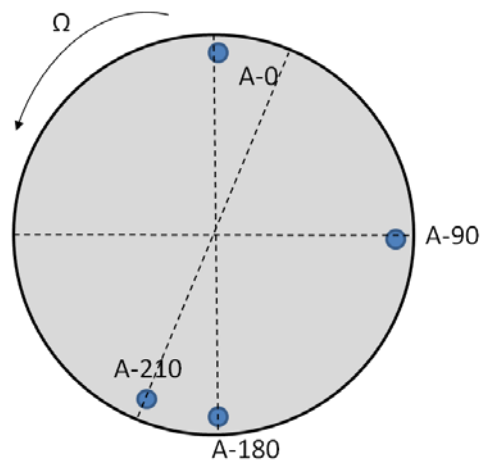


Figure 3: Rotating disk with the screwed accelerometers (viewed from the bottom)

4. Results and discussion

In order to determine the natural frequencies and mode shapes of the disk experimentally, the disk is impacted in the different situations tested. Here the discussion is centered on the mode $n=\pm 2$. This mode will be studied for the case that the disk is not rotating ($\Omega_{disk}=0\text{Hz}$) and for the disk rotating at the maximal rotating speed tested ($\Omega_{disk}=8\text{Hz}$). The aim of this study is to validate the main effects explained in the analytical model of section 2, i.e. when the disk is rotating in water. Furthermore, the case of the disk in air (rotating and not rotating) will be shown as comparison.

Figure 4, shows the modes $n=\pm 2$ for the non rotating case ($\Omega_{disk}=0\text{Hz}$) for the disk in air (figure 4a) and for the disk in water (figure 4b). Due to added mass effect of water, the value of the natural frequency when the disk is submerged is smaller. Analyzing the amplitudes of both cases, it can be seen that A-0, A-90 and A-180 have approximately the same amplitude since it is a mode with two nodal diameters (each maximum or minimum occurs every 90°). The accelerometer A-210 shows small amplitude, because it is located close to a nodal point.

Analyzing the phase of both cases it can be seen, that the difference between them is approximately 0° or 180° . This confirms that the corresponding mode shape is a standing wave, as predicted in the analytical model when $\Omega_{disk}=0\text{Hz}$.

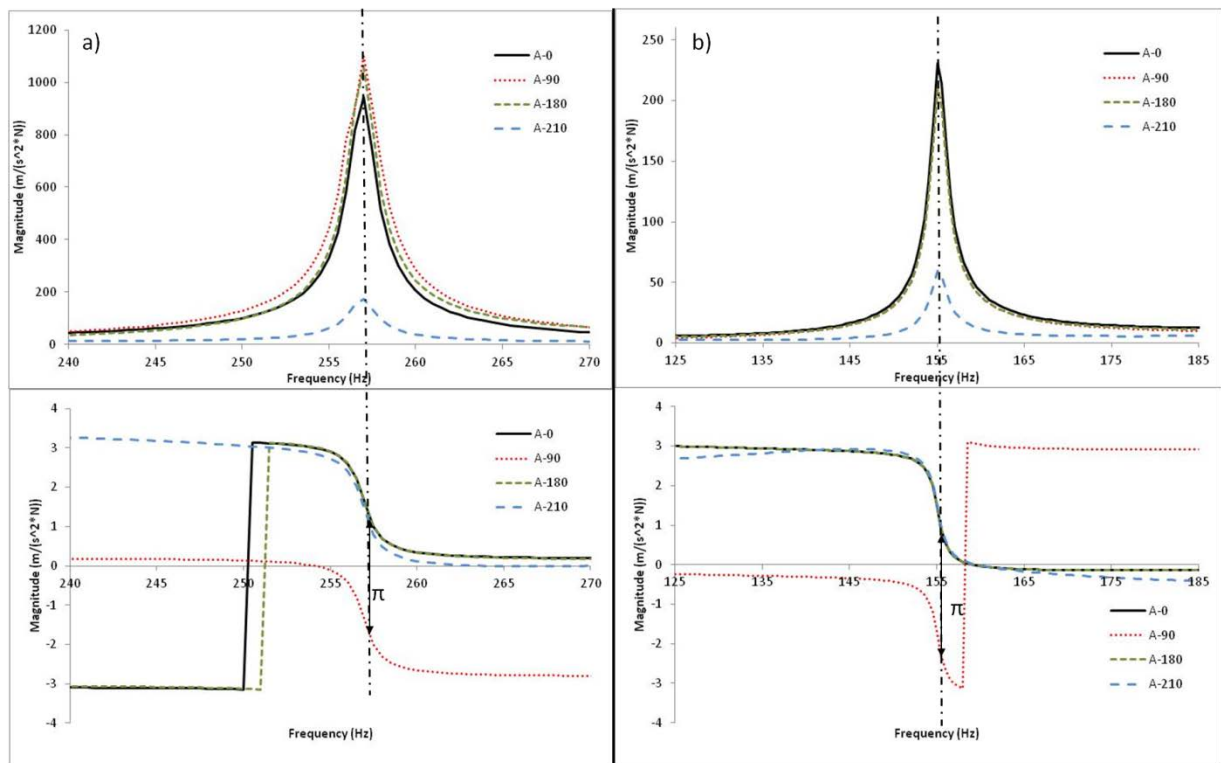


Figure 4:a) Natural frequency $n=\pm 2$ for the disk in air and $\Omega_{disk}=0\text{Hz}$ b) Natural frequency $n=\pm 2$ for the disk in water and $\Omega_{disk}=0\text{Hz}$

The same situation but for the disk rotating ($\Omega_{disk}=8\text{Hz}$) is shown in figure 5. When the disk rotates in air (figure 5a), the same behavior as the former cases is observed, i.e. all the points moving in phase or in counterphase with different amplitude of vibration, depending on their location. Only one peak is observed in this case as predicted with the analytical model (equation (6)), when the density of the surrounding fluid is negligibly small.

When the disk rotates in water (figure 5b) the behavior of the disk is totally different. The first observation is that two peaks appear instead of one. Analyzing the amplitudes, it can be seen that they are approximately the same for all the sensors used. Analyzing the phase shift between sensors, it is shown that now the phase between them is not 0° and 180° . This kind of motion (phase or counterphase to each other) is only observed between the accelerometers A-0, A-90 and A-180. According to equation 8, the phase shift for the modes $\pm n$ should be $\pm n\theta$. Therefore, these three sensors are moving in phase or in counterphase to each other. But observing the relative phase between A-180 and A-210 this is approximately $\pm 60^\circ$ as predicted with equation (8). All the mentioned facts, demonstrate that the mode shapes that appear on the disk for this n , are two travelling waves.

The travelling direction of the waves can be determined with the phase shift of the sensor A-210 with respect to A-180. In the first peak the sensor A-210 is advanced in phase with respect to A-180. Therefore this peak corresponds to a travelling wave travelling counterclockwise if the view of figure 4 is taken into account. As seen in this figure, this direction corresponds to the rotating direction of the disk. For the second peak the travelling direction is the opposite direction, since A-210 is delayed with respect to A-180.

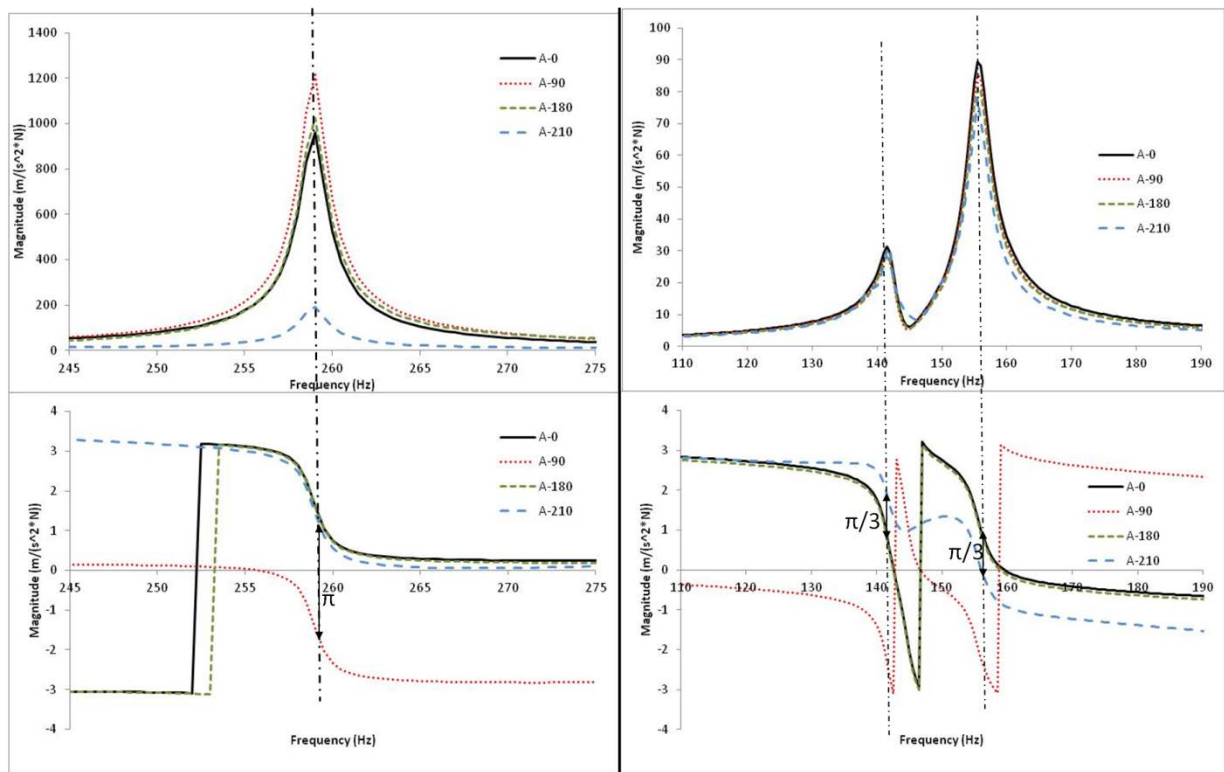


Figure 5: a) Natural frequency $n=\pm 2$ for the disk in air and $\Omega_{disk}=8\text{Hz}$ b) Natural frequency $n=\pm 2$ for the disk in water and $\Omega_{disk}=8\text{Hz}$

4.1. Discrepancy with the analytical model

The value of the natural frequency of the disk (rotating and not rotating) in air can be predicted with the analytical model proposed by Kubota in [7]. Nevertheless, when the disk is submerged in water the values predicted with the analytical model do not match the experimental values. When the disk is not rotating, the predicted value with the model is higher as the experimental value. This is because the model only considers the disk in contact with water in one side. In this case, the distance disk-upper cover is considered as H in the analytical model, since in the tested position the upper fluid has the most relevant effect on the added mass, because the disk is closer to the upper wall. The comparison between experimental and analytical results for these cases is shown in Table 1.

Table 1. Natural frequency of the mode $n=\pm 2$. Experimental, analytical and percentage error

	Experimental	Analytical	Percentage error
Air ($\Omega_{disk}=0\text{Hz}$)	257,25 Hz	255 Hz	0,87 %
Air ($\Omega_{disk}=8\text{Hz}$)	258,75 Hz	255 Hz	1,45%
Water ($\Omega_{disk}=0\text{Hz}$)	155,25 Hz	175,48 Hz	13,03%

The other source of discrepancy is the rotating speed of the fluid. The difference between the two natural frequency of each mode n ($f_{n-} - f_{n+}$) increases linearly with the rotating speed of the fluid (with respect to the disk). Since the model considers that the fluid rotates uniformly with respect to the disk (potential flow) with a velocity equal the rotating speed of the disk, the difference ($f_{n-} - f_{n+}$) is not well predicted by the model. In order to determine the rotating speed of the fluid with more accuracy, the viscosity has to be taken into account and the real flow field has to be considered. The comparison

between analytical and experimental results for the values f_{n-} , f_{n+} and $f_{n-} - f_{n+}$ when the disk is rotating ($\Omega_{disk}=8\text{Hz}$) in water is shown in Table 2.

Table 2. Natural frequency of the modes $n=2$, $n=-2$ and difference for the submerged and rotating disk ($\Omega_{disk}=8\text{Hz}$). Experimental, analytical and percentage error

	Experimental	Analytical	Percentage error
$f_{n=+2}$	141,75 Hz	166,88 Hz	17,04 %
$f_{n=-2}$	155,5 Hz	183,72 Hz	18,15 %
$f_{n=-2} - f_{n=+2}$	13,75 Hz	16,85 Hz	22,54 %

5. Concluding remarks and future work

In this paper a disk that rotates inside a tank full of water has been excited with an impact device installed on the casing and the response has been measured with miniature accelerometers attached on the disk.

The previous study of Kubota[7] has been used as the reference for the analytical model of a rotating disk with surrounding fluid water. In this paper, the main effects of the surrounding rotating water on the natural frequencies that were mentioned in that study have been checked experimentally. Nevertheless, some points have to be changed in the model to have a better prediction of the values of the natural frequencies.

As future work, in order to improve the analytical model the whole fluid field has to be considered (in case that a totally submerged disk is studied) instead of only one side of the disk. Furthermore, the viscosity effects on the velocity flow field have to be taken into account in order to determine the rotating speed of the fluid with respect to the disk with accuracy.

Acknowledgment

The authors acknowledge the Spanish Ministry of Economy and Competitiveness for the economic support received from Grant No. DPI2012-36264. The authors also wish to acknowledge Voith Hydro Holding GmbH & Co. KG for the technical and economic support received for developing this work.

References

- [1] Egusquiza E, Valero C, Huang X, Jou E, Guardo A and Rodriguez C 2012 Failure investigation of a large pump-turbine runner *Engineering Failure analysis* **23** 27-34
- [2] Kwak M K 1991 Vibration of circular plates in contact with water *Journal of Applied Mechanics* **58** 480-3
- [3] Lamb H 1921 On the vibrations of an elastic plate in contact with water *Proceedings of the Royal Society of London A* **98** 205-16
- [4] Rodriguez C G, Flores P, Pierart F G, Contzen L R and E.Egusquiza 2012 Capability of structural-acoustical FSI numerical model to predict natural frequencies of submerged structures with nearby rigid surfaces *Computers&Fluids* **64** 117-26
- [5] Harrison C, Tavernier E, Vancauwenberghe O, Donzier E, Hsu K, Goofwin A, Marty F and Mercier B 2007 On the response of a resonating plate in a liquid near a solid wall *Sensors and Actuators A: Physical* **134** 414-26
- [6] Askari E, Jeong K-H and Amabili M 2013 Hydroelastic vibration of circular plates immersed in a liquid-filled container with free surface *Journal of Sound and Vibration* **332** 3064-85
- [7] Kubota Y and Ohashi H 1991 A study on the natural frequencies of hydraulic pumps *1st ASME Joint International Conference on Nuclear Engineering* 589-93
- [8] Belvins R 1984 *Formulas for natural frequency and mode shape*: Krieger Publishing Company)
- [9] Leissa A W 1969 *Vibration of Plates* (Washington (USA): Government Printing Office)