

Influence of the added mass effect and boundary conditions on the dynamic response of submerged and confined structures

D Valentín¹, A Presas¹, E Egusquiza¹, C Valero¹

¹Centre de Diagnòstic Industrial i Fluidodinàmica (CDIF), Universitat Politècnica de Catalunya, Av. Diagonal, 647, ETSEIB, 08028 Barcelona

E-mail: david.valentin@mf.upc.edu

Abstract. The dynamic response of submerged and confined disk-like structures is of interest in the field of hydraulic machinery, especially in hydraulic turbine runners. This response is difficult to be estimated with accuracy due to the strong influence of the boundary conditions. Small radial gaps as well as short axial distances to rigid surfaces greatly modify the dynamic response because of the fact of the added mass and damping effects. Moreover, the effect of the shaft coupling is also important for certain mode-shapes of the structure. In the present study, the influence of the added mass effect and boundary conditions on the dynamic behavior of a submerged disk attached to a shaft is evaluated through experimental tests and structural-acoustic coupling numerical simulations. For the experimentation, a test rig has been developed. It consists of a confined disk attached to a shaft inside a cylindrical container full of water. The disk can be fixed at different axial positions along the shaft. Piezoelectric patches are used to excite the disk and the response is measured with submersible accelerometers. For each configuration tested, the natural frequencies of the disk and the shaft are studied. Numerical results have been compared with experimental results.

1. Introduction

Along with technology and manufacturing level of hydraulic turbines improvement, there is a trend to increase the power concentration of the units. Therefore, the head, fluid velocity and hydraulic forces are being considerably increased and, consequently, some severe vibration problems could be induced. Moreover, due to power regulation requirements imposed by the global energy production system, hydraulic turbines are increasingly working in off-design conditions causing serious vibration problems in the machine. If the natural frequencies of the structure are not accurately calculated during the stage of design, it is not possible to ensure that, under operation, a certain force could excite a natural frequency producing resonance. Important failures in hydraulic turbomachinery where resonance played an important role are documented (e.g. Ohashi [1] and Egusquiza et al. [2]).

When a structure is submerged in a dense fluid its natural frequencies, and consequently its dynamic behavior, drastically change in comparison with those in vacuum. Moreover, if the submerged structure is close to a rigid surface its dynamic response is also affected. Pump-turbines have submerged rotating parts extremely close to rigid static surfaces, such as the upper and lower case clearance, the radial gap between the runner and the volute or the labyrinth seals (see Figure 1).



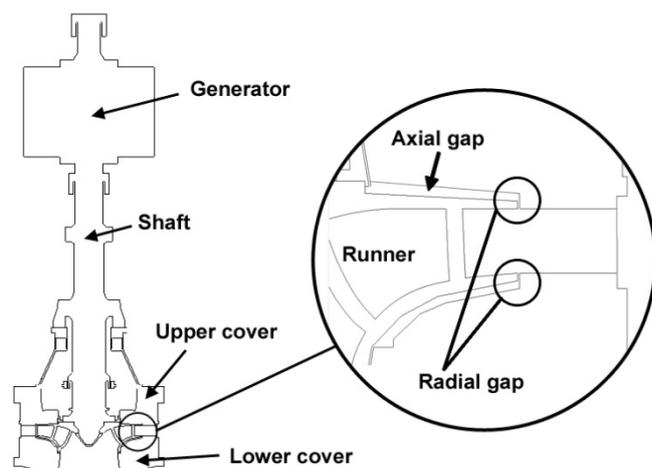


Figure 1. Example of the nearby rigid surfaces in a hydraulic turbine runner.

The effect of the hydraulic seals on the dynamic response in a Francis turbine is uncertain as it is shown in the numerical and experimental work of Liang et al. [3] and Rodriguez et al. [4]. Hübner et al. [5] confirmed the effect of these seals was not considered in the natural frequencies' calculation during the design stage of hydraulic turbine runners because the simulations were carried out assuming large distances between the runner submerged in a cylindrical water tank and the stationary parts. Moreover, these simulations did not consider the shaft-runner coupling effect, which is rather important for certain vibration modes where the structure is deformed both axially and radially.

Simpler models, such as cantilever plates or disks, have been used since the middle of last century in order to easily study the natural frequencies of submerged structures. Current analytical studies of natural frequencies for submerged structures are based on the added mass effect. The effect on the natural frequencies of a submerged structure is the same as considering an additional mass. Thus, according to the added mass theory, a structure submerged in water has lower natural frequencies than in vacuum or in the air. The added mass effect also explains that the closer to a rigid surface is the full-submerged structure, the lower natural frequency values it has.

The effect of the added mass for disks in an infinite medium of water has been investigated for many years. Ginsberg and Chu [6] developed a variational method to derive the mode-shapes of a circular plate in contact with a heavy fluid. Amabili and Kwak [7] applied the Rayleigh-Ritz method to study the free vibrations of circular plates inside liquids. Gorman and Horacek [8] presented other energy-based methods, such as the Laplace method, showing similar results than the Rayleigh-Ritz method. However, these investigations only considered submerged structures in an infinite medium of water for axial mode-shapes, neither radial mode-shapes nor nearby rigid surfaces were studied.

The effect of nearby rigid surfaces on the added mass of submerged disks was studied by Kubota and Suzuki [9], Kubota and Ohashi [10] and Askari et al. [11]. Kubota and Suzuki [9] and Kubota and Ohashi [10] analytically and experimentally studied the influence of the axial rigid surface of a submerged disk, however no radial gap between the disk and a rigid surface was considered. Askari et al. [11] studied the influence of the axial nearby rigid surface on the natural frequencies of two different submerged disks. They numerically estimated the effects of the radial gap between the disk and the nearby rigid surface considering the disk as a thin beam. They concluded that the nearby rigid surface had an important influence on the natural frequencies in the axial direction and a small contribution in the radial direction. However, all the mode-shapes studied in these works were axial mode-shapes because no shaft was considered.

In the present study, an experimental analysis of a submerged-confined disk is carried out. The disk is made of stainless steel and it is submerged in water and coupled to a shaft which is part of an aluminum tank. The disk can be moved up and down along the shaft in order to evaluate the influence of the shaft coupling position on the natural frequencies value. The radial gap between the disk and the tank is very small (0.35 ± 0.05 mm), the same proportion that there is approximately in a hydraulic

turbine against its runner radius. Moreover, structure-acoustic numerical simulations are performed in order to validate the experimental method and to prove the capability of the simulation to predict natural frequencies of submerged-confined structures.

2. Theoretical background

The dynamic behavior of a body vibrating in the vacuum can be studied assuming a simplified mass, spring and damper system of 1 DOF. The equation of motion that describes the vibration of this system is:

$$M\ddot{X} + C\dot{X} + KX = 0 \quad (1)$$

where M is the mass of the body, C is the structural damping, K is the spring constant and X is the displacement of the body from the equilibrium position. If dissipation effects are neglected, the damping term disappears, becoming Eq. (1) in:

$$M\ddot{X} + KX = 0 \quad (2)$$

From this equation the natural frequencies can be calculated for each mode shape as:

$$f_v = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \quad (3)$$

However, when a quiescent fluid is considered around the body, a fluid force (F_f) appears over the surface of the body, modifying the general motion equation (Eq.(2)) as:

$$M\ddot{X} + KX = F_f \quad (4)$$

The fluid force applied to the body (F_f) is mainly due to the pressure drag and it can be characterized by the assumption of considering the fluid as incompressible and Newtonian in the Navier-Stokes equations. If nonlinear terms in the Navier-Stokes equations are neglected, F_f is defined as:

$$F_f = -M_A\ddot{X} \quad (5)$$

The coefficient M_A is called the added mass and it is defined as the fluid force applied to the structure due to the inertia of the fluid entrained by the moving structure. Evaluating the fluid force (F_f) on the dynamic motion of the submerged body, Eq. (4) becomes:

$$M\ddot{X} + KX = -M_A\ddot{X} \quad (6)$$

which can be rewritten as in Eq. (2) in order to describe the free vibration of the body.

$$(M + M_A)\ddot{X} + KX = 0 \quad (7)$$

As in the case of vacuum, the natural frequencies value of the system for each corresponding mode-shape can be calculated from Eq.(7) as:

$$f_f = \frac{1}{2\pi} \sqrt{\frac{K}{M + M_A}} \quad (8)$$

Comparing Eq. (2) and Eq. (8), the natural frequency value of the body submerged in a fluid is lower than in the vacuum due to the added mass (M_A). Assuming that the natural frequencies in the vacuum are practically the same as in the air, a dimensionless added mass factor (λ) can be introduced. This factor is defined as the added mass (M_A) over the modal mass (M). In Eq. (9) this dimensionless factor is computed from natural frequencies in air and submerged in a fluid.

$$\lambda = \frac{M_A}{M} = \left(\frac{f_a}{f_f}\right)^2 - 1 \quad (10)$$

The previous formulation explains the dynamic behavior of a submerged body in an infinite medium of fluid. However, the boundary conditions of the fluid medium also affect the dynamic behavior of the structure but it is difficult to find an expression to calculate their influence due to the complex formulation of the Navier-Stokes equations. Moreover, combined mode-shapes (mode-shapes which have radial and axial displacement at the same time) are not easily described with these dynamics equations of 1DOF. Therefore, experimental results are necessary to understand the influence of boundary conditions even in simpler models.

3. Experimental investigation

3.1. Test rig description

The test rig used in the present work comprises a stainless steel disk coupled to a shaft and confined in an aluminum tank (see Figure 2). The disk has a diameter of 250 mm, whereas the diameter of the shaft is of 40 mm. The disk thickness is of 8 mm and the height of the shaft is 100 mm. The tank has a large thickness and a heavy mass in order to not influence on the natural frequencies of the disk. The disk can be moved up and down along the shaft to evaluate the influence of the disk position (H). There is a small radial gap (G) between the disk and the tank (0.35 ± 0.05 mm). The radial gap was selected as 0.35 mm in order to represent the same order of magnitude that there is in a hydraulic pump-turbine against its runner radius ($\frac{G}{R_D} \approx 10^{-3}$).

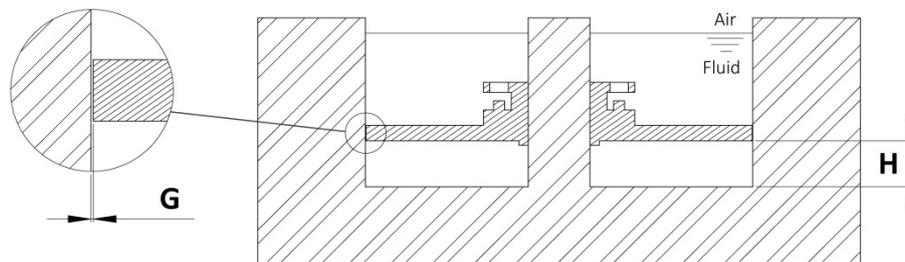


Figure 2. Cross-section view of the test rig: Disk position along the shaft (H) and radial gap (G).

3.2. Instrumentation

The natural frequencies and mode shapes of the disk were determined making use of experimental modal analysis (EMA). These natural frequencies were estimated in the air and submerged in water. The frequency response function (FRF), which is the ratio between the vibration and the force that produces that vibration, was computed to perform EMA. Piezoelectric patches have been used in order to excite the disk. The piezoelectric patches (P-876 DuraAct Patch Transducers) are contracted when they receive an electric signal. This contraction creates a force over the disk and consequently a small displacement of the structure. The electric signals were sent to the piezoelectric patches by means of a four channel signal generator (National Instruments 9263, ± 10 V) and a voltage amplifier (OM 835, 250V/-100V).

The dynamic response of the disk was measured using three miniature accelerometers of 2.9 g of weight (Brüel & Kjær 4394, 10 mV/g) which are installed on the disk surface with 90 degrees separation. Another miniature accelerometer was installed radially on the shaft to detect the radial vibration. Measurements of accelerometers were computed and monitored using an acquisition system (Brüel & Kjær Type 3053-B-120). Signals obtained were analyzed computing the FRF between the patches signal and the response of the accelerometers.

3.3. Procedure

Natural frequencies of the disk were estimated in the air and in water for different H values. The piezoelectric patches could excite the disk at a certain frequency sending a sinusoidal signal or doing a frequency sweep in a certain frequency band. To estimate natural frequencies of the disk, the frequency sweep method was used. FRFs for each accelerometer were calculated using the signal sent to the patch as a reference with 0.25 Hz of resolution. A Hanning window was applied to the signal of the accelerometers with an overlap of 95%. The peak-hold method was applied to see the maximum amplitude for each peak value. The parameter estimation methods of the PULSE Reflex® software [12] were applied to the computed FRFs to find the natural frequencies and the mode-shapes.

3.4. Mode-shapes studied

In hydraulic turbines, combined modes (mode-shapes with radial and axial displacement at the same time) are of interest because both runner and shaft are involved. Thus, this study is focused on this type of combined mode-shapes. The axial mode-shapes of a disk are defined with its number of nodal diameters (ND) and nodal circles (NC) [11]. Nodal diameters (ND) and nodal circles (NC) are formed by the points which remain stationary in the vibration cycle of a mode-shape. The radial mode-shapes of a one-side cylindrical clamped shaft are defined with its number of nodal nodes (nodes without displacement in the vibration cycle) [13]. The first combined mode-shape found in this test rig is a combination of a 1ND mode of the disk and a Mode 1 of the shaft (see Figure 3). Other axial modes, such as 2ND, 3ND or 4ND modes are independent from the shaft since the deformation at the disk center is practically zero. Therefore, the combined shaft-disk mode is selected for the study in the present work.

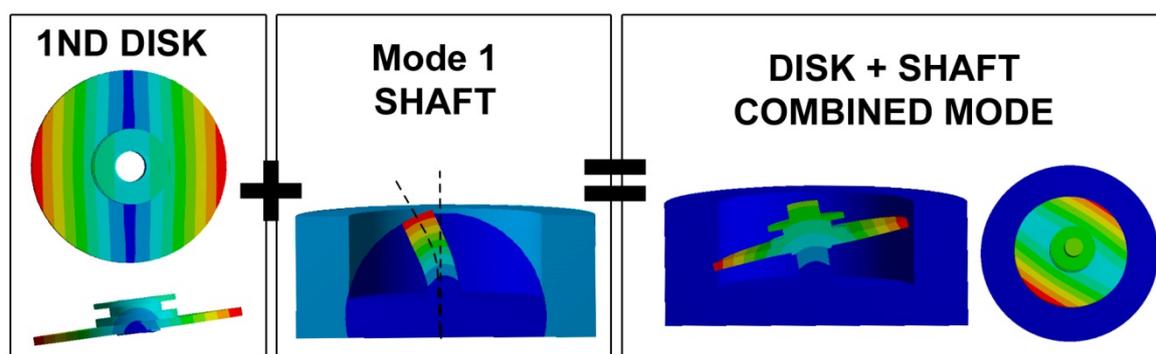


Figure 3. Combined shaft-disk mode-shape.

4. Numerical simulation

Structural-acoustic numerical simulations were carried out to estimate the dynamic behavior of the submerged disk. The commercial software ANSYS® [14] was used to solve the FEM (Finite Element Method) model (see Figure 4). The simulations considered the tank, the shaft and the disk in order to calculate with a good accuracy the natural frequencies. The material's properties (density, elastic module and Poisson module) were determined by the software database for a standard aluminum for the tank and the shaft ($\rho = 2770 \text{ kg/m}^3$, $E = 70 \text{ GPa}$, $\nu = 0.33$) and a standard stainless steel for the disk ($\rho = 7800 \text{ kg/m}^3$, $E = 200 \text{ GPa}$, $\nu = 0.3$). The fluid density was considered as 1000 kg/m^3 and the speed of sound in water was considered as 1430 m/s . For simulations in the air, the natural frequencies were estimated in the vacuum due to the deviation was considered negligible [15]. Structural-acoustical numerical simulations assume that the fluid is considered inviscid, irrotational and without mean flow.

The mesh was built using hexahedral elements (see Figure 4). Fluid and solid elements were of grade 2 with 8 points of integration (FLUID30 and SOLID185). The solid nodes in contact with the fluid were defined as FSI (Fluid Structure Interaction) Interface. The free surface condition was defined by zero pressure in the fluid nodes and sloshing effects were neglected. The stiffness of the

nodes of the disk in contact with the shaft had to be modified in order to simulate the real junction between both parts. In the experimental model the disk was coupled to the shaft by means of a single screw, which provided a junction stiffness of one thousand times lower than if they would be totally fixed. Therefore, the value of this junction stiffness was calibrated with experimental results. To determine the size of the numerical model, the number of elements was increased until the absolute percentage difference between the natural frequencies calculated with a number of elements in comparison to the densest mesh tested ($2 \cdot 10^5$ elements) was below 1% in all the mode-shapes. The optimal mesh selected approximately had $1.4 \cdot 10^5$ elements.

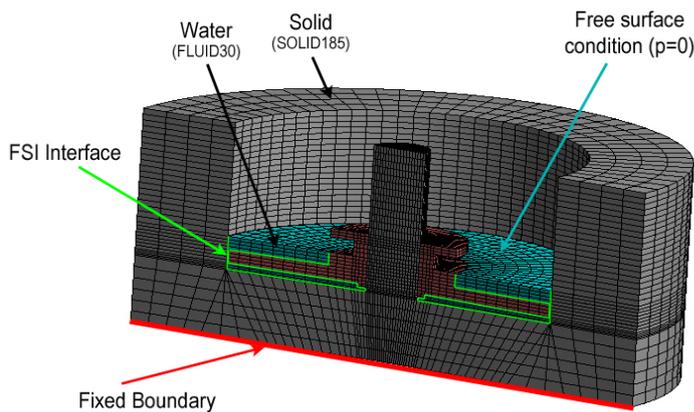


Figure 4. Cross-section view of the numerical model and applied boundary conditions.

5. Results and discussion

5.1. Results in the air

To determine the influence of the disk position along the shaft, different tests changing H (4, 10, 20, 30, 50 and 70 mm) were carried out in the air. Figure 5 shows an example of the natural frequency calculation for the studied mode-shape. In this figure, FRFs of each accelerometer are plotted with the electric signal of the piezoelectric patch as a reference. It is appreciated that the responses of accelerometers at 0 and 90 degrees are in phase among themselves but in countphase with the response of the accelerometer at 180 deg. This fact demonstrates that the disk is moving like a 1ND mode. Moreover, the accelerometer of the shaft demonstrates that in this natural frequency the shaft has an important contribution.

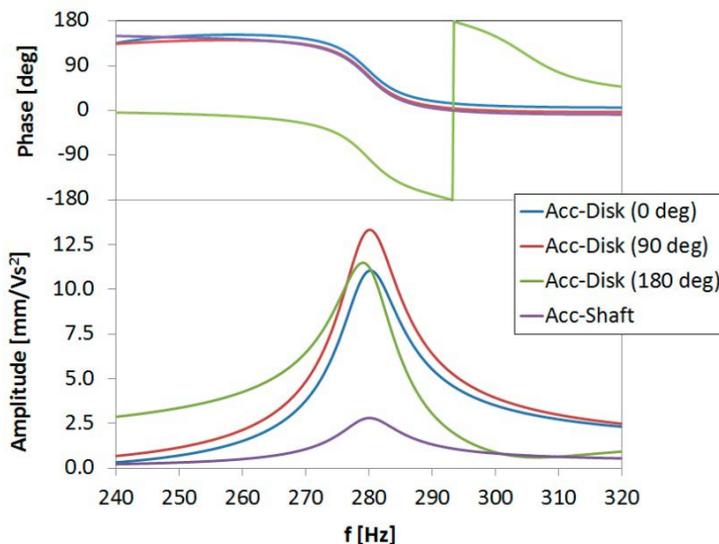


Figure 5. FRFs of the accelerometers with the piezoelectric patch as a reference. Amplitude and phase for $H = 30$ mm.

Comparing both experimental and numerical results for each configuration tested, an error less than 5% is found. Figure 6 shows the influence of the H distance on the natural frequency value for the studied mode-shape. The nearest to the tip of the shaft is the disk (highest H value), the lowest natural frequency value has the mode-shape. Therefore, it is demonstrated that the disk position along the shaft is important for the studied mode-shape even in the air.

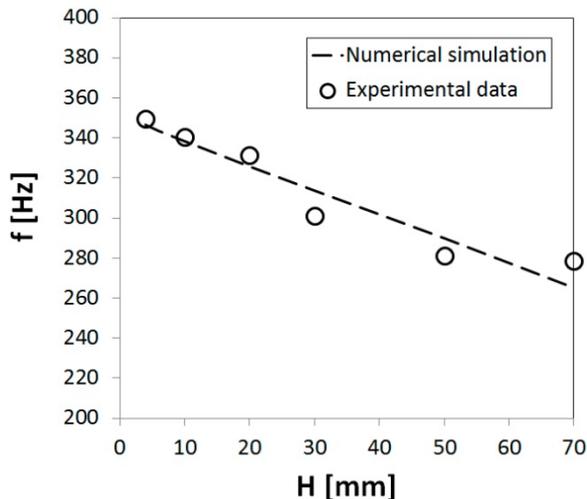


Figure 6. Influence of the disk position along the shaft (H) on the natural frequency of the studied mode-shape. Numerical and experimental results in the air.

5.2. Results in water

When the disk was submerged in water, more disk positions along the shaft (H) were tested (4, 6, 8, 10, 20, 30, 50 and 70 mm) in order to adequately capture the added mass effect. The water level above the disk was fixed always at 30 mm. Figure 7 shows the numerical and experimental results of the natural frequency of the studied mode-shape for different H distances. A reduction in the natural frequency value is observed when the disk is positioned near the rigid surface at the bottom. This phenomenon is due to the added mass effect [9-11]. After a certain distance ($H > 30$ mm), the natural frequency value can be considered approximately constant. Therefore, contrary to the case in the air, the natural frequency value increases when H increases. This fact demonstrates that in water the added mass effect is more important than the disk position along the shaft (H).

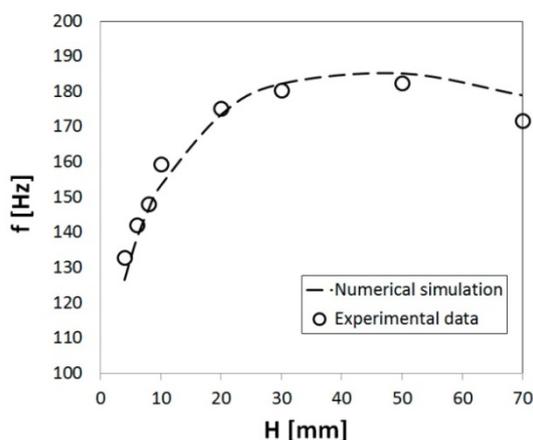


Figure 7. Influence of the disk position along the shaft (H) on the natural frequency of the studied mode-shape. Numerical and experimental results in water.

Similar results are obtained if the natural frequency values in water for different H distances are compared with those ones in the air (see Figure 8a). When the disk is near the rigid surface, the decrease in the natural frequency value with respect to the water is about 60%, whereas for the furthest

configuration to the rigid surface, the natural frequency value decreases only 30%. This behaviour is also demonstrated with the added mass factor (λ) (see Figure 8b). Experimental and numerical results accurately match (less than 6% of error).

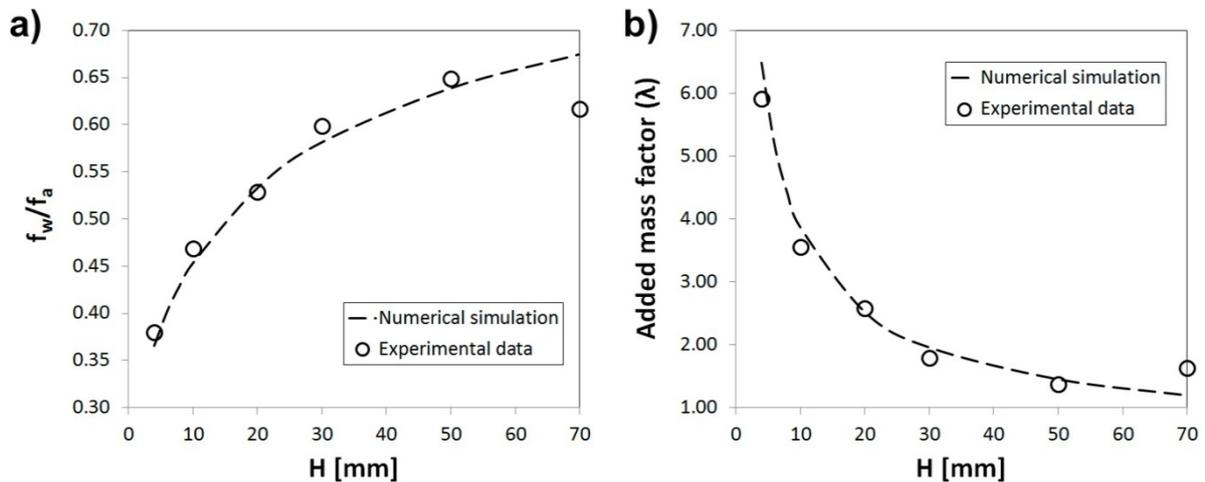


Figure 8. a) Natural frequency in water (f_w) over the natural frequency in the air (f_a) for different values of the disk position along the shaft (H). b) Added mass factor (λ) for different values of the disk position along the shaft (H).

As it is demonstrated that numerical simulation accurately agree with experimentation, in order to evaluate the influence of the radial gap width (G) on the natural frequency value, different numerical simulations changing this parameter were performed. Figure 9a shows the natural frequency in water (f_w) over the natural frequency in the air (f_a) for different radial gap values (G). It is found that, the smaller is the radial gap, the lowest natural frequency value has the structure. Moreover, when the radial gap is sufficiently large ($G > 30$ mm), the natural frequency value remains more or less constant. The opposite behaviour can be observed with the added mass factor (λ). Therefore, it is demonstrated that the added mass effect is more important when the radial gap is small.

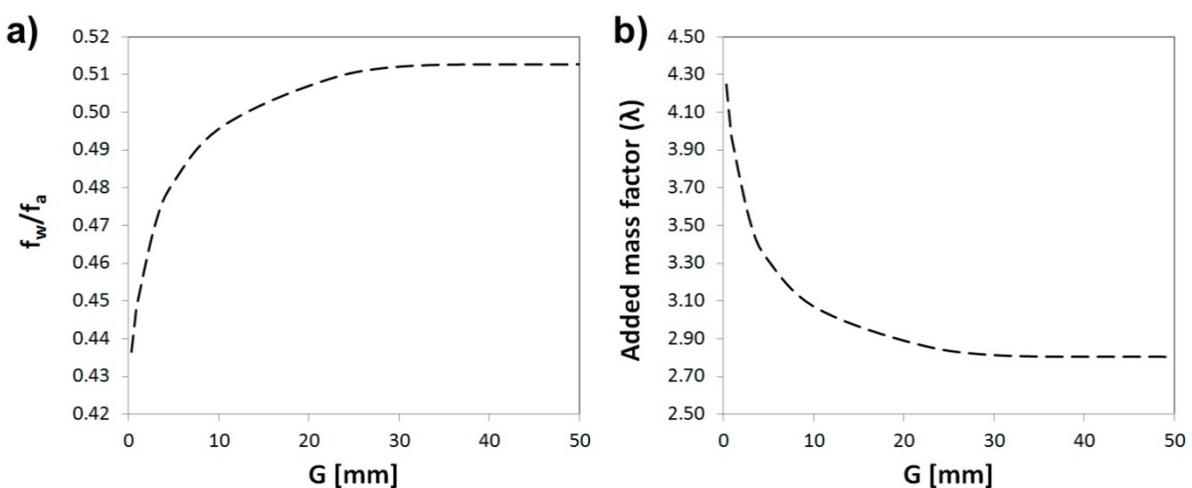


Figure 9. a) Natural frequency in water (f_w) over the natural frequency in the air (f_a) for different radial gap values (G). b) Added mass factor (λ) for different radial gap values (G). Numerical results.

6. Conclusions

The shaft coupling influence on the dynamic response of a submerged and confined disk with a small radial gap (0.35 mm) has been studied. This study is of interest in engineering applications such as in hydraulic turbine runners, which are attached to a shaft with small radial gaps between the runner and the turbine casing. To evaluate this dynamic response, a test rig has been developed. This test rig is made of a stainless steel disk (250 mm of diameter) attached to a shaft which is part of an aluminum tank. The natural frequency of a combined mode-shape, which has radial and axial deformation, has been experimentally measured for different configurations in air and in water. Moreover, the influence of the disk position along the shaft (H) is evaluated. Results of the disk submerged in water have been compared with the ones obtained in the air. A numerical model using structural-acoustical simulations has been built up to validate the experimental results.

First, the influence of the disk position along the shaft (H) was calculated in the air. Obtained results show that the natural frequency value decreases when H increases (disk near the tip of the shaft). Therefore, the disk position along the shaft influences the natural frequency of this combined-mode shape. Numerical and experimental results accurately match with less than 5% of error.

The same tests were carried out in water in order to evaluate the added mass influence on the disk position along the shaft (H). Results show that the natural frequency value in water decreases with respect to that in the air for all the configurations. When the disk is close to the rigid surface (small H distance values), a reduction of approximately 60% in the natural frequency is observed, whereas for large H distance values, the reduction is only about 30%. This behavior is also demonstrated with the added mass factor (λ), which leads us to think that the added mass effect is more important than the disk position along the shaft. Experimental and numerical results of natural frequencies match with accuracy (less than 6% of error).

To demonstrate the importance of the radial gap, numerical simulations changing the radial gap width have been carried out. Results show that there is a 5-10% of difference in the natural frequency value between the configuration with the smallest radial gap (0.35 mm) and the configuration with the largest one (50 mm). Moreover, it is demonstrated that the added mass effect is rather important for small radial gaps.

In conclusion, the effect of the radial gap and the shaft coupling is significant and should be considered to calculate natural frequencies of submerged and confined structures. This calculation can be accurately performed making use of structural-acoustic coupling simulations.

Nomenclature

f	Natural frequency [Hz]
G	Radial gap [mm]
H	Disk position along the shaft [mm]

Greek letters

λ	Added mass factor (dimensionless)
-----------	-----------------------------------

Subscripts

a	air
A	Added mass
f	fluid
v	vacuum
w	water

Acknowledgment

The authors acknowledge the Spanish Ministry of Economy and Competitiveness for the economic support received from Grant No. DPI2012-36264. The authors also wish to acknowledge Voith Hydro Holding GmbH & Co. KG for the technical and economic support received for developing this work.

References

- [1] Ohashi H 1994 Case study of pump failure due to rotor stator interaction *International Journal of Rotating Machinery* **1** 53-60
- [2] Egusquiza E, Valero C, Xingxing H, Jou E and Guardo A 2012 Failure investigation of a large pump-turbine runner *Engineering Failure Analysis* **23** 27-34
- [3] Liang Q, Rodriguez C, Egusquiza E, Escaler X, Farhat M and Avellan F 2007 Numerical simulation of fluid added mass effect on a Francis turbine runner *Computers & Fluids* **36** 1106-18
- [4] Rodriguez C, Egusquiza E, Escaler X, Liang Q and Avellan F 2006 Experimental investigation of added mass effects on a Francis turbine runner in still water *Journal of Fluids and Structures* **22** 699-712
- [5] Hübner B, Seidel U and Roth S 2010 Application of fluid-structure coupling to predict the dynamic behavior of turbine components *25th IAHR symposium on hydraulic machinery and systems. IOP Conf. Series: Earth Environ. Science* **12**
- [6] Ginsberg J H and Chu P 1992 Asymmetric vibration of heavily fluid-loaded circular plate using variational principles *Journal of Acoustic Society of America* **2** 894-906
- [7] Amabili M and Kwak M 1996 Free vibrations of circular plates coupled with liquids: revising the Lamb problem *Journal of Fluids and Structures* **10** 743-61
- [8] Gorman D and Horacek J 2007 Analysis of the free vibration of a coupled plate/fluid interacting system and interpretation using sub-system modal energy *Engineering Structures* **29** 754-62
- [9] Kubota Y and Suzuki T 1984 Added mass effect on disc vibrating in fluid *Transactions of the Japan Society of Mechanical Engineers* 243-8
- [10] Kubota Y and Ohashi H 1991 A study of the natural frequencies of hydraulic pumps *Proceedings of the 1st JSME/ASME Joint International Conference on Nuclear Engineering* 589-93
- [11] Askari E, Jeong K and Amabili M 2013 Hydroelastic vibration of circular plates immersed in a liquid filled container with free surface *Journal of Sound and Vibration* **332** 3064-85
- [12] Binda M, Harčarik T and Šimčák F 2011 Modal Parameters Estimation By Using PULSE Reflex Modal Analysis. In: *4th International Conference of Modelling of Mechanical and Mechatronic Systems*, (Faculty of Mechanical engineering, Technical university of Košice)
- [13] Blevins R D 1979 *Formulas for Natural Frequency and Mode Shapes* (Florida, USA)
- [14] ANSYS® 2010 *Ansys User's Manual 13.0* (Canonsburg, USA)
- [15] Rodriguez C G, Flores P, Pierart F G, Contzen L R and Egusquiza E 2012 Capability of structural-acoustical FSI numerical model to predict natural frequencies of submerged structures with nearby rigid surfaces. *Computers & Fluids* **64** 117-26