Statistical modeling of a joint metric of SAR interferogram's magnitude and phase using Fisher distribution for texture

Xianxiang Qin[1]**, Shilin Zhou, Huanxin Zou and Gui Gao**

School of Electronic Science and Engineering, National University of Defense Technology, Changsha 410073，PR China

qxxzhijia@126.com

Abstract. In this paper, aiming at the precise description of a recently proposed joint metric of SAR interferogram's magnitude and phase (IMP), especially for high resolution interferometric SAR images, a novel distribution denoted as S^1 distribution is proposed. At first, under the framework of product model, the Fisher distribution is introduced for modeling the textural component and the closed-form PDF of the novel distribution is presented. The proposed distribution generalizes the existing S^0 distribution which models the textural component by the inverse gamma distribution. Then, based on the Mellin transform, a parameter estimator of the proposed model is derived. Finally, the experimental results on real data have validated the efficiency of the proposed distribution and the parameter estimation.

1. Introduction

Due to its ability to be operated day and night regardless of weather conditions, synthetic aperture radar (SAR) has become increasing popular over the past decades. The interferometric SAR (InSAR) techniques which combine two complex SAR images to form both the complex interferogram and the coherence map have been widely applied to various fields, like providing the digital elevation model (DEM) generation and the ground moving target Indication (GMTI) [1]-[2].

Currently, focusing on the GMTI, a new joint metric of the SAR interferogram's magnitude and phase (IMP) has been proposed [3]. As the information from both the magnitude and phase are exploited effectively, this metric has shown an obvious advantage compared to that only based on magnitude or phase, and has great potential for many other applications. In the literature, the PDF of the IMP metric for homogeneous areas was derived. Besides, with the inverse gamma distribution for modeling the texture (backscattering RCS magnitude component), a distribution, named as $\mathcal{S}^0$ distribution, was derived for the heterogeneous areas. Nowadays, however, with the increasing resolution of SAR images, the property of the pixels especial in the extremely areas as urban areas has changed significantly. In this case, a more flexible model is required.

Recently, an empirical distribution, called Fisher distribution, has been introduced for statistical modeling of SAR images. Various theoretical analysis and experimental results have shown the advantage of this model compared with many other classical SAR image distributions [4]. To meet the need of modeling the IMP metric in high resolution SAR images, we propose a novel distribution by

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[1]   To whom any correspondence should be addressed.

using Fisher distribution for texture. Moreover, a corresponding estimator is derived based on the Mellin transform [5].

The paper is organized as follows. Section 2 introduces the recently proposed IMP metric. Section 3 presents two existing distributions of IMP and gives the detailed derivation of our proposed distribution for IMP. The parameter estimation of our distribution is derived in Section 4. Experimental results on real data are shown in Section 5. We give our conclusion in Section 6.

2. Joint metric of SAR IMP

2.1. Normalized multilook interferogram

The n-looks covariance matrix is the average of several independent samples, which is given as [2]-[3]

$$\hat{R} = \frac{1}{n}\sum_{k=1}^{n}\mathbf{Z}(k)\mathbf{Z}(k)^{H} = \frac{1}{n}\sum_{k=1}^{n}\begin{bmatrix} \left|z_{1}(k)\right|^{2} & z_{1}(k)z_{2}(k)^{*} \\ z_{1}(k)^{*}z_{2}(k) & \left|z_{2}(k)\right|^{2} \end{bmatrix}, \tag{1}$$

where n is the number of looks, $\mathbf{Z}(k) = \left[z_{1}(k), z_{2}(k)\right]^{T}$ is the kth single-look snapshot, the superscript * denotes complex conjugate and H conjugate complex transpose. The off-diagonal elements $\eta e^{j\psi}$ indicate the complex multilook interferogram I_{n}, i.e., $I_{n} = \eta e^{j\psi} = (1/n)\sum_{k=1}^{n}\left[z_{1}(k)z_{2}(k)^{*}\right]$, where η and ψ are the multilook interferogram magnitude and phase respectively. Then, the normalized complex multilook interferogram is given as [3]

$$\xi = \frac{1}{n}\left|\sum_{k=1}^{n}\left[z_{1}(k)z_{2}(k)^{*}\right]\right| / \sqrt{E\left(\left|z_{1}\right|^{2}\right)E\left(\left|z_{2}\right|^{2}\right)} \tag{2}$$

2.2. IMP Metric

In order to form the IMP, the multilook interferogram phase is transformed firstly by $\vartheta' = e^{j\psi} - 1$ which represents the difference of the interferogram's phase in the complex domain. The relationship between ϑ' and ψ is illustrated in the complex domain in Figure 1. As ϑ' is symmetrical on the real part axis [3], it can be simplified as $\vartheta = 1 - \cos\psi$. Then the metric named IMP metric is formed as [3]

$$\varsigma = \xi \cdot (1 - \cos\psi) = \xi \cdot \vartheta. \tag{3}$$
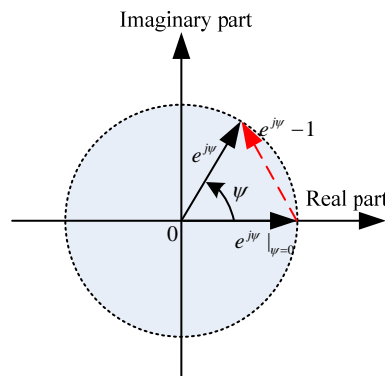

Figure 1. Relationship between ψ and $e^{j\psi} - 1$ in complex domain

3. Statistical modeling of IMP

3.1. Existing IMP distributions

In literature [3], a distribution of IMP for homogeneous areas is proposed as

$$p_\varsigma(\varsigma) = \sqrt{\upsilon_0 / (\pi\varsigma)} \exp(-\upsilon_0\varsigma), \ \varsigma, \upsilon_0 > 0. \tag{4}$$

For modeling the IMP metric of heterogeneous areas where the RCS is fluctuant, the product model is introduced, i.e., [2]

$$Y_i = A_i X_i, i = 1,2 \tag{5}$$

where i is the independent receiving channel, A_i represents the backscattering RCS magnitude component (texture), X_i is the speckle component that follows the zero-mean and unit variance complex Gaussian distribution. Assuming that the energy of dual-channel is balanced, and then the IMP metric is given by

$$\eta = A^2 \cdot \varsigma = W \cdot \varsigma. \tag{6}$$

In [3], the inverse gamma distribution [6] is introduced to describe the texture, of which the PDF is

$$p_W(\omega) = \frac{\gamma^{-\alpha}}{\Gamma(-\alpha)} \omega^{\alpha-1} \exp\left(-\frac{\gamma}{\omega}\right), -\alpha, \gamma > 0, \tag{7}$$

where γ and α are the scale and shape parameters respectively. Under the framework of product model, a distribution for the IMP metric, named as $\mathcal{S}^0$ distribution, is derived as [3]

$$p_\eta(\eta) = \sqrt{\frac{\upsilon_1}{\pi}} \frac{\Gamma(-\alpha+1/2)}{\Gamma(-\alpha)} \frac{1}{\sqrt{\eta}} (\upsilon_1\eta+1)^{\alpha-\frac{1}{2}}, \tag{8}$$

where $\upsilon_1 = \upsilon_0/\gamma$.

3.2. New IMP distribution

Nowadays, with the increasing resolution of SAR images, the property of the pixels has changed significantly. In this case, the inverse gamma distribution may become insufficient and a more flexible model is required. Herein, we employ a flexible empirical distribution, Fisher distribution, to model the texture, of which the PDF is given as [4]

$$p_W(\omega) = \frac{1}{B(M,L)} \frac{L}{M\mu} \left(\frac{L\omega}{M\mu}\right)^{L-1} \left(1 + \frac{L\omega}{M\mu}\right)^{-(L+M)}, \tag{9}$$

where μ is the scale parameter, $L > 0$ and $M > 0$ are two shape parameters, and $B(M,L) = \Gamma(L)\Gamma(M)/\Gamma(L+M)$ is the Beta function. For the product model $\eta = \varsigma\omega$, according to the relationship that $p_\eta(\eta) = \int_0^\infty (1/t) p_W(t) p_\varsigma(\eta/t) dt$, we can derived the PDF of η as

$$p_\eta(\eta) = \sqrt{\frac{L\upsilon_2}{M\pi}} \frac{\Gamma(M+1/2)}{B(M,L)} \frac{1}{\sqrt{\eta}} \cdot U\left(M+\frac{1}{2}, -L+\frac{3}{2}, \frac{L\upsilon_2}{M}\eta\right), \tag{10}$$

where $\upsilon_2 = \upsilon_0/\mu$, and $U(a,b,z)$ is the confluent hypergeometric function of the second kind (also referred to as the KummerU function) defined as [7]-[8]

$$U(a,b,z) = (1/\Gamma(a)) \int_0^\infty t^{a-1} (1+t)^{b-a-1} e^{-zt} dt. \tag{11}$$

For simplicity, we note this novel model as $\mathcal{S}^1$ distribution in this paper. As the Fisher distribution tends toward the inverse gamma distribution when the shape parameter $\mathcal{L}$ tends toward infinity [8]. Therefore, in this case, $\mathcal{S}^1$ model would reduce to $\mathcal{S}^0$ distribution too.

4. Parameter estimation

The method of log-cumulant (MoLC) based on the Mellin transform is an efficient method of various distributions of SAR images [5]. The idea of the MoLC is to estimate parameters by solving a system of the log-cumulants of the distribution. The first three orders log-cumulants of ς and ω can be calculated as (12) and (13) respectively.

$$\begin{cases} \kappa_{\varsigma 1} = \Psi(1/2) - \ln \upsilon_0 \\ \kappa_{\varsigma 2} = \Psi(1,1/2) \\ \kappa_{\varsigma 3} = \Psi(2,1/2) \end{cases} . \tag{12}$$

$$\begin{cases} \kappa_{W1} = \Psi(L) - \Psi(M) - \log(L/(\mu M)) \\ \kappa_{W2} = \Psi(1,L) + \Psi(1,M) \\ \kappa_{W3} = \Psi(2,L) - \Psi(2,M) \end{cases} . \tag{13}$$

According the property of Mellin transform, the log-cumulants of η are the sum of that of ς and ω [5], namely

$$\begin{cases} \kappa_{\eta 1} = \Psi(L) - \Psi(M) - \log(\upsilon_2 L/M) + \Psi(1/2) \\ \kappa_{\eta 2} = \Psi(1,L) + \Psi(1,M) + \Psi(1,1/2) \\ \kappa_{\eta 3} = \Psi(2,L) - \Psi(2,M) + \Psi(2,1/2) \end{cases} \tag{14}$$

Then the parameters of the proposed distribution can be estimated by solving the equations (14). In practice, the first three orders l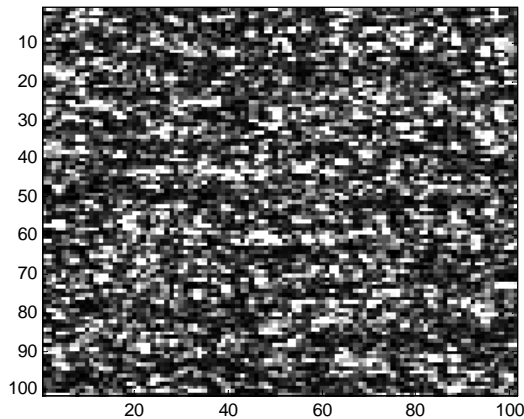og-cumulants would be replaced by the empirical ones, namely $\hat{\kappa}_1 = \frac{1}{N}\sum_{i=1}^{N}\ln X_i$, $\hat{\kappa}_2 = \frac{1}{N}\sum_{i=1}^{N}(\ln X_i - \hat{\kappa}_1)^2$ and $\hat{\kappa}_3 = \frac{1}{N}\sum_{i=1}^{N}(\ln X_i - \hat{\kappa}_1)^3$ for the observed sample set of IMP $\mathbf{X} = \{X_1, X_2, ..., X_N\}$.
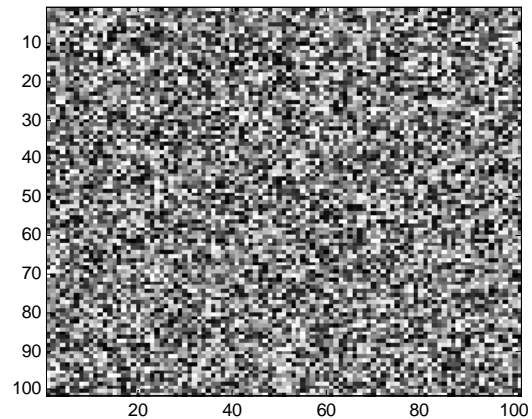
5. Experimental results and analysis

In this study, two C-band dual-channel SAR data of different terrains acquired by the RADARSAT-2 system over San Francisco is employed, of which the spatial resolution is 8m×8m. Figure 2 (a) and Figure 2 (c) present the normalized magnitude images of a ocean and a land areas, and Figure 2 (b) and Figure (d) give the corresponding phase images. Then, according to (3), the IMP metrics are formed, of which the images are given in Figure 3 (a) and Figure 3 (c). By applying the proposed distribution to model the IMP metric, the fitting results are illustrated in Figure 3 (b) and Figure 3 (d) respectively. It can be seen from Figure 3 (b) and Figure 3 (d) that the proposed distribution fits the histograms of the IMP metrics precisely, which has validated the proposed model and the parameter estimation.

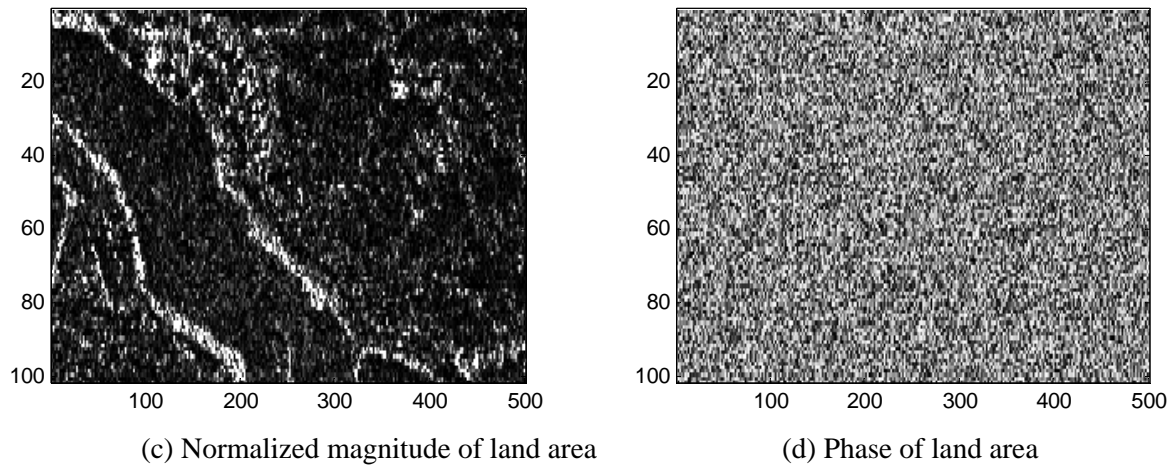(a) Normalized magnitude of ocean area   (b) Phase of ocean area

(c) Normalized magnitude of land area   (d) Phase of land area

Figure 2. Images of the normalized magnitude and the phase of the experimental data

(a) IMP of ocean area   (b) Fitting result of ocean area

(c) IMP of land area   (d) Fitting result of land area
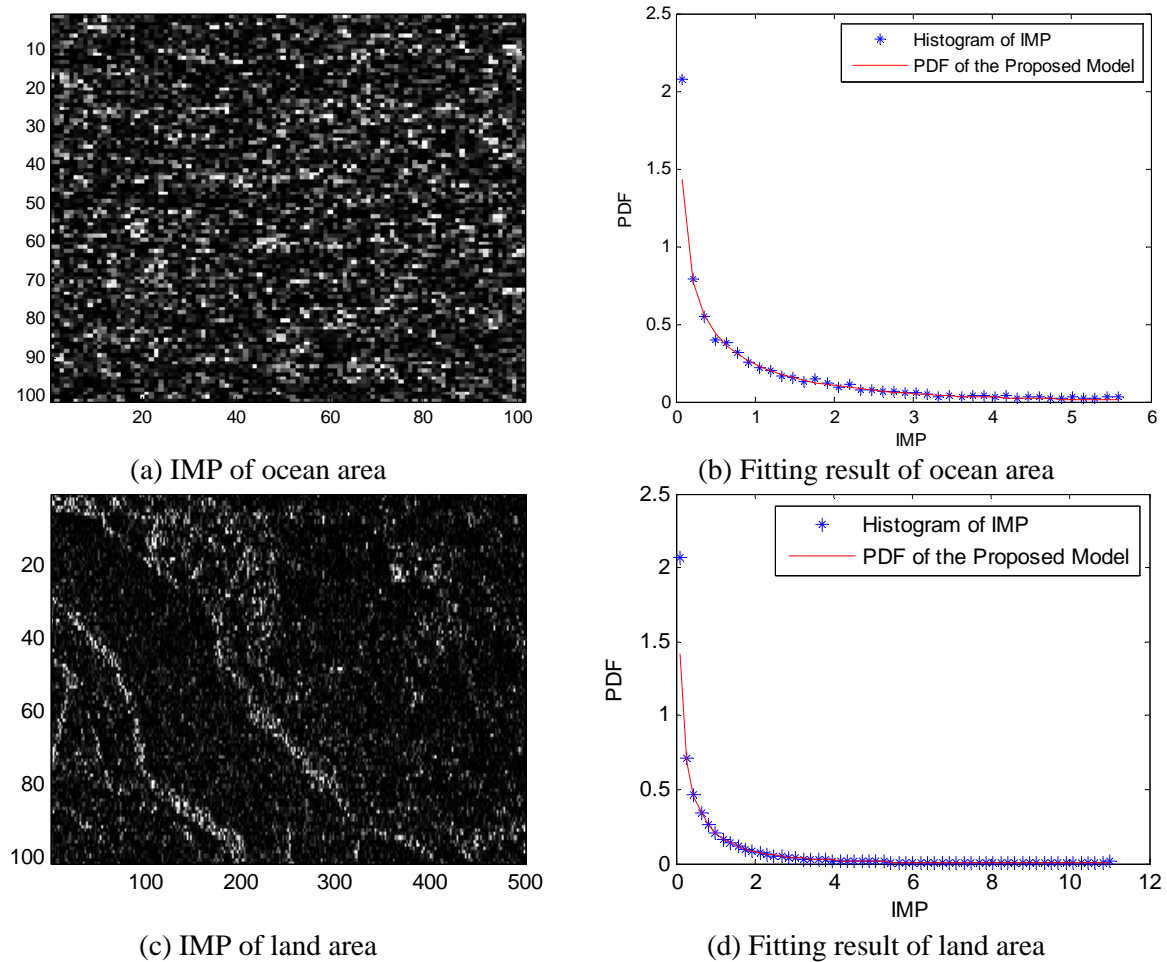
Figure 3. Image of the IMP and the fitting results

6. Conclusion

In this paper, to meet the need of modeling the IMP metric in the high resolution SAR images, a novel distribution, denoted as $\mathcal{S}^1$ distribution, is proposed under the product model where the Fisher distribution is used for texture component. A closed-form PDF of the novel distribution is derived, which generalizes the existing $\mathcal{S}^0$ distribution too. Moreover, a parameter estimator of the proposed

distribution is derived based on the Mellin transform. Results on real data have validated the proposed model and the parameter estimation.

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