

Feature extraction for SAR target recognition based on supervised manifold learning

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Abstract. On the basis of manifold learning theory, a new feature extraction method for Synthetic aperture radar (SAR) target recognition is proposed. First, the proposed algorithm estimates the within-class and between-class local neighbourhood surrounding each SAR sample. After computing the local tangent space for each neighbourhood, the proposed algorithm seeks for the optimal projecting matrix by preserving the local within-class property and simultaneously maximizing the local between-class separability. The use of uncorrelated constraint can also enhance the discriminating power of the optimal projecting matrix. Finally, the nearest neighbour classifier is applied to recognize SAR targets in the projected feature subspace. Experimental results on MSTAR datasets demonstrate that the proposed method can provide a higher recognition rate than traditional feature extraction algorithms in SAR target recognition.

1. Introduction

SAR automatic target recognition plays an important role in environmental monitoring and battlefield awareness. As a typical recognition problem associated with high dimensionality and limited number of samples, target recognition in SAR images usually needs extracting useful low-dimensional features before classification.

Some traditional dimensionality reduction algorithms, such as principal component analysis (PCA) and linear discriminant analysis (LDA), have been extensively used to extract features for SAR target recognition [1-2]. However, since these algorithms are linear in nature, they are not appropriate to handle SAR dataset which is inherently nonlinear.

More recently, nonlinear feature extraction techniques have drawn much more attention [3-5]. Among them, manifold learning-based algorithms were extensively studied because of their geometric intuition and computational feasibility. Locally linear embedding (LLE) [6], laplacian eigenmaps (LE) [7], isometric feature mapping (ISOMAP) [8] and local tangent space alignment (LTSA) [9] are several typical manifold learning algorithms. Previous works have shown that these algorithms can successfully derive the low-dimensional embedding coordinates of the nonlinear observation data. However, some limitations still exist in such algorithms for target recognition task. Firstly, traditional manifold learning algorithms often suffer from the out-of-sample problem. As the low-dimensional embedding results are derived from a fixed training data set and the nonlinear map is implicit, when apply to a new sample, traditional manifold learning algorithms cannot find the new sample's embedding directly. This limits the applications of manifold learning algorithms to target recognition problems. Secondly, traditional manifold learning algorithms mainly focus on preserving the local



property of the data rather than the class information, which will inevitably weaken the performance of target recognition.

To overcome these limitations, this paper proposes a new supervised manifold learning algorithm called supervised local tangent space alignment (SLTSA) for SAR feature extraction. Compared with the original LTSA algorithm, SLTSA attempts to enhance the recognition performance from two aspects. On the one hand, SLTSA aims to preserve the within-class local property and simultaneously maximize the between-class separability. The use of class information can boost the discriminating power of SLTSA. On the other hand, instead of correlated ones, SLTSA extends LTSA by using an optimal set of uncorrelated discriminant features. As indicated in [10], uncorrelated features contain minimum redundancy and ensure independence of features. Therefore the features generated by SLTSA algorithm have better discriminating power than that generated by LTSA.

The remainder of the paper is organized as follows. Section 2 presents a brief review of original LTSA. Section 3 describes the proposed SLTSA algorithm. The experimental results on SAR data set will be presented in Section 4, followed by the conclusions in Section 5.

2. Local tangent space alignment

LTSA is a well-known manifold learning algorithm. Given a D -dimensional data set $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\} \in \mathbb{R}^{D \times N}$, sampled from a d -dimensional manifold M ($d < D$). LTSA aims to map the high-dimensional data $\mathbf{X}$ to the low-dimensional embedding $\mathbf{Y} = \{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N\} \in \mathbb{R}^{d \times N}$ in a low-dimension Euclidean space. The procedure for LTSA is described in the following:

Step1: Set neighborhoods. For each sample $\mathbf{x}_i$, determine its k nearest neighborhood $\mathbf{X}_i = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, \dots, \mathbf{x}_{ik})$ on the basis of Euclidean distance between $\mathbf{x}_i$ and other samples.

Step2: Compute local tangent space. For each neighborhood $\mathbf{X}_i$, use PCA method to seek for a projection matrix $\mathbf{V} \in \mathbb{R}^{D \times d}$ such that the mean squared error is minimized, i.e.

$$\arg \min_{\mathbf{V}} \sum_{j=1}^k \left\| \mathbf{x}_{ij} - \bar{\mathbf{x}}_i - \mathbf{V}\mathbf{V}^T(\mathbf{x}_{ij} - \bar{\mathbf{x}}_i) \right\|_2^2 \quad (1)$$

$$\text{s.t. } \mathbf{V}^T\mathbf{V} = \mathbf{I}$$

where $\bar{\mathbf{x}}_i = \frac{1}{k} \sum_{j=1}^k \mathbf{x}_{ij}$ is the mean of neighborhood $\mathbf{X}_i$, $\mathbf{I}$ is the identity matrix, $j = 1, 2, \dots, k$. Then, one can

obtain the local coordinates $\boldsymbol{\theta}_i = \{\boldsymbol{\theta}_{i1}, \boldsymbol{\theta}_{i2}, \dots, \boldsymbol{\theta}_{ik}\} \in \mathbb{R}^{d \times k}$ for each neighborhood $\mathbf{X}_i$, where $\boldsymbol{\theta}_{ij} = \mathbf{V}^T(\mathbf{x}_{ij} - \bar{\mathbf{x}}_i)$ is the coordinate of $\mathbf{x}_{ij}$ in the local tangent space.

Step3: Align local coordinates. Assume that the global low-dimensional coordinates $\mathbf{Y}_i = \{\mathbf{y}_{i1}, \mathbf{y}_{i2}, \dots, \mathbf{y}_{ik}\} \in \mathbb{R}^{d \times k}$ and local coordinates $\boldsymbol{\theta}_i$ satisfy the affine transformation $\mathbf{y}_{ij} = \mathbf{L}_i \boldsymbol{\theta}_{ij} + \mathbf{c}_i$. LTSA aligns the local coordinates to obtain the global coordinates by minimizing the global reconstruction error as follows

$$\mathbf{Y} = \arg \min_{\mathbf{Y}} \sum_{i=1}^N \|\mathbf{E}_i\| \quad (2)$$

where $\mathbf{E}_i = \sum_{j=1}^k \left\| \mathbf{y}_{ij} - (\mathbf{L}_i \boldsymbol{\theta}_{ij} + \mathbf{c}_i) \right\|^2 = \left\| \mathbf{Y}_i - (\mathbf{L}_i \boldsymbol{\theta}_i + \mathbf{c}_i \mathbf{1}^T) \right\|^2$. After some algebra, equation (2) can be rewritten as

$$\mathbf{Y} = \arg \min_{\mathbf{Y}} \sum_{i=1}^N \|\mathbf{E}_i\| = \arg \min_{\mathbf{Y}} \|\mathbf{Y}\mathbf{S}\mathbf{W}\|^2 = \arg \min_{\mathbf{Y}} \text{tr}(\mathbf{Y}\boldsymbol{\Phi}\mathbf{Y}^T) \quad (3)$$

where $\mathbf{S} = [\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_N]$, $\mathbf{S}_i$ is the 0-1 selection matrix such that $\mathbf{Y}\mathbf{S}_i = \mathbf{Y}_i$, and $\boldsymbol{\Phi} = \mathbf{S}\mathbf{W}\mathbf{W}^T\mathbf{S}^T$ is called as alignment matrix. The matrix $\mathbf{W}$ is a diagonal matrix and its diagonal element $\mathbf{W}_i = \mathbf{H}_k(\mathbf{I} - \boldsymbol{\theta}_i^+ \boldsymbol{\theta}_i)$,

where $\mathbf{H}_k = \mathbf{I} - \mathbf{e}\mathbf{e}^T / k$ is the centering operator and $\mathbf{\Theta}_i^+$ is the Moore–Penrose generalized inverse of $\mathbf{\Theta}_i$.

3. Supervised local tangent space alignment

LTSA algorithm can extract features from high-dimensional data. However, since class information of data is ignored, LTSA is unsupervised in nature and cannot be applied to recognition tasks directly. To enhance the recognition performance of LTSA, we present a supervised version of LTSA. The proposed SLTSA aims to make the best of class information and uncorrelated feature space to improve discriminant power of the original LTSA.

3.1. Objective function of SLTSA

Suppose that the data set $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\} \in R^{D \times N}$ belongs to c classes, and let $l(\mathbf{x}_i)$ denote the class label of data point $\mathbf{x}_i$. For each $\mathbf{x}_i$, we use the class information to build two nearest neighborhoods of $\mathbf{x}_i$: the within-class neighborhood $N_w(\mathbf{x}_i)$ and between-class neighborhood $N_B(\mathbf{x}_i)$. $N_w(\mathbf{x}_i)$ contains k nearest neighbours sharing same labels with $\mathbf{x}_i$, while $N_B(\mathbf{x}_i)$ contains k nearest neighbours with different labels from $\mathbf{x}_i$.

The original LTSA algorithm can preserve the local property of the data. Similar to LTSA, we can define the within-class objective function by minimizing the sum of the local within-class reconstruction error E_i^W , i.e.

$$\min \sum_{i=1}^N \|E_i^W\| = \min \text{tr}(\mathbf{Y}\mathbf{\Phi}^W\mathbf{Y}^T) \quad (4)$$

where the alignment matrix $\mathbf{\Phi}^W$ is computed like LTSA. The difference is that the nearest neighbors for each sample are from $N_w(\mathbf{x}_i)$.

For recognition tasks, preserving the local within-class property in the low-dimensional feature space usually cannot guarantee to obtain good recognition results. To extract more effective features for recognition, we should consider the between-class information. Inspired by the idea of Fisher discriminant analysis, we attempt to enlarge the local between-class separability by maximizing the sum of the local between-class reconstruction error E_i^B , i.e.

$$\max \sum_{i=1}^N \|E_i^B\| = \max \text{tr}(\mathbf{Y}\mathbf{\Phi}^B\mathbf{Y}^T) \quad (5)$$

where the alignment matrix $\mathbf{\Phi}^B$ is computed like LTSA. The difference is that the nearest neighbors for each sample are from $N_B(\mathbf{x}_i)$.

For the purpose of recognition, we expect to search for a projection that maximizes the local between-class separability and preserves the local within-class property. From this point of view, a desirable projection should meet two optimization criterions as follows

$$\begin{cases} \max \text{tr}(\mathbf{Y}\mathbf{\Phi}^B\mathbf{Y}^T) \\ \min \text{tr}(\mathbf{Y}\mathbf{\Phi}^W\mathbf{Y}^T) \end{cases} \quad (6)$$

Obviously, the projection obtained from equation (6) is implicit, which means that this method inevitably suffers from the out-of-sample problem. In order to overcome this shortcoming, we introduce an explicit linear mapping $\mathbf{Y} = \mathbf{V}^T \mathbf{X}$ on the above optimization criterions. Thus equation (6) can be converted to the following form

$$\begin{cases} \max \text{tr}(\mathbf{V}^T \mathbf{X} \mathbf{\Phi}^B \mathbf{X}^T \mathbf{V}) \\ \min \text{tr}(\mathbf{V}^T \mathbf{X} \mathbf{\Phi}^W \mathbf{X}^T \mathbf{V}) \end{cases} \quad (7)$$

The equation (7) can be solved by difference criterion and the quotient criterion. In this paper, we use difference criterion and define the objective function of SLTSA as follows

$$\arg \max_{\mathbf{V}} = \alpha J_B - (1 - \alpha) J_W \quad (8)$$

where α is a trade-off parameter, $J_B = \text{tr}(\mathbf{V}^T \mathbf{X} \Phi^B \mathbf{X}^T \mathbf{V})$ and $J_W = \text{tr}(\mathbf{V}^T \mathbf{X} \Phi^W \mathbf{X}^T \mathbf{V})$.

3.2. Uncorrelated feature extraction

In fact, the features obtained by equation (8) are statistically correlated. It means that the projected feature space may contain redundancy, which will affect the recognition performance. In this section, we impose the statistically uncorrelated constraint on the obtained feature space.

Assume that any two different features $\mathbf{y}_i$ and $\mathbf{y}_j$ are statistically uncorrelated, then

$$E\{[\mathbf{y}_i - E(\mathbf{y}_i)][\mathbf{y}_j - E(\mathbf{y}_j)]^T\} = \mathbf{v}_i^T \mathbf{S}_t \mathbf{v}_j = 0 \quad (9)$$

where $\mathbf{v}_i$ and $\mathbf{v}_j$ are the i th and j th column of projection matrix $\mathbf{V}$, and $\mathbf{S}_t = E\{[\mathbf{x}_i - E(\mathbf{x}_i)][\mathbf{x}_j - E(\mathbf{x}_j)]^T\}$ is the total scatter matrix of the training set $\mathbf{X}$. If we let $\mathbf{v}_i^T \mathbf{S}_t \mathbf{v}_i = 1$, then equation (9) can be summarized as follows

$$\mathbf{V}^T \mathbf{S}_t \mathbf{V} = \mathbf{I} \quad (10)$$

As a result, SLTSA algorithm can extract uncorrelated features by maximizing the following objective function

$$\arg \max_{\mathbf{V}} J = \alpha J_B - (1 - \alpha) J_W \quad (11)$$

$$\text{s.t. } \mathbf{V}^T \mathbf{S}_t \mathbf{V} = \mathbf{I}$$

With some mathematical derivation, the constrained maximization problem shown in equation (11) can be reduced to a generalized eigenvalue problem

$$\mathbf{X}[\alpha \Phi^B - (1 - \alpha) \Phi^W] \mathbf{X}^T \mathbf{v} = \lambda \mathbf{S}_t \mathbf{v} \quad (12)$$

Finally, the projection matrix $\mathbf{V}$ can be computed by the d eigenvectors corresponding to the first d largest eigenvalues of equation (12). For each data point $\mathbf{x}_i$, the discriminative and uncorrelated feature can be given as $\mathbf{y}_i = \mathbf{V}^T \mathbf{x}_i$.

4. Experiment

To verify the effectiveness of our proposed method for feature extraction, experiments based on the Moving and Stationary Target Acquisition and Recognition (MSTAR) dataset have been done. The MSTAR dataset contains three types of SAR target including T72, BMP2, and BTR70. The original target images in MSTAR are all sized 128×128 pixels and the resolution is $0.3\text{m} \times 0.3\text{m}$. In this paper, the SAR target images obtained at the depression angle 17° are collected as training set and the SAR target images obtained at the depression angle 15° are collected as test set. The total number of training samples is 698 and the total number of test samples is 1365. The algorithmic procedure of SAR target recognition using SLTSA is summarized as follows:

(1) Image pre-processing. First, the redundant background of original SAR target image is excluded and the cropped image is 44×44 pixels with the target at the center. Second, we normalize the amplitude of the cropped image and use the cropped and normalized image as our experimental dataset.

(2) Feature extraction. The proposed SLTSA algorithm is used to extract features of MSTAR data. In our experiments, the trade-off parameter α in SLTSA is fixed as 0.1. To demonstrate the effectiveness of SLTSA, several state-of-the-art dimensionality reduction methods including PCA, LDA, LTSA and MMC are also utilized to extract features of MSTAR data.

(3) Target recognition. In the projected low-dimensional feature space, the nearest neighbor classifier is used to evaluate the classification performances.

Table 1 gives the best recognition rate obtained from five different algorithms and the corresponding number of feature dimension. Obviously, the proposed SLTSA algorithm performs better than PCA, LDA, LTSA and MMC[11]. The best SAR target recognition rate of SLTSA is 96.8%, which is 2.6% higher than PCA, 15.9% higher than LDA, 8.1% higher than MMC and 9.6% higher than original LTSA algorithm. Due to taking full advantage of class information and uncorrelated feature property, SLTSA is more discriminative than other four feature extraction algorithms.

Figure 1 presents the plot of recognition rate versus the variation of feature dimension for PCA, LDA, LTSA, MMC and the proposed SLTSA. We can see that the variation of feature dimension can affect the recognition performance. For SLTSA, only small feature dimension can achieve good recognition performance. This will be helpful to save a mass of computing time and storage space in SAR target recognition.

Table 1. Best recognition result obtained by each algorithm.

Algorithm	Best recognition rate (%)	Feature dimension
PCA	94.2	50
LDA	80.9	2
LTSA	87.2	100
MMC	88.7	100
SLTSA	96.8	60

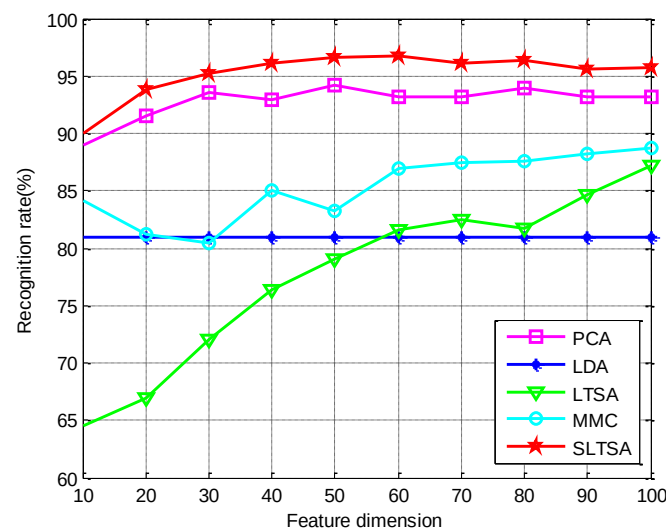


Figure 1. Recognition rate versus the variation of feature dimension for five different algorithms.

5. Conclusion

A novel SAR feature extraction algorithm called SLTSA is presented in this paper. The characteristics of SLTSA are described as follows: first, SLTSA considers not only the local manifold structure but also the class information, which makes it more discriminative than traditional manifold learning algorithms. Second, SLTSA introduces an uncorrelated constraint to make the extracted features statistically uncorrelated, which can improve the recognition rate of SAR target recognition. Experimental results on MSTAR demonstrate the effectiveness and feasibility of SLTSA.

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