

# Semi-physical Simulation of the Airborne InSAR based on Rigorous Geometric Model and Real Navigation Data

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**Abstract.** Raw signal simulation is a useful tool for the system design, mission planning, processing algorithm testing, and inversion algorithm design of Synthetic Aperture Radar (SAR). Due to the wide and high frequent variation of aircraft's trajectory and attitude, and the low accuracy of the Position and Orientation System (POS)'s recording data, it's difficult to quantitatively study the sensitivity of the key parameters, i.e., the baseline length and inclination, absolute phase and the orientation of the antennas etc., of the airborne Interferometric SAR (InSAR) system, resulting in challenges for its applications. Furthermore, the imprecise estimation of the installation offset between the Global Positioning System (GPS), Inertial Measurement Unit (IMU) and the InSAR antennas compounds the issue. An airborne interferometric SAR (InSAR) simulation based on the rigorous geometric model and real navigation data is proposed in this paper, providing a way for quantitatively studying the key parameters and for evaluating the effect from the parameters on the applications of airborne InSAR, as photogrammetric mapping, high-resolution Digital Elevation Model (DEM) generation, and surface deformation by Differential InSAR technology, etc. The simulation can also provide reference for the optimal design of the InSAR system and the improvement of InSAR data processing technologies such as motion compensation, imaging, image co-registration, and application parameter retrieval, etc.

## 1. Introduction

With the first application in Earth observation in early 1970s, the airborne Synthetic Aperture Radar Interferometry (InSAR) has been widely used for photogrammetric mapping and geophysical remote sensing. The geographic information and geometric relationship are crucial to the application of the InSAR system. For the space borne InSAR system, the geolocation information of the target can be derived from the location and velocity of the platform which is immune to the sensor's orientation and the installation offset position comparing to the center of mass of the spacecraft/satellite [1]. While, for the airborne InSAR system, the geolocation is sensitive to attitude variation and installation offset locations of the antenna. Furthermore, the inaccurate information of the trajectory and attitude of the

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aircraft reported by the aerial Position and Orientation System (POS) and the relative positions of the GPS and IMU with respect to the InSAR antennas, which bring much more difficulties, are also needed to be considered in the geolocation calculation of airborne InSAR applications.

Airborne InSAR is well known for its flexibility of the flight path design and a high spatial and temporal resolution, while affected by the variation of aircraft's trajectory and altitude, it's difficult to obtain the precise geolocation of the irradiated targets. As a result, the wide application of the airborne InSAR is confined. The geometric relationship between the antenna and irradiated target is critical to the InSAR application and is derived from the measurement of POS, installation offset position of the GPS, IMU and the antenna, and the baseline length and inclination, etc, which are still difficult to be reported or estimated precisely. The effects of the measurement on the final products can not be evaluated accurately as well. With the purpose of dealing with these difficulties, a simulation of airborne InSAR is proposed in this paper. Based on the rigorous geometric imaging model and effective navigation data, the sensitivity of key parameter, i.e., baseline length and inclination, absolute phase and the orientation of the antennas, of the InSAR system can be studied by means of the simulation. It is also indicated that the simulation can provide reference for the optimal design of the InSAR system and the improvement of InSAR data processing technologies such as motion compensation, imaging, image co-registration, and application parameter retrieval etc.

## 2. Rigorous Geometric Model for airborne InSAR System

### 2.1. Geometric Imaging Model principle

It is known that the location of a target in a two dimensional image can be determined by means of a Range-Doppler (RD) model [2]. The location in the cross-track (range) direction is determined through measuring the time it takes a radar pulse to propagate to the target and return to the radar, and that in along-track (azimuth) is determined from the Doppler frequency shift resulted from the relative velocity between the radar and target. For a pixel  $(i, j)$  (range, azimuth) in an SAR image, its location in slant range and azimuth direction can be determined as:

$$R_i = c \cdot t_i = \frac{c}{2} \cdot \left( t_0 + \frac{i}{RSR} \right) \quad (1)$$

$$f_D = \frac{2}{\lambda |\bar{R}_A|} \bar{V}_A (\bar{R}_A - \bar{R}_T) \quad (2)$$

where  $c$  is the light velocity,  $t_i$  and  $t_0$  are the two way times from the antenna to  $i^{th}$  pixel and the first (near) pixel range respectively. RSR is the range sampling rate of the radar system.  $f_D$  is the Doppler frequency which can be determined by the phase history function in the radar raw data processing.  $\lambda$  is the wavelength.  $\bar{V}_A$  is the velocity of the aircraft.  $\bar{R}_A$  and  $\bar{R}_T$  are the antenna and target location vector in the local projected mapping coordinate system (the coordinate systems will be introduced in the following section) respectively.  $||$  represents the modulus operator. Using the radar beam direction and sign of the Doppler shift, the target location can be derived from Eqs. (1) and (2). However, for the side looking geometry of SAR system, the targets in the same range but with different height can cause the location ambiguity, although the locus are distributed in the plane formed by the radar Line of Sight (LOS) to the target and vector pointing from the radar to nadir.

Utilizing the property that a small slant range difference results in a phase shift, which can be obtained by measuring the difference along the LOS of the target in the two or multiple SAR images acquired by the antennas in slight different look angles, InSAR technology can provide elevation information of the irradiated target. As shown in Fig. 1, taken the master antenna as a reference, the phase can be expressed by:

$$\begin{aligned}
\phi &= \phi_S - \phi_M \\
&= \frac{2\pi p}{\lambda} (|\bar{\rho}_S| - |\bar{\rho}_M|) \\
&\approx -\hat{n}_{LOS} \cdot \bar{B}
\end{aligned} \tag{3}$$

Where  $\bar{\rho}_M$  and  $\bar{\rho}_S$  are the vectors from master and slave antennas to target respectively.  $\hat{n}_{LOS}$  is the unit vector in the LOS direction from the master antenna.  $\bar{B}$  represents the baseline vector.

With the assumption that the SAR system is working at the ortho-sidelooking model, Eq.(3) can be re-written as:

$$\phi = -\frac{2\pi p}{\lambda} \cdot |\bar{B}| \cdot \sin(\theta - \alpha) \tag{4}$$

where  $\theta$  is the radar look angle and  $\alpha$  is the inclination angle of the baseline. According to the geometry in Imaging coordinate system, the target can be constructed as:

$$\bar{T} = \bar{A}_M + |\bar{\rho}_M| \cdot \hat{n}_{LOS} \tag{5}$$

$\hat{n}_{LOS}$  will be determined in the following section.  $\theta$  can be derived by Eq.(4), and  $\phi$  is the measured interferometric phase derived from the SAR images. The reconstruction of the height requires the accurate position of antenna, the time delay of the transmitted wave, the length and inclination angle of the baseline, and interferometric phase derived from the co-registered SAR data.

## 2.2. Coordinate System transformation and 3D information construction

To accurately describe the rigorous geometric relationship of the InSAR system, nine coordinate systems (CSs) are required, including Earth-Rotation and Earth-Fixed (EREF), Geodetic (Geo), Navigation (Nav), Aircraft Carried Navigation (ACN), Aircraft Body-Fixed (ABF), Imaging (Ima), Local Projected Mapping (LPM), Inertial Measurement Unit (IMU) and InSAR Antenna (IA) CS (The abbreviation of the CS names will be used in the equations for CS transformation). The definitions of the coordinate systems are summarized in Table 1.

Table 1. Definitions of the Coordinate Systems	
Coordinate System	Defination
Earth Centered, Earth-fixed (ECEF)	Origin is the Earth's center of mass; z-axis is coincident with the Earth's principal rotation axis and positive toward the Conventional International Origin (CIO); positive x-axis passes through the intersection of the equatorial plane and the meridian; y-axis completes the right-handed Cartesian system.
Geodetic (Geo)	This CS is used by the GPS measurement, Position in this CS is described as latitude, longitude and height.
Navigation (Nav)	This CS is fixed to the Earth's surface with its origin choosed as the intersection of the Earth surface and the line from the Earth's center of mass to the mass point of the aircraft; x-axis points to geodetic north; y-axis points to geodetic east; z points downward along the local ellipsoid normal.
Aircraft Carried Navigation (ACN)	Comparing with Nav CS, the only difference is the origin of ACN CS is up shifted to the mass point of the aircraft.
Aircraft Body-Fixed (ABF)	This CS is rigidly attached to the aircraft with origin at the mass point; x-axis points forward, lying in the symmetric plane of the plane; y-axis points to starboard and z-axis points downward.
Imaging (Ima)	This CS has same origin with Nav CS, z-axis is parallal with the z-axis of Nav CS but in opposite direction; x-axis points along the craft flying direction; y-axis completes the right handed Cartesian system.
Local Projected Mapping (LPM)	This CS shares the same x-y plane with Nav CS, and z-axis is in the same direction; origin is arbitrarily fixed to a point on the x-y plane; x-axis positively points to the trajectory of the craft; y-axis completes

right-handed Cartesian system.

Inertial Measurement Unit (IMU)	The origin locates at the geometric center and x-, y- and z-axis line along the symmetric axis of the IMU box. Ideally, this CS should perfectly align with ABF CS.
InSAR Antenna (IA)	The origin is at the geometric center and x-, y-axis line along the symmetric axis of the antenna plate; Ideally, the x-axis aligns with the x-axis of ABF CS; z-axis points to the irradiated target.

The goal of the InSAR is to construct the 3D information in LPM CS, which is different from the CS for the navigation sensors. For instance, the relative geometric relationships of the navigation sensors (IMU and GPS) antennas and InSAR antennas are defined in ABF CS and the aircraft location is in Geodetic CS. To perform SAR image geocoding, all the measurements and geometric relationships should be transferred into a same CS (i.e., LPM CS). Similar with the aerial optical sensors, location and orientation are used to model the SAR LOS emitted from the antennas.

Based on the triangular relationship, the location of the SAR antenna can be calculated using the GPS antenna location and the relative position between GPS antenna and InSAR antennas with the former is in Geodetic CS and the latter in ABF CS. Using the converted GPS location and relative location offsets in LPM CS, the location of SAR antenna can be obtained. The transformation of position of GPS antenna can be presented as:

$$\overline{P}_{GPS\ LPM} = C_{Ima}^{LPM} C_{Nav}^{Ima} C_{EREF}^{Nav} C_{Geo}^{EREF} \begin{pmatrix} \lambda \\ \varphi \\ h \end{pmatrix} \quad (6)$$

where  $\overline{P}_{GPS\ LPM}$  represents the converted position of GPS antenna.  $C_{from}^{to}$  represents the transformation from “from” to “to” CS.  $(\lambda, \varphi, h)$  are the latitude, longitude, and altitude measured by the GPS antenna, respectively.

The transformation of the offset position between the GPS and SAR antennas can be presented as:

$$\overline{P}_{GPS-A_M\ LPM} = C_{Ima}^{LPM} C_{Nav}^{Ima} C_{ACN}^{Nav} C_{ABF}^{ACN} \overline{P}_{GPS-A_M\ ABF} \quad (7)$$

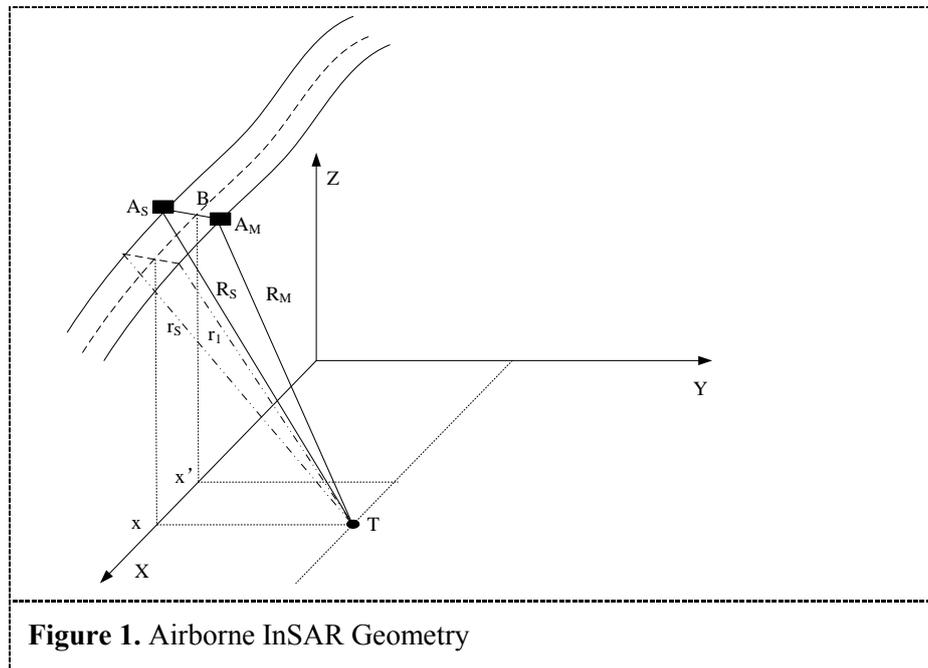
where  $\overline{P}_{GPS-A_M\ LPM}$  and  $\overline{P}_{GPS-A_M\ ABF}$  are the offset positions between GPS antenna and the SAR antenna  $A_M$  (here, we just take master antenna as an example and the transformation for slave antenna is the same) in LPM and ABF CS respectively. The antenna's position in CS LPM is given by:

$$\overline{P}_{A_M\ LPM} = \overline{P}_{GPS\ LPM} + \overline{P}_{GPS-A_M\ LPM} \quad (8)$$

The orientation of radar LOS, described as (omega, phi, kappa) angles for optical sensor, should be transformed from CS IA to CS LPM. The radar LOS emitted from the master antenna is defined as a unit vector ( $\hat{n}_{IA} = (0, 0, 1)$ ) in IA CS and is transformed into LPM CS:

$$\hat{n}_{LPM} = C_{Nav}^{Ima} C_{ABF}^{Nav} C_{IA}^{ABF} \hat{n}_{IA} \quad (9)$$

According to Eqs (6) - (9), the target location can be calculated using airborne InSAR imaging geometric model proposed in this study.



**Figure 1.** Airborne InSAR Geometry

### 3. Airborne InSAR Simulation

The objective of the simulation is to generate the SAR raw signal pairs of an illuminated scene, providing the reference for studying the sensitivity of the key parameters (i.e., baseline length and inclination, absolute phase and the orientation of the antennas) of the airborne InSAR system and their impact on the InSAR applications. The semi-physical InSAR simulation is based on the rigorous airborne InSAR geometric model, appropriate electromagnetic model of the backscattered fields and the accurate inclusion of the SAR system transfer function (TF) [3]. The rigorous geometric model has been introduced in the section 2.2. The electromagnetic model is based on a two-scale composite model for the scattering of the surface described by planar facets with known electromagnetic parameters. The TF is implemented by means of a two-dimensional (2-D) formulation [4]. The inputs are the navigation data from experiment and DEM of the test field generated by the real airborne InSAR system in the airborne remote sensing center of Institute of Remote Sensing and Digital Earth (RADI), Chinese Academy of Sciences (CAS).

The geometry of the airborne InSAR system is detailed in figure 1. The two InSAR (across-track mode) antennas ( $A_M$ ,  $A_S$ ) both illuminate the generic point ( $T$ ) with the distance of  $R_M$  and  $R_S$ , respectively, and the distance from  $T$  to the antennas' flight paths are  $r_M$  and  $r_S$ . The raw signal can be written as:

$$s_{M,S}(x', r'_{M,S}) = \iint \Upsilon_{M,S}(x, r_{M,S}) g(x'-x, r'_{M,S} - r_{M,S}) dx dr_{M,S} \quad (10)$$

Where  $\Upsilon(\square)$  is the backscattering coefficient of the scene.  $g(\square)$  is the impulse response function, given by:

$$g(x'-x, r'-r; r) = w^2 \left( \frac{x'-x}{X} \right) \text{rect} \left[ \frac{2(r'-r)}{c\tau} \right] \exp(j\phi) \quad (11)$$

Where  $\phi = -\frac{4\pi}{\lambda} \Delta R - \frac{2f}{c^2} (r'-r - \Delta R)^2$ ;  $\Delta R = R - r = \sqrt{(x'-x)^2 + r^2} - r$ .  $w(\square)$  is the normalized illumination function of the real antenna over the scene.  $X$  is the distance in azimuth.  $\tau$  is the chirp duration.  $f$  is the chirp frequency rate.

The geometric relationship of the InSAR system can be described by the geometric model and the distance used in Eqs. (10) and (11) is defined in LPM CS which can be calculated by the antenna installation information, aircraft navigation and DEM data along with a set of CS transformation. The

two returns from a single facet were modeled only by surface scattering and the backscattering coefficients of the elementary facet can be written as follows:

$$\begin{aligned} \Upsilon_M &= \exp(-2jkR_M)S_M \int_A \exp(-2jk_M \cdot t) dA \\ \Upsilon_S &= \exp(-2jk(R_M + R_S))S_S \int_A \exp(-2j(k_M + k_S) \cdot t) dA \end{aligned} \quad (12)$$

$S$  is a function of the average Fresnel reflection coefficients over the facet [4].  $K_M$  and  $K_S$  are the wave number vectors.  $t$  is the vector describing the generic point of the facet.  $A$  is the facet surface.

#### 4. Concluding remarks and future work

We have presented a semi-physical method for the simulation of the airborne InSAR raw signal in this study. The simulation is based on rigorous geometric model and effective navigation data. In order to accurately describe the geometric relationship of the system, nine coordinate systems are defined in the InSAR system, from which the distance used in the simulation is computed. The appropriate electromagnetic model of the backscattered fields was used to calculate the backscattering coefficients of the illuminated scene. The SAR system transfer function (TF) was implemented by means of a two-dimensional formulation. The inputs are the navigation data and the fine-resolution DEM data generated by the InSAR system in RAD1, CAS. The future work includes accomplishing airborne InSAR data processing procedure by the simulated raw dataset and thus investigating the sensitivity of the key parameters of the InSAR system.

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