

# Calibration and Validation of Airborne InSAR Geometric Model

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**Abstract.** The image registration or geo-coding is a very important step for many applications of airborne interferometric Synthetic Aperture Radar (InSAR), especially for those involving Digital Surface Model (DSM) generation, which requires an accurate knowledge of the geometry of the InSAR system. While the trajectory and attitude instabilities of the aircraft introduce severe distortions in three dimensional (3-D) geometric model. The 3-D geometrical model of an airborne SAR image depends on the SAR processor itself. Working at squinted model, i.e., with an offset angle (squint angle) of the radar beam from broadside direction, the aircraft motion instabilities may produce distortions in airborne InSAR geometric relationship, which, if not properly being compensated for during SAR imaging, may damage the image registration. The determination of locations of the SAR image depends on the irradiated topography and the exact knowledge of all signal delays: range delay and chirp delay (being adjusted by the radar operator) and internal delays which are unknown *a priori*. Hence, in order to obtain reliable results, these parameters must be properly calibrated. An Airborne InSAR mapping system has been developed by the Institute of Remote Sensing and Digital Earth (RADI), Chinese Academy of Sciences (CAS) to acquire three-dimensional geo-spatial data with high resolution and accuracy. To test the performance of the InSAR system, the Validation/Calibration (Val/Cal) campaign has carried out in Sichun province, south-west China, whose results will be reported in this paper.

## 1. Introduction

The image registration or geo-coding is a very important step for many applications of airborne interferometric Synthetic Aperture Radar (InSAR), especially for those involving Digital Surface Model (DSM) generation, which requires an accurate knowledge of the geometry of the InSAR system. While the uncertainties variation of the aircraft's trajectory and attitude damage achievement of the precise geolocation of the irradiated targets. The geometric relationship of the antennas and irradiated target is critical to the InSAR application which is derived from the measurement of Position and Orientation System (POS), estimation of the installation offset position of the GPS, IMU and the InSAR antennas, and the baseline length and inclination, etc. As with the common concept in the photogrammetry, it's difficult to obtain the real value of the measurements and estimations, but these aforementioned values are critical to the 3-D geometric model of the airborne InSAR system.

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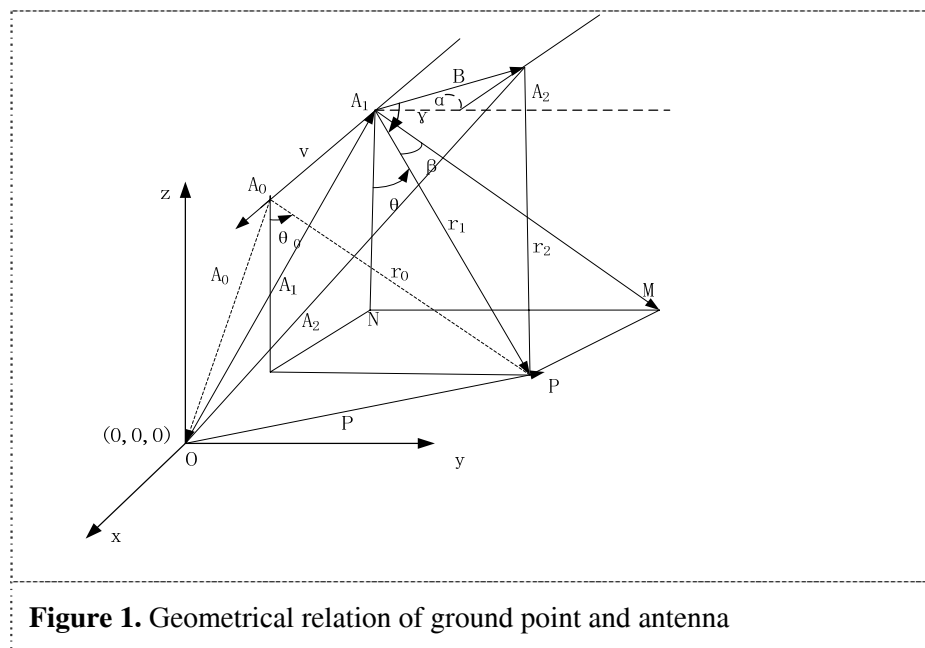
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Working at squinted model, i.e., with an offset angle (squint angle) of the radar beam from broadside direction, the aircraft motion instabilities may produce distortions in airborne InSAR geometric relationship, which, if not properly being compensated for during SAR imaging, may damage the image registration. Thus, in order to obtain desirable registration results, the parameters of the airborne InSAR system should be precisely calibrated, including the exact knowledge of all signal delays: range propagation delay, chirp delay (being adjusted by the radar operator) and internal delays which are unknown *a priori*, etc. To test the performance of the InSAR mapping system developed by the Institute of Remote Sensing and Digital Earth (RADI), Chinese Academy of Sciences (CAS), a Validation/Calibration (Val/Cal) campaign has carried out in Sichun province, south-west China in 2011. This paper will describe how to calibrate and validate an InSAR geometrical model based on the data acquired by the Val/Cal campaign.

## 2. Calibration of airborne InSAR geometrical model

For the most of airborne InSAR applications, it's assumed that the radar beam has a constant squint angle with respect to the nominal trajectory of the aircraft, and the squint angle is determined by a constant Doppler frequency. The geometric relation of the irradiated target and the InSAR antennas is shown in figure 1, which involved two different coordinate systems with the first one is fixed to the nominal trajectory and the second one is in the tangent plane of the Earth's ellipsoid. The geographic location of the target ( $\vec{P}$ ) can be determined by the location of InSAR master antenna ( $\vec{A}_1$ ) and the radar look vector ( $\vec{\rho}$ ), which can be derived from the interferometric phase ( $\Phi$ ), the baseline information (length and incline angle  $\alpha$ ) and the aircraft attitude angles.



**Figure 1.** Geometrical relation of ground point and antenna

The airborne InSAR geometric model can be described by Range Equation, Doppler Equation and Phase Equation, as shown in the following:

$$\vec{P} - \vec{A}_i = \vec{r}_i \quad (1)$$

$$\frac{\lambda f_{dol}}{2} = \frac{(\vec{P} - \vec{A}_1) \cdot (\vec{P} - \vec{A}_1)}{|\vec{P} - \vec{A}_1|} \quad (2)$$

$$\phi = \frac{2\pi Q(r_2 - r_1)}{\lambda} = \frac{2\pi Q r_1}{\lambda} \left[ \left(1 - \frac{2\vec{r}_1 \cdot \vec{B}}{r_1^2} + \left(\frac{B}{r_1}\right)^2\right)^{\frac{1}{2}} - 1 \right] \quad (3)$$

The geographic location of the target can be solved:

$$\vec{P} = \vec{A}_1 + r_1 \times \begin{bmatrix} \sin \theta_y \sin(\alpha + \theta_1 + \theta_r) + \cos \theta_y \sin \theta_p \cos(\alpha + \theta_1 + \theta_r) \\ \cos \theta_y \sin(\alpha + \theta_1 + \theta_r) - \sin \theta_y \sin \theta_p \cos(\alpha + \theta_1 + \theta_r) \\ -\cos \theta_p \cos(\alpha + \theta_1 + \theta_r) \end{bmatrix} \quad (4)$$

Wherein,  $\vec{P}$  is target location vector of,  $\vec{A}_1$  is InSAR master antenna location vector,  $r_1$  is range,  $\theta_y$  is heading angle,  $\alpha$  is baseline incline angle,  $\theta_y$ ,  $\theta_r$  and  $\theta_p$  is the heading, roll and pitch angles respectively which are determined by POS system carried on the aircraft.  $\theta_1$  can be written as:

$$\theta_1 = \arcsin \left[ \frac{B}{2r_1} - \frac{\lambda \phi}{2\pi Q B} - \frac{\lambda^2 \phi^2}{8\pi^2 Q^2 B r_1} \right] \quad (5)$$

Where B is baseline length,  $\lambda$  is wave length,  $\phi$  is unwrapped phase. The sensitivity equations are derived by differentiating the basic location equations with respect to the different parameters of the interferometer (Eq. (4)). There are two different error sources related to the parameters to be taken into account, one corresponding to the static parameters that are not expected to change with time, i.e. baseline length and inclination, and another is related to the different data recorded in-flight, such as attitude angles, Doppler frequency, etc. In both cases, errors on its estimation or measurement bias will lead to errors on the final target locations.

The absolute time delay is that for the interferometric channel identified as the reference. Errors on its calibration lead to range errors:

$$\begin{bmatrix} \frac{\partial X_p}{\partial \tau} \\ \frac{\partial Y_p}{\partial \tau} \\ \frac{\partial Z_p}{\partial \tau} \end{bmatrix} = \begin{bmatrix} -\frac{r_1}{\sin \theta_1} \frac{\text{sol}}{2} (\cos \theta_y \sin \theta_p \cos(\theta_1 - \alpha + \theta_r) + \sin \theta_y \sin(\theta_1 - \alpha + \theta_r)) \left(-\frac{B}{2r_1^2} + \frac{\lambda^2 \phi^2}{8\pi^2 Q^2 B r_1^2}\right) \\ \frac{\text{sol}}{2} (\sin \theta_y \sin \theta_p \sin(\theta_1 - \alpha + \theta_r) + \cos \theta_y \cos(\theta_1 - \alpha + \theta_r)) + \\ \left(-\frac{r_1}{\sin \theta_1} \frac{\text{sol}}{2} \left[-\frac{B}{2r_1^2} + \frac{\lambda^2 \phi^2}{8\pi^2 Q^2 B r_1^2}\right]\right) (\sin \theta_y \sin \theta_p \cos(\theta_1 - \alpha + \theta_r) - \cos \theta_y \sin(\theta_1 - \alpha + \theta_r)) \\ -\frac{\text{sol}}{2} \cos \theta + \left(-\frac{B \text{sol}}{4r_1} + \frac{\lambda^2 \phi^2 \text{sol}}{16\pi^2 Q^2 B r_1}\right) (\csc \theta_1) \cos \theta_p \cos(\theta_1 - \alpha + \theta_r) \end{bmatrix} \quad (6)$$

Baseline length is one of the most critical parameters of the target locations. Small errors on its estimation lead to large errors on target location:

$$\begin{bmatrix} \frac{\partial X_p}{\partial B} \\ \frac{\partial Y_p}{\partial B} \\ \frac{\partial Z_p}{\partial B} \end{bmatrix} = \begin{bmatrix} -\frac{r_1}{\sin \theta_1} (\cos \theta_y \sin \theta_p \cos(\theta_1 - \alpha + \theta_r) + \sin \theta_y \sin(\theta_1 - \alpha + \theta_r)) \left(\frac{1}{2r_1} + \frac{\lambda \phi}{2\pi Q B^2} + \frac{\lambda^2 \phi^2}{8\pi^2 Q^2 B^2 r_1}\right) \\ -\frac{r_1}{\sqrt{\sin^2 \theta - \sin^2 \beta}} \left( \frac{\cos \theta \cos \theta_p \cos(\theta_1 - \alpha + \theta_r)}{\sin \theta_1} \left(\frac{1}{2r_1} + \frac{\lambda \phi}{2\pi Q B^2} + \frac{\lambda^2 \phi^2}{8\pi^2 Q^2 B^2 r_1}\right) \right. \\ \left. + \sin \beta (\cos \theta_y \sin \theta_p \cos(\theta_1 - \alpha + \theta_r) + \sin \theta_y \sin(\theta_1 - \alpha + \theta_r)) \frac{1}{\sin \theta_1} \left(\frac{1}{2r_1} + \frac{\lambda \phi}{2\pi Q B^2} + \frac{\lambda^2 \phi^2}{8\pi^2 Q^2 B^2 r_1}\right) \right) \\ \cos \theta_p \cos(\theta_1 - \alpha + \theta_r) \left(\frac{1}{\sin \theta_1}\right) \left(\frac{1}{2} + \frac{\lambda \phi r_1}{2\pi Q B^2} + \frac{\lambda^2 \phi^2}{8\pi^2 Q^2 B^2}\right) \end{bmatrix} \quad (7)$$

The sensitivity of baseline incline angle is:

$$\begin{bmatrix} \frac{\partial X_p}{\partial \alpha} \\ \frac{\partial Y_p}{\partial \alpha} \\ \frac{\partial Z_p}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} r_1(-\cos \theta_y \sin \theta_p \cos(\theta_1 - \alpha + \theta_r) - \sin \theta_y \sin(\theta_1 - \alpha + \theta_r)) \\ -\frac{r_1}{\sqrt{\sin^2 \theta - \sin^2 \beta}} (\cos \theta \cos \theta_p \cos(\theta_1 - \alpha + \theta_r) \\ + \sin \beta (\cos \theta_y \sin \theta_p \cos(\theta_1 - \alpha) + \sin \theta_y \sin(\theta_1 - \alpha))) \\ r_1 \cos \theta_p \cos(\theta_1 - \alpha + \theta_r) \end{bmatrix} \quad (8)$$

The number of integer cycles on the unwrapped phase can be easily derived from the *a priori* information of the terrain elevation and the master antenna location. But there still is a remaining phase offset errors:

$$\begin{bmatrix} \frac{\partial X_p}{\partial \phi} \\ \frac{\partial Y_p}{\partial \phi} \\ \frac{\partial Z_p}{\partial \phi} \end{bmatrix} = \begin{bmatrix} -\frac{r_1}{\sin \theta_1} (\cos \theta_y \sin \theta_p \cos(\theta_1 - \alpha + \theta_r) + \sin \theta_y \sin(\theta_1 - \alpha + \theta_r)) \left( -\frac{\lambda}{2\pi QB} - \frac{\lambda^2 \phi}{4\pi^2 Q^2 B r_1} \right) \\ -\frac{r_1}{\sqrt{\sin^2 \theta - \sin^2 \beta}} \frac{1}{\sin \theta_1} \left( \frac{\lambda}{2\pi QB} + \frac{\lambda^2 \phi}{4\pi^2 Q^2 B r_1} \right) (-\cos \theta \cos \theta_p \cos(\theta_1 - \alpha) \\ - \sin \beta (\cos \theta_y \sin \theta_p \cos(\theta_1 - \alpha) + \sin \theta_y \sin(\theta_1 - \alpha))) \\ \cos \theta_p \cos(\theta_1 - \alpha + \theta_r) \left( \frac{1}{\sin \theta_1} \right) \left( -\frac{\lambda r_1}{2\pi QB} - \frac{\lambda^2 \phi}{4\pi^2 Q^2 B} \right) \end{bmatrix} \quad (9)$$

The sensitivity equations show the impact of the interferometric parameters on the location errors, which can be used to calibrate the interferometric parameters from the errors on the location of several well-located Corner Reflectors. The first estimation of the target location can be obtained by the initial interferometer parameter estimation and the unwrapped interferometric phase implemented the location equations. Then an iterative process is performed until the target location errors go into a constant domain.

### 3. Validation of airborne InSAR geometrical model

To test the performance of the InSAR mapping system, a Validation/Calibration (Val/Cal) campaign has carried out in Sichun province, south-west China in 2011, in which nine well located corner reflectors were used to test our method. As shown in Table 1 and 2, the results demonstrate the efficiency of our method. In the campaign, the aircraft flew in the direction of west to east or reverse. It can be detected in Table 2 that the eastern errors are smaller than northern errors, namely the errors in range are larger than that in azimuth direction, which mainly caused by the absolute time delay variation or the inaccuracy of the measurement in motion compensation processing. The accuracy of geometrical model is qualified for 1:10000 map productions.

**Table 1.** Corner Reflectors locations obtained by GPS and InSAR

PointName	Height survey	Height imaging	DX	DY	DZ	DXYZ
C000	457.731	457.6889	0.057548	-0.07288	0.042107	0.101962
C001	459.381	459.4236	0.004723	0.048565	-0.0426	0.064773
C002	459.568	459.6493	-0.01389	0.102847	-0.08129	0.13183
C003	460.339	460.3709	-0.10804	-0.12995	-0.03195	0.171989

C004	461.395	461.2756	-0.31188	0.081917	0.119366	0.343839
C005	462.19	462.1209	-0.1846	0.019952	0.069124	0.198121
C006	462.999	463.0534	-0.04427	0.04288	-0.0544	0.082207
C007	463.038	463.0099	-0.06084	0.018714	0.028099	0.069581
C008	463.671	463.7207	0.070466	-0.08024	-0.04969	0.117784

**Table 2.** Corner Reflectors locations errors of check points.

PointName	Northern errors	Eastern errors	Height errors
X03A04	1.322	0.053	-0.328
X04A05	0.247	0.103	0.058
X05A06G	1.809	-0.101	-0.986
X06G	1.954	-0.219	-0.770
X07	0.547	-0.319	-0.221
XE102	-0.152	-0.211	0.415
XE103	0.772	-0.314	-0.201

#### 4. Conclusion

As Airborne InSAR geometrical model is foundation of InSAR mapping system, the Cal/Val of an airborne InSAR geometric model are discussed in this paper. Based on the test data acquired by an airborne InSAR mapping system developed by RADI, CAS, the calibration results, obtained by our method, suggest that the accuracy of the geometric model is qualified for 1:10000 map productions.

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