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Bayesian Inversion for Layered Spherical Symmetric Earth Conductivity Model from Global Magnetic Data

H Grandis[1]* and P Tarits[2]

[1] Faculty of Mining and Petroleum Engineering, Institut Teknologi Bandung (ITB), Bandung, Indonesia

[2] Institut Universitaire Européen de la Mer, Université de Bretagne Occidentale (UBO), Brest, France

*e-mail: grandis@geoph.itb.ac.id

Abstract. In the Bayesian perspective, inference on model parameters from observed data is performed by calculating the likelihood of the data given prior model parameters, i.e. to estimate the posterior probability of model parameters. With the advent of computational resources, there are increasing interests in resolving full non-linear inverse problems using global approach. Although the current trends are geared towards algorithms to efficiently explore the model space, we employed the classical "pure" Monte Carlo method to resolve the inverse problem in the global scale induction study. Observatory and satellite magnetic data are used to provide insight on the deep mantle conductivity. In this case, layered (1D) spherical symmetric conductivity model can be considered as adequate to represent the Earth's conductivity variation with depth. Model parameters (resistivities and thicknesses) with uniform probabilities over predefined intervals are drawn as samples of the model space. Reliable posterior estimates are derived from a large number of samples which are still manageable with the current PC technology. Relatively small uncertainties of the posterior estimates suggest that the Monte Carlo method is adequately sampled the model space with a small number of model parameters. Our results are consistent with a monotonic increase of conductivity with depth with a marked inflexion at about 700-900 km, while discontinuities at 410 km and 660 km known from seismic and petrology data seem unresovable directly from EM data.

1. Introduction

In electromagnetic (EM) induction studies, currents induced by temporal variations of the Earth's main magnetic field can be used to infer the Earth's electrical conductivity. In global induction studies, magnetic data from a global network of observatories allow probing the conductivity down to depths of 1000 km or more. These magnetic observatories are sparse and irregularly distributed such that laterally averaged conductivity profiles tend to represent continental regions only [1,2]. More recent works employed data from satelites [3-5] and combined data from both observatories and satellites as well [6]. Global induction studies are also extended to investigate deep electrical conductivity of other planets, especially Mars [7,8]. In terms of modelling, both layered or 1-D [3-8] and 3-D [3,9] models are used to represent deep mantle conductivity.

The paper describes the modelling of the global EM induction data derived from measurements from the global network of geomagnetic observatories combined with those from Ørsted, CHAMP, SAC-C and the Swarm trio satellites [6]. This data set represents the largest ever used for mantle conductivity studies and readers are referred to Püthe et al. [6] for more detailed data processing and analyses. We used Monte Carlo method to resolve the inverse problem recast in the Bayesian paradigm [10] with layered or 1-D conductivity model for a spherically symmetric earth.

2. Method

In global induction studies, the Earth is considered as a spherically symmetric body having conductivity variations as a function of its radius. The model response is complex admittance function C (in km) converted from the frequency dependent transfer function (see [3] and [7] for more details). For our inversion purpose, the conductivity variations with Earth's radius is reparameterized as resistivity of layers with interface at depths.

In the Bayesian inference, the inverse problem solution is the conditional probability of the model given the data. We are interested in the minimum information required by the data. In contrast with smooth (i.e. Occam-like) inversion where models with minimum features are sought, we consider models with a minimum number of parameters. The model space was defined a priori by a uniform probability over a certain interval for each model parameter. Unlike techniques based on Markov chains [e.g. 11] and equivalent, we compute the full range of variation of the parameter and control the accuracy of the computation of the a posteriori marginal probabilities of the model parameters [10,12].

Two class of models were considered, one with a small number of layers where the model parameters are both resistivity and depth of the layer. In the other models, we fixed the depth of the transitions such that the model parameters are resistivity of layers only. With a small number of model parameters and relatively simple analytical forward modelling function, 10s million models can be generated quite fast with the current personal computer technology.

3. Results

First, an inversion was performed to infer both resistivity and interface depth of a four-layer model with minimal "a priori" information, i.e. number of layers and interval (minimum and maximum values) of model parameters. Those intervals were determined in view of recover physical property transitions known from other geophysical data, i.e. seismology. The marginal probability density function (PDF) for each model parameter is shown in Figure 1. Only resistivity and depth of 4-th layer at about 800 km are very well constrained. The marginal probabilities for resistivity of the transitions above 800 km are one side bounded and provide only the lower limit for the parameter. The resistivity versus depth profile and data fitting are presented in Figure 2. For unresolved or partially resolved parameters, the mean values and uncertainties are in fact determined also by their "a priori" intervals. The model response (complex admittance) appears to be very smooth such that the resolution of the data is considerably very low for the frequency or period interval considered in this study.
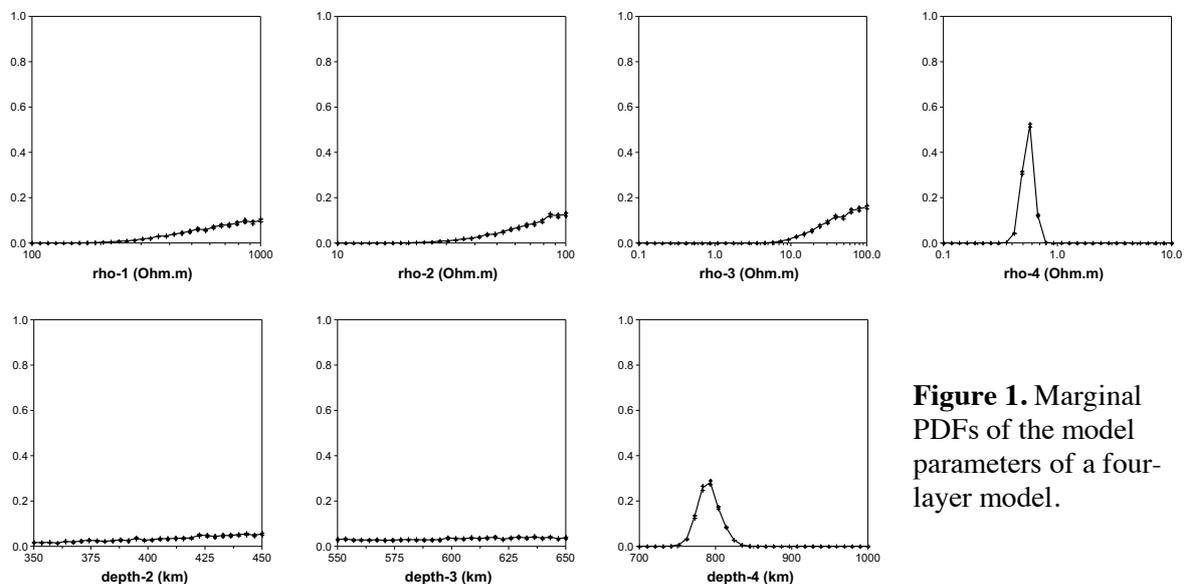

**Figure 1.** Marginal PDFs of the model parameters of a four-layer model.
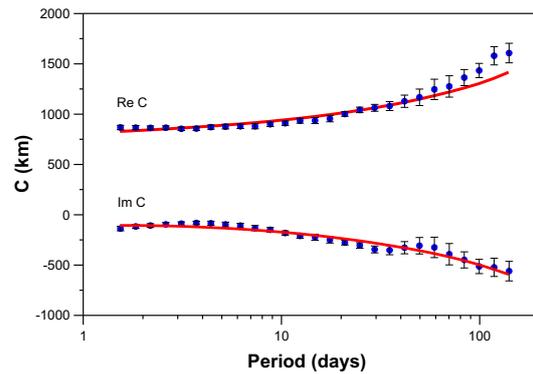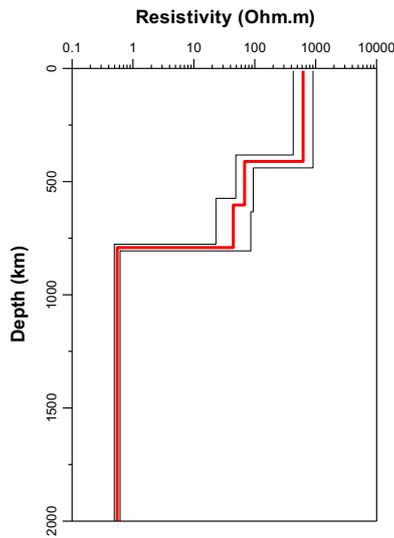
Figure 2. Inverse model as resistivity-depth profile with the mean values in red and their uncertainties envelop in black (left panel), observed complex admittance in blue and model fit in red (top panel)

The minimum structure required by the data is one transition at about 800 km. Therefore, an inversion with only two-layer model was performed. In this case only resistivity of the first layer is partly constrained, while other parameters are well represented by their expected (mean) values and the square root of variance, since their marginal probabilities are bell-like (Figure 3). There is no significant difference from the previous result in terms of inverse model response to observed data fit (Figure 4).
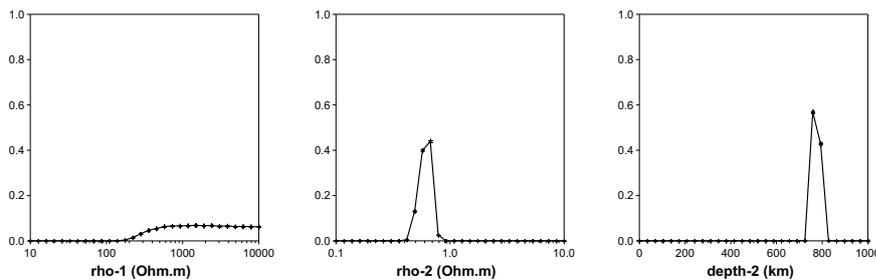

Figure 3. Marginal PDFs of the model parameters of a two-layer model.
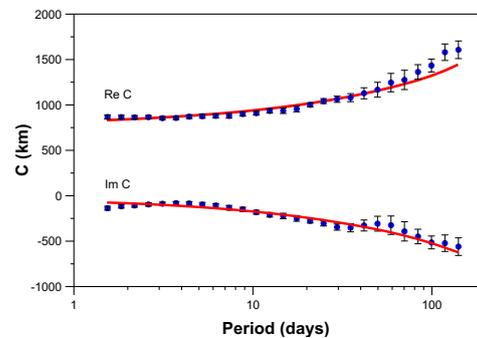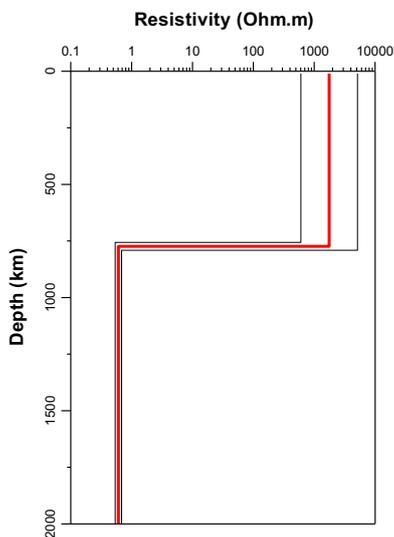

Figure 4. Resistivity-depth profile for a two-layer model with the mean values in red and their uncertainties envelop in black (left panel), observed complex admittance in blue and model fit in red (top panel)

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Seismology and petro-physical data state that the mantle is layered with limits at around 410, 520 and 660 km on average. Laboratory conductivity data show rapid changes of resistivity values across the mantle discontinuity. However no discontinuity is infered from seismology at around 800 km. To accommodate such information, another inversion was conducted by fixing the mantle discontinuities at 410 and 660 km. The 520 km transition was considered to close to 660 km such that the layer thickness is too thin to be resolved by the data. Only lower mantle discontinuity is inferred along with all resistivities. Again only the lower mantle discontinuity parametres are fully resolved while the upper mantle resistivities' lower limit are bounded (Figure 5).
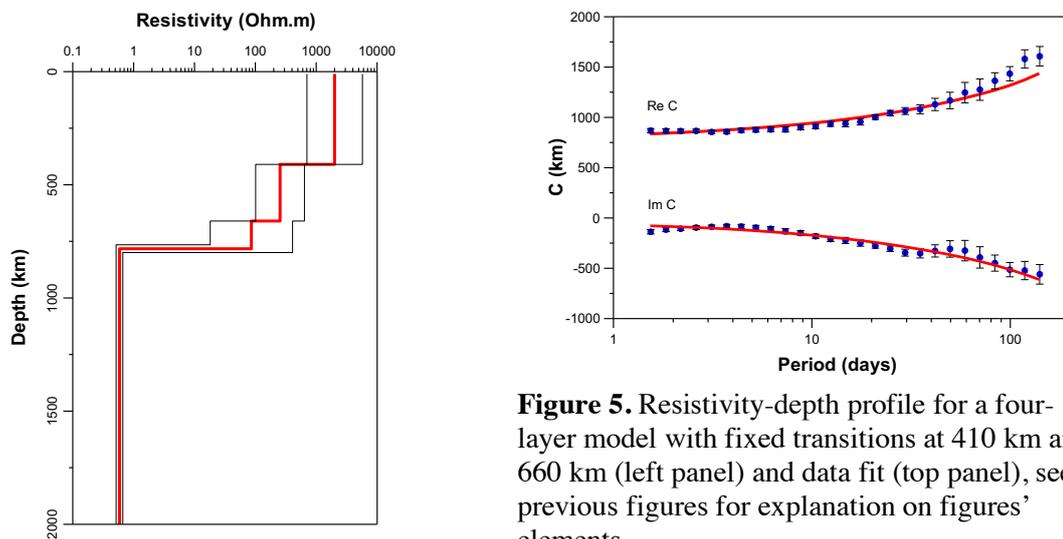
Figure 5. Resistivity-depth profile for a four-layer model with fixed transitions at 410 km and 660 km (left panel) and data fit (top panel), see previous figures for explanation on figures' elements.

Several other inversions were also performed during this study. One of them is with the depth discretized into 100 km thick layers and fixed, all resistivites are inferred with "a priori" model parameter interval chosen from previous results. The inverse model only shows gradual resistivity transition above 800 km still with an abrupt resistivity change at around 800 km as before. We also fixed all known transition, i.e. 410, 660 and 780 km in order to obtain the marginal laws only on resistivities. The results are quite similar with those presented in Figure 5, except for the fixed last (4-th) layer transition. The marginal probability on resistivites is presented in Figure 6 for comparison with results from mineral physics of Xu et al. [13] and Khan & Shankland [14].
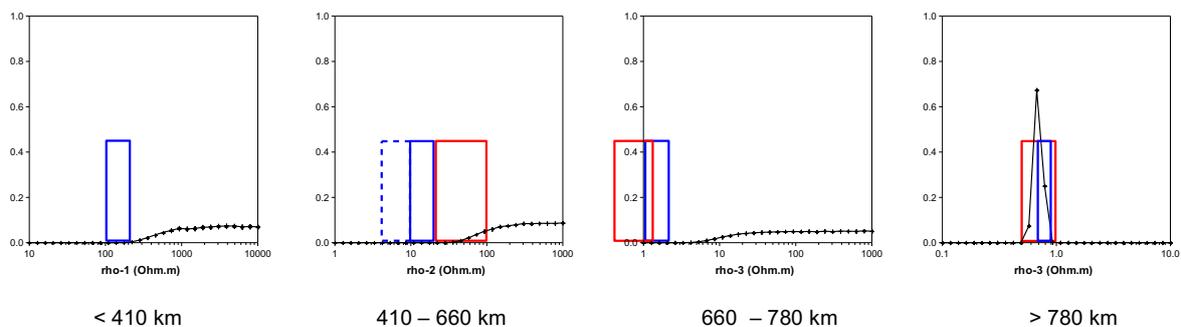
Figure 6. Marginal PDFs for resistivities from inversion with fixed transitions at 410, 660 and 780 km. The blue and red boxes are results from Xu et al. [13] and Khan & Shankland [14] respectively. See text on discussion and conclusion for more details.

4. Discussion and Conclusion

For the stratified mantle model, each resistivity can be interpreted as the mean value across a given layer. These GDS (Geomagnetic Deep Sounding) data bound very accurately the change in resistivity in the uppermost lower mantle (770 ± 10km) and the resistivity below that limit (around 0.6-0.7 Ohm.m). Mineral physics [13,14] agree well with this result.

Above 780 km and in the upper mantle, only the lowest value of the resistivity is bounded. The marginal law of the posterior model provides a quantitative bound for mineral physics models. Thus for lower limit of 10% for the cumulative PDF from the lowest "a priori" resistivity value should be more than 500-700 Ohm.m below 410 km while in the transition zone it should be more than 110-130 Ohm.m, a value significantly larger than proposed by mineral physics unless low temperature adiabat is considered [14]. In the uppermost lower mantle, the resistivity should be more than around 7 Ohm.m, again more resistive than predicted by mineral physics.

The global data set obtained by Puthe et al. [6] used in this study provides valuable bounds on the mantle resistivity for the seismological and petro-physical stratified mantle. These bounds should provide insight in the conduction processes and physical state of the mantle.

References

[1]   Olsen N 1998 *Geophys. J. Int.* **133** 298-308.
[2]   Koyama T, Khan A and Kuvshinov A 2014 *Geophys. J. Int.* **196** 1330-1350.
[3]   Constable S and Constable C 2004 *Geochem. Geophys. Geosyst.* **5** Q01006.
[4]   Civet F, Thébault E, Verhoeven O, Langlais B and Saturnino D 2015 *Geophys. Res. Lett.* **42** 3338-3346.
[5]   Püthe C and Kuvshinov A 2013 *Earth Planet Space* **65** 1233-1237.
[6]   Püthe C, Kuvshinov A, Khan A and Olsen N 2015 *Geophys. J. Int.* **203** 1864-1872.
[7]   Civet F and Tarits P 2013 *Planet. Space Sci.* **84** 102-111.
[8]   Civet F and Tarits P 2014 *Earth Planet Space* **66** 85.
[9]   Püthe C and Kuvshinov A 2014 *Geophys. J. Int.* **197** 768-784.
[10]   Tarits P, Jouanne V, Menvielle M and Roussignol M 1994 *Geophys. J. Int.* **119** 353-368.
[11]   Grandis H, Menvielle M and Roussignol M 2013 *Int. J. Geoph.* 531473.
[12]   Maulinadya S and Grandis H 2017 *Proc. Asia-Pasific Near Surface Conf.*
[13]   Xu Y, Shankland T J and Poe B T 2000 *J. Geophys. Res.* **105**, 27, 865-875.
[14]   Khan A and Shankland T J 2012 *Earth Planet Sci. Lett.* **317-318**, 27-43.