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Modeling the optimal decision-making strategy of an organization using mathematical methods

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Abstract. The paper builds and studies a stochastic decision-making model under the influence of both negative and positive factors in a competitive environment. There is a finite set of company states, which is estimated by the value of the firm's capitalization per unit of time. Depending on the decision taken, the company goes into one or another state. In each state, the set of valid strategies depends on this state. The purpose of the simulation is to find the optimal strategy that maximizes the expected capitalization of the company from the transition process to various states, having a finite number of stages.

1. Introduction

During the life of the company is constantly exposed to external and internal environment. Impact can be both positive and negative. Environmental factors can be divided into direct and indirect. The first group of elements of external factors usually include consumers, competitors, suppliers, laws and state authorities, trade unions. The second group consists of the state of the economy, scientific and technological progress, political and socio-cultural factors, international events. The elements of internal factors of a firm typically include the structure, goals, objectives, technology and human resources.

1.1. Informal statement of the problem of constructing and analyzing a stochastic decision-making model

A model $\Gamma = \langle I = \{1, 2, \dots, n\} = \{i\}_1^n, K_i = \{1, 2, \dots, k_i\}, S_i = \{s_i\}_1^n, R_i \rangle$ is considered, in which the decision-making process is represented by a finite number of states $i, i=1, 2, \dots, n$.

Let each state $i \in S = \{1, \dots, N\}$ correspond to a finite set K_i of solutions (or alternatives), whose elements are denoted by numbers $1, \dots, k_i$. The space of strategies K is the direct product of the sets of solutions $K = K_1 \times K_2 \times \dots \times K_N$.



There is a matrix $P = (p_{ij})$ of transition probabilities between states. The remuneration structure is represented in the form of a matrix $R(q) = (r_{ij}(q))$, the elements of which are the income values. The transition probability matrix P and the income matrix $R(q)$ depend on the decision policies k that the decision maker has.

Depending on what decision is made (management at the current stage), the development of a company takes place according to one or another scheme, which implies a combination of positive factors α_i and negative factors β_i on the state of the company Ω .

There is a finite set of states of the company i , which is estimated by the value of the firm's capitalization per unit time t . Depending on the decision taken, the company goes into one or another state i . In each state i the set of admissible strategies k depends on this state.

The goal of the task is to find the optimal strategy that maximizes the expected capitalization of the company from a process that has a finite number of stages. By capitalization we mean the process during which, under the influence of certain factors, there is an increase in the money supply due to their investment in any active income-generating volumes.

2. Formalization of the problem of constructing and analyzing a stochastic decision-making model

Let the process of transition from state to state occur, not deterministic, but stochastically controlled by the transition matrix $P = (p_{ij})$, where p_{ij} is the probability that the system at the time $t+1$ is in the state j , if it is known that at the time t it was in the state i .

Consider the case when the matrix does not depend on time, and decisions are made at each step. Suppose that at each step one of a set of such matrices can be chosen as a transition matrix, and we denote the policy-corresponding matrix q by $P(q) = (p_{ij}(q))$.

Suppose further that not only the state at each step changes, but also the capitalization of the firm, which is a function of the initial and final states and the solution. Let $R(q) = (r_{ij}(q))$ means the corresponding matrix of the firm's capitalization.

Note that this model is not perfect, since the same factor can be both positive and negative. An example is the change of technology, personnel, investment, etc. For a more detailed solution of specific problems, consider the stochastic model on a specific example

3. Classification of the state of the company in the Markov model of management of its activities

We introduce a classification based on the key performance indicators of the company (for example, sales of goods and / or services, productivity, profits, customer satisfaction, etc.). In this case, we will consider the growth of workers' labor productivity $x(t)$. We introduce the following indicators: growth rate $V(t) = x(t)/t$; growth acceleration $a(t) = V(t)/t$.

Thus, if the speed is zero, then the firm is in a stable state (there is no growth and drop in labor productivity); if the speed is greater than zero, then the company is in a state of growth, if less than zero, then in a crisis state. Acceleration more characterizes the stable state not only of the company, but also of the market as a whole, because shows spasmodic processes of growth or decline in the productivity of workers.

3.1. Example

Suppose that a firm implements its activities in five sectors: agriculture, industry, commerce, construction and transport. In any industry, you can invest money in labor resources in the following areas: 1 - staff development; 2 - retraining of workers; 3 - the use of third-party workers; 4 - movement of workers from industry to industry; 5 - active search for employees.

For each fixed policy in each industry, the probabilities are set that the next employee of the company will find it in one of five industries. The corresponding incomes in monetary units associated

with the introduction of an employee into the industry are known. Because each policy has a different set of employees, transition probabilities and incomes depend on the policies. Income in this case is expressed as a fall or increase in the capitalization of the company. These tasks can be written as table 1.

Table 1. Baseline.

The state	Policy	Transient probability					Income				
		j=1	2	3	4	5	i=1	2	3	4	5
1	1	0,5	0,1	0,1	0	0,3	-10	50	30	0	-20
	2	0,2	0,3	0,2	0,1	0,2	-50	50	300	-20	150
	3	0,4	0,2	0	0,1	0,2	200	-250	0	150	130
	4	0,3	0,2	0,1	0,2	0,2	-100	260	-230	300	180
	5	0,1	0,2	0,3	0,1	0,3	150	180	-120	180	280
2	1	0,3	0,2	0,1	0,2	0,2	100	130	160	-300	-20
	2	0,2	0,2	0,2	0,2	0,2	-30	100	-200	250	200
	3	0,2	0,4	0,2	0	0,2	600	700	100	0	630
	4	0	0,2	0,3	0,1	0,3	0	1200	240	280	640
	5	0,4	0,1	0,1	0,1	0,3	-10	430	480	790	640
3	1	0,3	0,2	0,1	0,2	0,2	20	-20	710	150	-10
	2	0,1	0,1	0,3	0,2	0,3	350	450	-50	150	-100
	3	0,2	0,3	0,2	0	0,3	-580	400	650	0	270
	4	0,1	0,2	0,3	0,1	0,3	160	-20	-30	160	540
	5	0,3	0,2	0,1	0,2	0,2	260	450	590	540	350
4	1	0	0,3	0,2	0,1	0,4	0	530	710	260	460
	2	0,1	0,1	0,6	0,1	0,1	350	-400	-60	210	280
	3	0,4	0,1	0,1	0,1	0,2	250	400	650	460	670
	4	0,1	0,1	0,3	0,2	0,3	160	-50	-80	120	680
	5	0,1	0,4	0,3	0,2	0	460	250	490	250	0
5	1	0,1	0,2	0,1	0,3	0,3	-400	30	-10	570	-60
	2	0,1	0	0,3	0,2	0,4	-10	0	-40	260	370
	3	0,3	0,2	0,1	0,2	0,2	250	400	650	640	590
	4	0,2	0,3	0,2	0,1	0,2	-30	150	-60	290	-20
	5	0,4	0,1	0,1	0	0,3	-80	120	450	0	350

Let us explain the table by the example of the last row. It shows that if in the fifth industry to invest money in the first policy (advanced training of workers), then with a probability of 0.1, an employee will be acquired in industry 1, and the capitalization of the company will fall by 400 units. With a probability of 0.2, an employee will be acquired in industry 2 with an income of 30 units, and with a probability of 0.1, an employee will be acquired in industry 3, with an income of -10 units, with a probability of 0.3 employee acquisition in industry 4, with an income in 570 units and with a probability of 0.3 employee acquisition in industry 5, and the fall of the capitalization of the company will be 60 units. There are five states in this problem, i.e. $N = 5$, and five policies in states 1, 2, 3, 4, 5, i.e. $n_1=5, n_2=5, n_3=5, n_4=5, n_5=5$. So, there is $5*5*5*5*5=3125$ possible policies.

To solve the problem, we choose the policy vector as the initial approximation

$$D = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

which means that we work in all sectors. This is a policy that maximizes the mathematical expectation of a firm’s capitalization growth. For her, we have a transition probability matrix

$$[p_{ij}] = \begin{bmatrix} 0.5 & 0.1 & 0.1 & 0 & 0.3 \\ 0.3 & 0.2 & 0.1 & 0.2 & 0.2 \\ 0.3 & 0.2 & 0.1 & 0.2 & 0.2 \\ 0 & 0.3 & 0.2 & 0.1 & 0.4 \\ 0.1 & 0.2 & 0.1 & 0.3 & 0.3 \end{bmatrix} \text{ and a growth vector for the firm’s capitalization}$$

$$\left[\sum_{j=1}^N p_{ij} r_{ij} \right] = \begin{bmatrix} -3 \\ 8 \\ 101 \\ 511 \\ 118 \end{bmatrix}.$$

We denote the sum $\sum_{j=1}^N p_{ij} r_{ij}$ by q_i , since the expected capitalization of the firm depends only on i .

The equation for determining the values v_i , assuming a value v_3 of zero, is written as

$$g + v_1 = -3 + 0.5v_1 + 0.1v_2 + 0.3v_3 + 0v_4, \quad g + v_2 = 8 + 0.3v_1 + 0.2v_2 + 0.1v_3 + 0.2v_4,$$

$$g + v_3 = 101 + 0.3v_1 + 0.2v_2 + 0.1v_3 + 0.2v_4, \quad g + v_4 = 511 + 0v_1 + 0.3v_2 + 0.2v_3 + 0.1v_4,$$

$$g = 118 + 0.1v_1 + 0.2v_2 + 0.1v_3 + 0.3v_4 \text{ and have a solution } v_1 = -334.45, \quad v_2 = -209.71,$$

$$v_3 = -116.71, \quad v_4 = 328.17, \quad v_5 = 0, \quad g = 129.4.$$

Turning to the policy improvement procedure, we calculate the values $q_i^k + \sum_{j=1}^N p_{ij}^k v_j$ for all i and k .

i	k	$q_i^k + \sum_{j=1}^N p_{ij}^k v_j$
1	1	-202,867
	2	-27,328
	3	-71,905
	4	6,686
	5	39,417 +
2	1	-80,314
	2	-2,54
	3	371,884
	4	487,862 +
	5	224,395
3	1	12,686
	2	41,205

	3	61,855	
	4	103,417	
	5	316,686	+
4	1	457,562	+
	2	-83,625	
	3	251,395	
	4	191,205	
	5	256,292	
5	1	129,393	
	2	184,176	
	3	377,686	+
	4	-68,328	
	5	-36,422	

The results show that for $i = 1$ the value in the right column is maximum for $k=5$. Similarly, for $i = 3$. For $i = 2$ it is maximal for $k=4$. So the new policy will be determined by the vector

$$D = \begin{bmatrix} 5 \\ 4 \\ 5 \\ 1 \\ 3 \end{bmatrix} .$$

This means that in the first and third industries it is most advantageous to use the 5th policy, for the 2nd the fourth policy, for the fourth the first policy, and for the fifth the third.

Based on the above, we have

$$[p_{ij}] = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.1 & 0.3 \\ 0 & 0.2 & 0.3 & 0.1 & 0.3 \\ 0.3 & 0.2 & 0.1 & 0.2 & 0.2 \\ 0 & 0.3 & 0.2 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.1 & 0.2 & 0.2 \end{bmatrix} , [q_i] = \begin{bmatrix} 117 \\ 532 \\ 405 \\ 511 \\ 466 \end{bmatrix} .$$

Turning to the operation of determining the value, we solve the equations

$$\begin{aligned} g + v_1 &= 117 + 0.1v_1 + 0.2v_2 + 0.3v_3 + 0.1v_4 , & g + v_2 &= 532 + 0.2v_2 + 0.3v_3 + 0.1v_4 , \\ g + v_3 &= 405 + 0.3v_1 + 0.2v_2 + 0.1v_3 + 0.2v_4 , & g + v_4 &= 511 + 0.3v_2 + 0.2v_3 + 0.1v_4 , \\ g &= 466 + 0.3v_1 + 0.2v_2 + 0.1v_3 + 0.2v_4 . \end{aligned}$$

Assuming v_5 equals zero again, we get $v_1 = -311.82$, $v_2 = 134.366$, $v_3 = -61$, $v_4 = 129.84$, $v_5 = 0$, $g = 419.2$. Note that g increased from 129.4 to 419.2, which was achieved. So the firm earns an average of 419.2 units per transaction. Using the policy improvement procedure for these values, we calculate the values $q_i^k + \sum_{j=1}^N p_{ij}^k v_j$ for all i and k .

The new policy is thus determined by the vector $D = \begin{bmatrix} 5 \\ 4 \\ 5 \\ 1 \\ 3 \end{bmatrix}$.

Vector D coincides with the vector of the previous policy. So the process has converged, and the value has reached its maximum equal to 419.2. The company should in any industry provide the opportunity for retraining of workers. The application of such a policy will give, on average, per employee an income of 419.2 units.

4. Conclusion

Thus, using a stochastic decision-making model, it becomes possible to find an optimal strategy that maximizes the expected capitalization of a firm from the transition process to various states that has a finite number of stages. The model allows, using a finite set of company states, to investigate a set of admissible strategies depending on a given state. A firm, depending on the decision made, goes into one or another state, which is estimated by the value of the firm's capitalization per unit time t .

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References

- [1] Kriulina E, Tarasenko N, Miroshnitchenko N, Zaitseva I and Dedyukhina I 2016 Environmental Management in Agriculture: Problems and Solutions *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(3)** 1908-12
- [2] Zaitseva I and Popova M 2013 Technique to study the employment potential of the region: economic-mathematical aspect *World Applied Sciences Journal* **22 (1)** 22-5
- [3] Zaitseva I, Kriulina E, Ermakova A, Shevchenko E and Vorokhobina Y 2016 *Application of Factor Analysis to Study the Labour Capacity of Stavropol Krai* *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(4)** 2183-6
- [4] Zaitseva I, Popova M, Ermakova A, Bogdanova S and Rezenkov D 2016 Determination Prospects Of Development Labor Potential In Agriculture Stavropol Territory Based On Assessment His Condition *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(3)** 2592-5
- [5] Zaitseva I, Malafeyev O, Strekopytov S, Ermakova A and Shlaev D 2018 Game-theoretical model of labour force training *Journal of Theoretical and Applied Information Technology*
- [6] Zaitseva I, Malafeyev O, Strekopytov S, Bondarenko G and Lovyannikov D 2018 Mathematic model of regional economy development by the final result of labor resources *AIP Conference Proceedings*
- [7] Zaitseva I, Ermakova A, Shlaev D, Shevchenko E and Lugovskoy S 2017 Workforce planning redistribution of the region's results *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **8(1)** 1862-6
- [8] Kostyukov K, Zaitseva I, Bondarenko G, Svehinskaya T and Nechayeva S 2016 Workforce

- Planning as An Element of Control System” *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(6)** 2315–9
- [9] Zaitseva I, Malafeyev O, Poddubnaya N, Vanina A and Novikova E 2019 Solving a dynamic assignment problem in the socio-economic system *Journal of Physics: Conference Series*
- [10] Zaitseva I, Malafeyev O, Marenchuk Y, Kolesov D and Bogdanova S 2019 Competitive Mechanism for the Distribution of Labor Resources in the Transport Objective *Journal of Physics: Conference Series*
- [11] Malafeyev O, Lakhina J, Zaitseva I, Redinskikh N, Smirnova T and Smirnov N 2019 A mathematical model of production facilities location *Journal of Physics: Conference Series*
- [12] Malafeyev O, Redinskikh N, Nemnyugin S, Kolesin I and Zaitseva I 2018 The optimization problem of preventive equipment repair planning *AIP Conference Proceedings* **1978** 100013
- [13] Malafeyev O, Awasthi A, Zaitseva I, Rezenkov D and Bogdanova S 2018 A dynamic model of functioning of a bank *AIP Conference Proceedings*
- [14] Malafeyev O, Rylow D, Zaitseva I, Popova M and Novozhilova L 2017 Game-theoretic model of dispersed material drying process *AIP Conference Proceedings*
- [15] Malafeyev O, Saifullina D, Ivaniukovich G, Marakhov V and Zaitseva I 2017 The model of multi-agent interaction in a transportation problem with a corruption component *AIP Conference Proceedings*
- [16] Malafeyev O, Farvazov K, Zenovich O, Zaitseva I, Kostyukov K and Svecinskaya T 2018 Geopolitical model of investment power station construction project implementation *AIP Conference Proceedings*
- [17] Zaitseva I, Malafeyev O, Kolesin I, Ermakova A and Shlaev D 2018 Modeling of the labour force redistribution in investment projects with account of their delay” *IEEE International Conference on Power, Control, Signals and Instrumentation Engineering, ICPCSI* **2017** pp 68-70
- [18] Pichugin Y, Malafeyev O, Rylow D and Zaitseva I 2018 A statistical method for corrupt agents detection *AIP Conference Proceedings*
- [19] Malafeyev O, Rylow D, Zaitseva I, Ermakova A and Shlaev D 2018 Multistage voting model with alternative elimination *AIP Conference Proceedings*
- [20] Grigorieva X and Malafeev O 2014 A competitive many-period postman problem with varying parameters *Applied Mathematical Sciences* **8(145-148)** 7249-58
- [21] Malafeev O and Nemnyugin S 1996 Generalized dynamic model of a system moving in an external field with stochastic components *Theoretical and Mathematical Physics* **107(3)** 770
- [22] Pichugin Y and Malafeyev O 2016 Statistical estimation of corruption indicators in the firm *Applied Mathematical Sciences* **10 (41-44)** 2065-73
- [23] Drozdov G, Malafeyev O and Nemnyugin S 2015 Multicomponent dynamics of competitive single-sector economy development *International Conference on "Stability and Control Processes" in Memory of V.I. Zubov (SCP 2015 – Proceedings)*