Geometric tortuosity analysis of porous medium using simple neurite tracer

View the article online for updates and enhancements.

Geometric tortuosity analysis of porous medium using simple neurite tracer

Muhammad Nashih Amien, Genki Tanod Pantouw, Hessel Juliust, Fourier Dzar Eljabbar Latief*

Laboratory of Rock Physics, Physics of Earth and Complex Systems, Institut Teknologi Bandung, Jl. Ganesha no. 10 Bandung 40132, Indonesia

E-mail: fourier@fi.itb.ac.id

Abstract. The concept of tortuosity is used to measure and characterize the structure of porous media. Geometric tortuosity is the ratio between the shortest pathway in the medium to the distance between the inlet and outlet plane. Simple Neurite Tracer method is used to track the connected pores by tracing the tube structures of pores in 3D image stacks. In this analysis, the method is directly applied to the pore structure itself as well as the skeleton of the pores which represent the essential structure of the pore. The method is applied to four simple models of pores with different complexity levels, both the actual pore structure and the skeleton, to determine the effect of complexity to the tortuosity. The analysis is the applied on a digital sample of porous rock model with the size of 256 × 256 pixels. The interconnected pore for both the actual pore structure and the skeleton representation is traced by the Simple Neurite Tracer method. It was observed that the skeleton is a good representation of the actual pore because the pattern of tortuosity obtained between this two approaches are very similar. For all synthetic samples, the more complex the pore structure, the greater the tortuosity.

1. Introduction

Porous medium such as rock is described by several physical parameter that defines the characteristics, which can later be related to the nature of the structure as well as the flow of the fluids inside the structure. One of the physical properties is tortuosity, which is used to be poorly understood as a concept which compared with another physical properties such as porosity of permeability. The concept of tortuosity was first introduced by Carman as the ratio between the actual length of the paths that fluid travel through the pore to the length of porous media [1]. Later, Adler [2] defined tortuosity as the ratio between shortest pathways to the straight-line length. Other literature has defined tortuosity as either a geometric parameter, hydraulic parameter, electric or diffusive [3]. These types of tortuosity have a similarity on the definition and the value obtained is dimensionless.

The concept of tortuosity by Carman is widely used because of the simplicity to analyse the pore structure. Unfortunately, this concept does not always hold for complex porous media because in some cases this concept cannot distinguish several path with different complexity especially in the case when the paths

---

* To whom any correspondence should be addressed

has the same length [3]. Based on this problem, a new definition of tortuosity is needed so that the tortuosity can be more suitable to represent the complexity of the path.

1.1 Geometric Tortuosity
Geometric tortuosity is defined as the ratio of the average length of geometric flow paths in the medium to the straight line across the medium (sidelength). Based on the tortuosity concept by Carman, tortuosity is calculated as follows:

$$\tau_g = \frac{L_g}{L} \tag{1}$$

Compared to hydraulic tortuosity, geometrical tortuosity has smaller value, $\tau_g < \tau_h$, because the hydraulic tortuosity is calculated based on the flow path which lies precisely along the streamline and thus practically has a smooth line. Meanwhile, geometric tortuosity is calculated on a pathway which take the shortest possible path that cross the streamlines [4]. As can be seen in Fig.1, compared with the path taken by hydraulic tortuosity, geometrical path takes the shortest possible distance.

(a)   (b)

Figure 1. a. Hydraulic tortuosity flow path; b. Geometric tortuosity pathway.

2. Methods and Samples

2.1 Simple Neurite Tracer
In order to calculate the tortuosity, the interconnected pore must first be traced from inlet to outlet surface. In this study, simple neurite tracer is used to perform the tracing. Simple neurite tracer is an open source framework plugin on Fiji/ImageJ application for precise annotation of path from a tubular structure in an image stacks [5]. It is designed to follow the structure semi-automatically, and it contain data about the exact coordinates of the pathway. This application is widely used in biology and medical such as for tracking blood vessels or neurons. Simple neurite tracer can be used to tracing the path on porous media due to the tube-like structures of the pores between two end points.
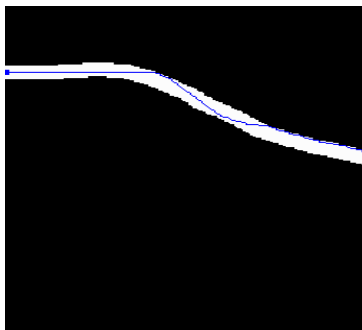

Figure 2. 2D tube-like structure (white line) and the path traced by simple neurite tracer (blue line)

After the exact path between entry and end-point is determined, the application can generate either the length of the path as a .csv file or the coordinates of the path as an .swc file and it can be used as input in another program for further analysis.

2.2 Skeletonization

Skeletonization is a method to represent a 3D structure to a thinned curve, usually useful to visualize a high dimensional structure to a simple navigation [6]. The skeleton can be produced with morphological thinning which erodes the structure until it obtains the actual curve that can represent the shape of the object.

In this study, the skeletonization method is used to generate a representation of the actual interconnected pore paths in the medium. The Skeletonization method is applied using the Skeletonize2D/3D plugin in the Fiji-ImageJ. Subsequently, simple neurite tracer is used to trace the interconnected path which is represented by the skeleton. In contrast to this, SNT can also be applied in tracing the path directly on the actual pore structure.

2.3 Calculating Tortuosity

Based on Carman definition, tortuosity calculation is as defined by Eq. 1. To include complexity of the path as a factor for tortuosity, in this study we used a tortuosity calculation as defined by Dougherty [7] which include path deflection angle parameter θ_i, as follows:

$$\tau = \frac{\sum_{i=0}^{n} \theta_i}{L} \, [\mathrm{m^{-1}}] \tag{2}$$

This calculation values has ($\mathrm{m^{-1}}$) dimension, but according to Hillel [8], tortuosity is a dimensionless geometrical parameter, thus we propose a more suitable definition as follows:

$$\tau = \frac{\sum_{i=0}^{n} \theta_i dl_i}{L}. \tag{3}$$

where dl_i is the length segment for the corresponding deflection angle as shown in Fig. 3. Path deflection angle is calculated by the definition of the angle between two vectors in a 3 dimensional space. The vectors are generated from segmented path, where the coordinates of the points of the path is listed in the .swc file obtained from tracing the path using the SNT. The angle of deflection is :

$$\theta_i = \cos^{-1}\left(\frac{v_{ix}v_{(i+1)x} + v_{iy}v_{(i+1)y} + v_{iz}v_{(i+1)z}}{\left(\sqrt{v_{ix}^2 + v_{iy}^2 + v_{iz}^2} + \sqrt{v_{(i+1)x}^2 + v_{(i+1)y}^2 + v_{(i+1)z}^2}\right)}\right). \tag{4}$$

2.4 Samples

In this study, simple path models are used to determine whether Eq. (4) can describe the complexity level of a path. The simple paths shown in Fig. 4 can be qualitatively distinguished by the complexity level (simple to complex path). The paths were generated with 10 pixels of width in a 256×256 2D image. A direct tracing of the pores as well as the tracing of the skeleton (Fig. 5) were done to compare the pattern from each approach, and also to analyse whether the skeleton can represent the actual pore.
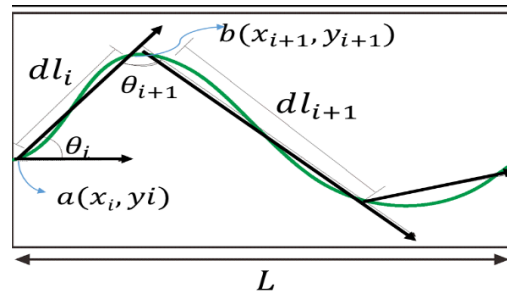
Figure 3. Path deflection angle and length of segment *dl* on a tortuous path.
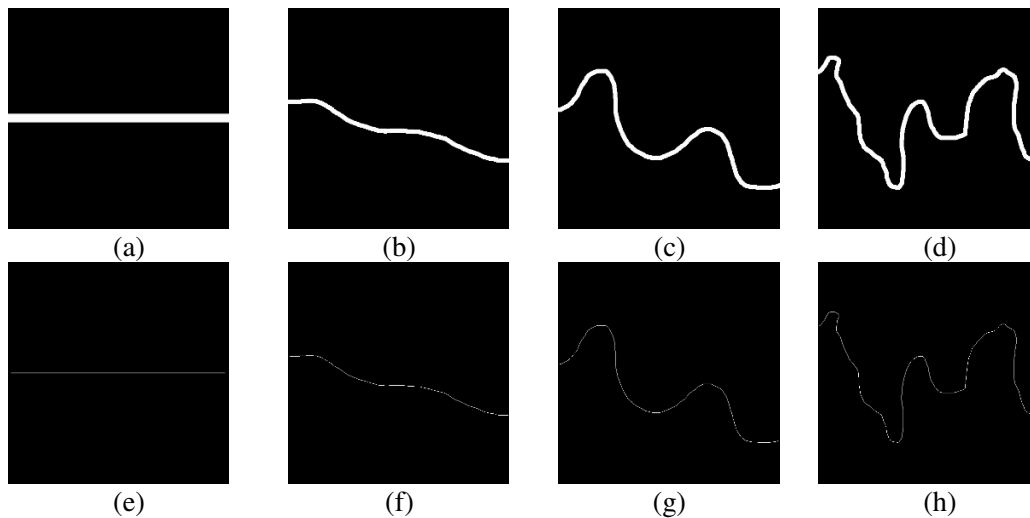

Figure 4. Simple pore model (a-d) and skeleton (e-f) with variation of complexity, (a)(e) A. straight line, (b)(f) B. slightly complex, (c)(g) C. more complex, (d)(h) D. most complex.

In this study, several models of porous medium were also generated. The synthetic complex porous media with spatial dimension of 256^3 pixels with variation in porosity of 0.1, 0.15, and 0.2 were generated using grain based model (3 ellipsoid model and a polyhedral model). The models were generated using a randomly deposited grains with the size varies from 10-16 pixels.

As also performed for the simple path models, the interconnected pore of these synthetic porous media were also traced directly on the actual pore as well as on the skeleton curves. For these synthetic samples, only nine paths are selected. From all the paths, the tracing yields the coordinates of each paths and geometrical tortuosity can be calculated based on Eq. (3). The tortuosity of these paths were then averaged by geometrical, arithmetical and harmonic averaging to obtain the representative tortuosity of the medium.
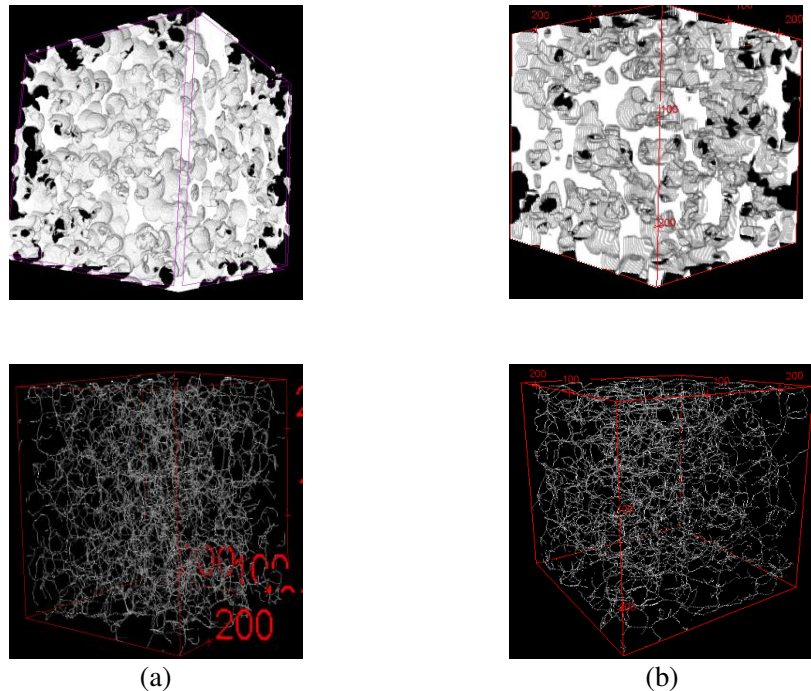
| (a) | (b) |

Figure 5. (a) Ellipsoid model synthetic sample and skeleton of the sample,
(b) Polyhedral model synthetic sample and skeleton of the sample .

3. Results and Discussions

Table 1 lists the tortuosity calculation from Eq. (3) for the simple path models which show in Figure 4. , obtained from direct tracing of the actual pore and the skeleton. For the simple path models, tortuosity and complexity shows very good correlation. Higher complexity make tortuosity higher because of when it calculate on Eq. (3) it has more angle of deflection. The results from both approach are quite similar on straight line (sample A) because of similar path took by the tracer. On sample B-D with variation of complexity, tortuosity from the skeleton is higher than tortuosity obtained from direct actual pore due to difference of the path took by SNT methods.

Table 1. Result of tortuosity calculation for simple path models.

Sample	Tortuosity (direct actual pore)	Tortuosity (skeleton)
A	0.0037	0.0022
B	0.2968	0.4410
C	0.4490	0.6861
D	0.5544	0.9874

Graphic on Figure 6 shows the average of tortuosity calculation for the synthetic porous medium models, with interconnected pore pathway traced by both direct tracing of the actual pore and the skeleton, for ellipsoid grain model and the polyhedral grain model respectively.However, tortuosity from the skeleton is higher than tortuosity obtained from direct actual pore. The difference between these two methods is because that the direct tracing of the actual pore is often yield a shorter (and less complex) trace.

For the synthetic complex porous media models, tortuosity obtained from the skeleton is ~4-5 times higher than that of the ones obtained from direct tracing of the actual pore. The paths which are generated by the skeletonization method are more tortuous compared to the path generated by tracing the actual pore.

Even though the result is higher, the two approaches produce a similar pattern as shown in Fig. 6., i.e., higher porosity produces lower tortuosity.
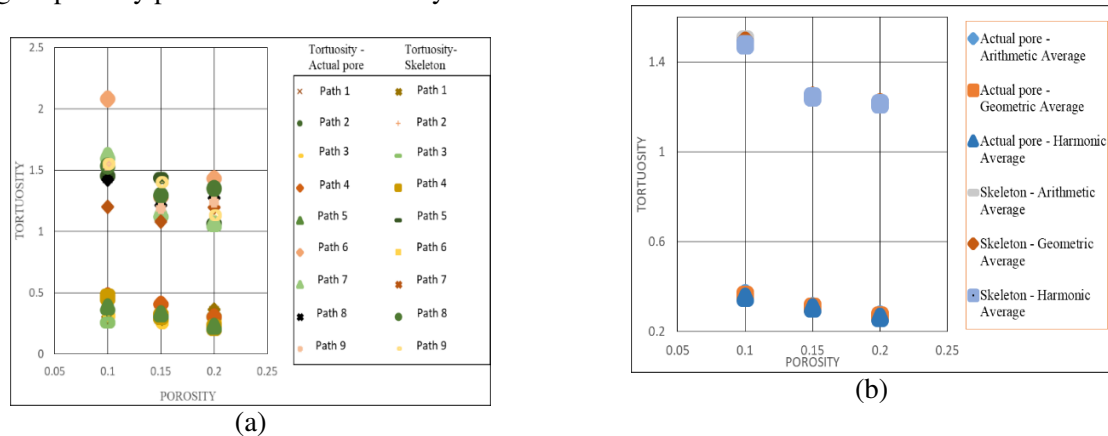

Figure 6. (a) Distribution of tortuosity based on all samples. Skeletonization methods give tortuosity results above 1.0 meanwhile tortuosity obtained by direct tracing to actual pore range about 0.2-0.5 (b) distribution of tortuosity average with 3 averaging method, all of the method give nearly identical tortuosity value.

4. Conclusion and Future Works

From the performed analysis, we can draw several conclusions. Equation (3) can be used to describe the complexity of the pathway. When it applied to a porous media sample, variation of porosity, which makes a variation of complexity, will have a different tortuosity value. Skeletonization method give higher tortuosity value rather than the actual pore which using the definition of geometrical tortuosity about shortest pathway on a pores, making the possibilities to study the actual path by skeleton compared to another flow path. Even though the results is higher, these two approaches have a similar pattern, making skeleton as a good representation of the pore. For future works, a more reliable correlation between tortuosity and porosity can be formulated, and tortuosity can also be correlated to other properties of porous media.

References
[1]   Carman P, "Fluid flow through granular beds," *Trans. Inst. Chem. Eng.*, vol. **15**, pp. 150–166, 1937.
[2]   Adler P M 1992 *Porous Media: Geometry and Transport* (Stoneham, MA: Butterworth- Heinemann)
[3]   Matyka M, Khalili A, and Koza Z. 2008 "Tortuosity-porosity relation in porous media flow" *Physical Revies E* **78**
[4]   Ghanbarian B, Hunt A, Ewing R, and Sahimi M 2013 "Tortuosity in porous media: a critical review" *Soil Sci. Soc. Am. J.* 2013a, **77**, 1461-1477.
[5]   Longair M H, Baker D, and Armstrong J 2011 "Simple Neurite Tracer: open source software for reconstruction, visualization, and analysis of neuronal processes*" Bioinformatics Application Note,* vol **27** no 17, pp 2453-2454
[6]   Cornea N D and Silver D 2007 "Curve Skeleton Properties, Applications, and Algorithms" *IEEE Transactions on Visualization and Computer Graphics,* vol **13** No.3
[7]   Fawziah U Z, Rochmatulloh A K, Sumaryono R F, Feranie S, and Latief F. D. E., "ANALISIS KOMPLEKSITAS STRUKTUR PORI PADA SAMPEL DIGITAL MEDIUM BERPORI," *Spektra J. Fis. dan Apl.*, vol. **2**, no. 3, pp. 203–210, Dec. 2017.
[8]   Hillel D 2004 *Introduction to environmental soil physics.* (San Diego: Academic Press) p 432