PAPER • OPEN ACCESS

Combination-Focusing Model of Freak Wave Based on Boussinesq Equation

To cite this article: Xiang-jun Yu *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **310** 022031

IOP Conf. Series: Earth and Environmental Science **310** (2019) 022031 doi:10.1088/1755-1315/310/2/022031

Combination-Focusing Model of Freak Wave Based on Boussinesq Equation

Xiang-jun YU [1,*], Qing-hong LI[1], and Hua WANG [1]

[1] Department of Military Oceanography and Hydrography, Dalian Navy Academy, Dalian, Liaoning, 116018, China

[*] Corresponding author: dl_yuxj@163.com

Abstract. Freak wave is a kind of instantaneous disastrous wave with large wave height, which has great destructive effect on the navigation of ships at sea. Based on the layered Boussinesq equation, a wave model for focusing simulation of freak wave is established, and the numerical generation of freak wave is realized. Through the calculation results, the evolution process and the non-linear characteristic parameters of the freak wave are analyzed and discussed. The following conclusions are drawn: The influence of different energy distribution modes on the simulation of the freak wave is analyzed. Focusing model of simulated freak wave based on layered Boussinesq equation can reproduce freak wave events on random wave field when energy input of freak wave train reaches 10%. The simulation efficiency is high, the influence on skewness is small, and the influence on kurtosis is large.

1. Introduction

As a special kind of disastrous wave, freak wave, because of its sudden occurrence and harmfulness to the safety of naval vessel activities, urgently needs us to improve our understanding of this threatening wave in order to protect the environment for human survival, predict the occurrence of such natural disasters and reduce the losses caused by it. Therefore, freak wave has become a hot topic in wave theory and application.

At present, the research on the mechanism of freak wave formation is mostly carried out through the angle of energy focus. Kharif and Pelinovsky[1] summarized the generating mechanism of freak waves, believing that the generation of freak waves may be caused by one or more of the following factors: wave superposition, wave-current interaction, topographic change, wind action, Benjamin-Feir instability and so on.

In order to studying the mechanism and influencing factors of freak wave, it is an effective way to reproduce the freak wave events in the laboratory. It is the most effective way to study freak wave in laboratory by focusing wave energy. Among them, the most commonly used method is the phase velocity method. According to the linear wave theory, the waves of different periods and amplitudes are combined in the form of linear superposition, and the initial phase of each component wave is artificially modulated to reach the maximum peak at the given position and time, so that the linear superposition of each component wave can produce large waves.

In order to overcome the disadvantage that the wavefront in the focusing position is still before and after the wave is focused, and the probability of extreme wave is very low, Kriebel[2] and others divide the energy spectrum into two parts: background spectrum and singular spectrum. The background spectrum is used to generate random wave field, simulate the real sea surface, and use singular spectrum. The results show that only 15% or 20% of the total energy can be used to generate the abnormal wave, that is, only a small part of the wave component can generate the abnormal wave. Huang[3] used a model to obtain the time series of wave surface containing abnormal waves by manual intervention of the random initial phase of the composed waves, but the simulation efficiency was low. Pei[4] developed the two-wave train combination model to three-wave train superposition, which improved the efficiency of abnormal wave simulation. Based on this method, the measured freak wave was reconstructed, and the feasibility of this simulation method was demonstrated. Liu[5] simulated the generation of freak wave in three-dimensional wave field based on the above combined focusing model. Zhao[6] carried out an experimental study of abnormal waves in a two-dimensional wave flume. The effects of dimensionless water depth, wave steepness, spectral peak elevation factor and spectral peak period on the statistical characteristics of random wave and the appearance of abnormal waves were studied. Based on the high-order spectral method, a numerical model for fast simulation of wave non-linear motion was established, and the spectral model of abnormal waves was simulated by focusing. The wave focusing at experimental scale and the simulation of abnormal waves in open sea area were carried out.

At present, the main mathematical models for wave problems are the mild slope equation ,the Boussinesq equation and the non-linear wave model based on potential flow theory by solving Laplace equation. In this paper, a numerical Boussinesq equation model for simulating deformed waves is established by using the Boussinesq equation mathematical model and the dispersion of waves, and the numerical simulation of the generation of deformed waves is realized.

2. Control Equation and Boundary Conditions

In this paper, the layered Boussinesq equation proposed by Lynett and Philip Liu[7-8] is used to study the numerical generation of freak wave. It is used to stratify the water cylinder again on the basis of the traditional Boussinesq equation. At the same time, it is assumed that the vertical velocity in each layer is linearly distributed, and the vertical velocity is integrated in each layer separately to calculate its average velocity, which is used to replace the vertical velocity in each layer. This method not only improves the accuracy of the Boussinesq equation model, but also improves the dispersion of the model, so that the model equation can be applied to larger water depth.

The theoretical system of the model is shown in Figure 1. The water column is decomposed into multi-layers (N layers). A depth integral model is used to replace the higher order polynomial approximation of the vertical flow field. The multiple quadratic polynomials are used to match the user-defined interface. Since there is no higher order space derivative and polynomial approximation, this method can produce an accurate model. A series of model equations can be obtained by piecewise integration of the original equation of motion with arbitrary N layers. In this way, the velocity profile of the whole water column is decomposed into N layers, and a velocity profile is determined independently in each layer, together with the interface between layers, so a series of solutions of the equation are composed of (2N-1) free parameters, which can optimize the known analytical solutions of water waves. By integrating the original equation of motion, a series of equations are formed, and the integration will be carried out in segments.

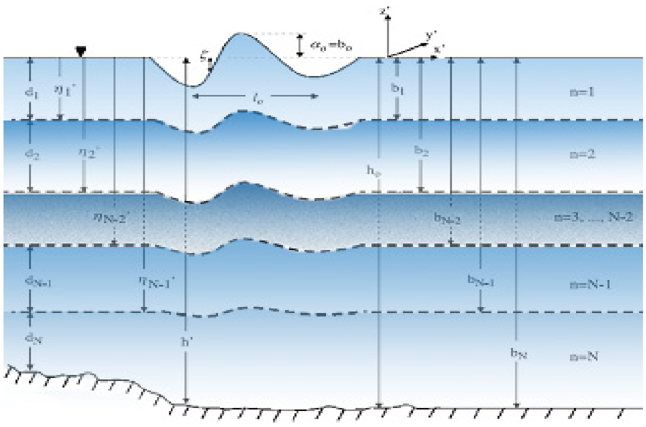


Figure 1. N-Layer Model of layered Boussinesq equation

Through a series of deductions, the following three governing equations can be obtained:

$$\begin{aligned}&\frac{1}{\varepsilon_0}\frac{\partial h}{\partial t}+\frac{\partial \zeta}{\partial t}+\nabla\cdot\sum_{n=1}^{N}\left(\frac{b_{n-1}}{h_0}\eta_{n-1}-\frac{b_n}{h_0}\eta_n\right)u_n-\\&\nabla\cdot\sum_{n=1}^{N}\mu_n^2\{[\frac{(\frac{b_{n-1}}{d_n}\eta_{n-1})^3-(\frac{b_n}{d_n}\eta_n)^3}{6}-(\frac{b_{n-1}}{d_n}\eta_{n-1}-\frac{b_n}{d_n}\eta_n)A_n]\nabla S_n\\&+[\frac{(\frac{b_{n-1}}{d_n}\eta_{n-1})^2-(\frac{b_n}{d_n}\eta_n)^2}{2}-(\frac{b_{n-1}}{d_n}\eta_{n-1}-\frac{b_n}{d_n}\eta_n)B_n]\nabla S_n\}=O(\mu_n^4)\end{aligned} \tag{1}$$

$$\begin{aligned}&\frac{\partial u_1}{\partial t}+\frac{\varepsilon_0}{2}\nabla(u_1\cdot u_1)+\nabla\zeta+\mu_1^2\frac{\partial}{\partial t}\{A_1\nabla S_1+B_1\nabla T_1]+\\&\varepsilon_0\mu_1^2\nabla(B_1u_1\cdot\nabla T_1+A_1u_1\cdot\nabla S_1)+\\&\varepsilon_0^2\mu_0^2\nabla(\zeta S_1T_1-\frac{h_0}{d_1}\frac{\zeta^2}{2}\frac{\partial S_1}{\partial t}-\zeta u_1\cdot\nabla T_1)+\\&\varepsilon_0^2\varepsilon_1\mu_0^2\nabla[\frac{\zeta^2}{2}(S_1^2-\frac{h_0}{d_1}u_1\cdot\nabla S_1)]=O(\mu_0^2\mu_1^2)\end{aligned} \tag{2}$$

$$\begin{aligned}&u_n+\mu_n^2\{[A_n-\frac{(\frac{b_{n-1}}{d_n}\eta_{n-1})^2}{2}]\nabla S_n+(B_n-\frac{b_{n-1}}{d_n}\eta_{n-1})\nabla T_n\}\\&=u_{n-1}+\mu_{n-1}^2\{[A_{n-1}-\frac{(\frac{b_{n-1}}{d_{n-1}}\eta_{n-1})^2}{2}]\nabla S_{n-1}\\&+(B_{n-1}-\frac{b_{n-1}}{d_{n-1}}\eta_{n-1})\nabla T_{n-1}\}+O(\mu_{n-1}^4,\mu_n^4)\end{aligned} \tag{3}$$

3. Theoretical Model of Wave Focusing

According to Longuet-Higgins[9] wave model theory, the wave surface at any point can be expressed as a linear superposition of waves with different frequencies and initial phases.

$$\eta(x,t) = \sum_{n=1}^{n=N} a_n \cos(k_n x - \omega_n t + \varepsilon_n) \qquad (4)$$

a_n is wave amplitude, k_n is wave number, ω_n is circular frequency, ε_n is random phase.

By improving the above-mentioned wave model, the extreme wave focusing model is obtained by selecting the initial phase of the wave and superimposing the wave at a specific time and position.

$$\eta(x,t) = \sum_{n=1}^{n=N} a_n \cos(k_n (x - x_c) - \omega_n (t - t_c)) \quad (5)$$

By using the extreme wave focusing model, the waves satisfying the definition requirements can be obtained in a numerical flume. However, in the extreme wave focusing model, the energy is concentrated in the focusing region, and the wave surface in the non-focusing region is almost zero. This is quite different from the actual wave surface, and the probability of this phenomenon is very low. When the wave meets the minimum criterion, the probability of freak wave occurrence is 1.0×10^{-3} at this time. Even if the freak wave is found in random wave, it still needs a long enough simulation length, which is difficult to achieve under experimental conditions, and the parameters of wave abnormality are difficult to control.

In order to simulate freak wave in Limited space-time domain, Kriebel[9] gives the combination of extreme wave focusing model and random wave model, which improves the efficiency of freak wave simulation; Pei[10] analyses the influence of energy distribution in the above combined model on the formation of freak waves; Liu[11] studies the abnormal wave events in three-dimensional wave field based on the combined model. The above research shows that the combination of limit wave model and random wave model can simulate and generate controllable abnormal waves in limited time and space. In this paper, the theoretical model of wave focusing is used to study the characteristics of abnormal waves generated by Boussinesq equation.

In the study of freak wave characteristics, the commonly used statistical features are:

(1) Maximum wave height $H_{\max}$, Significant Wave Height H_s and Wave deformity parameter $H_{\max} / H_s$

Maximum wave height is the maximum of wave height in continuous wave recording, and it is an important reference for engineering design. Significant Wave Height $H_s = 4\sqrt{m_0}$, m_0 is variance of wave surface elevation. $H_{\max} / H_s$ is often used as a parameter to indicate the degree of wave anomalies, is also a representation of the probability of wave occurrence. In linear wave theory, $H_{\max} / H_s > 2.0$ is often used as a criterion for determining freak wave events. In non-linear wave theory, $H_{\max} / H_s > 2.2$ is often used as a criterion for determining freak wave events. Because the simulation time is limited, $H_{\max} / H_s > 2.0$ adopts as a criterion for determining freak waves.

(2) skewness μ_3 and kurtosis μ_4

Assuming that the wave is a normal stochastic process, the wave surface elevation can be regarded as in accordance with the Gauss distribution. However, due to the influence of nonlinearity, there are some deviations between the wave surface elevation and the Gauss distribution. The deviation can be measured by skewness and kurtosis.

Skewness is a statistic describing the symmetry of the horizontal distribution of waves. Kurtosis is a statistic describing the steepness of the distribution of all the values of waves. It is compared with the normal distribution. Their definitions are as follows:

$$\mu_3 = \frac{1}{N}\sum_{n=1}^{N}\frac{(\eta_n - \overline{\eta})^3}{\eta_{rms}^3} \qquad (6)$$

$$\mu_4 = \frac{1}{N}\sum_{n=1}^{N}\frac{(\eta_n - \overline{\eta})^4}{\eta_{rms}^4} \qquad (7)$$

N is the number of measuring points, η_n is wave surface fluctuation at the nth measuring point, η_{rms} is RMS value of wave surface elevation at measuring point. For the Gauss distribution, $\mu_3 = 0$ and $\mu_4 = 3$.

4. Combination-Focusing Model of Freak Wave

In order to simulate the freak waves in the real ocean, the focus model and the random wave model are superimposed by different energy ratios. Setting a spectrum, one part of the spectrum energy enters the random wave train and the other part enters the focused wave train. E_R is energy ratio entering random wave train, E_P is energy ratio entering the focus model, $E_R + E_P = 1$.

Therefore, the free surface can be expressed as:

$$\begin{aligned}\eta(x,t) = &\sum_{n=1}^{n=N} A_{Rn}\cos(k_n x - \omega_n t + \varepsilon_n) + \\ &\sum_{n=1}^{n=N} A_{Pn}\cos(k_n(x - x_c) - \omega_n(t - t_c))\end{aligned} \qquad (8)$$

According to the set energy ratio, the amplitudes of random and focused waves are calculated from the wave spectrum.

$$A_{Rn} = \sqrt{2E_R S(\omega_n)\Delta\omega} \qquad (9)$$

$$A_{Pn} = \sqrt{2E_P S(\omega_n)\Delta\omega} \qquad (10)$$

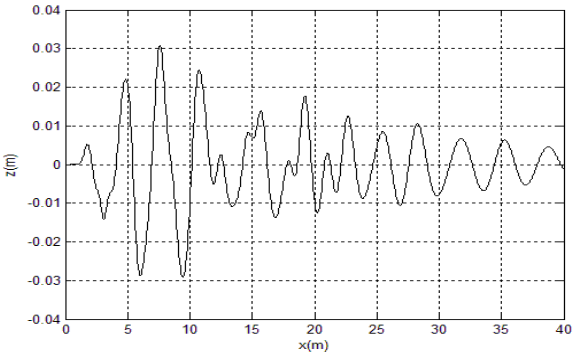
The target spectrum is JONSWAP spectrum.

$$S(\omega) = \beta_J H_{1/3}^2 T_P^{-4}\exp\left[-1.25(T_P f)^{-4}\right]\gamma^{\exp\left[-\frac{(\omega - f_P)^2}{2\sigma^2 f_P^2}\right]} \qquad (11)$$

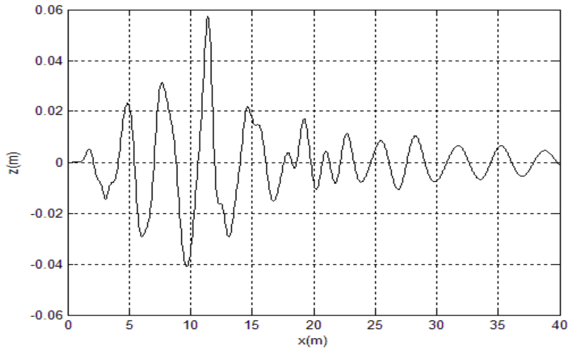
Among them, $\beta_J = \dfrac{0.06238(1.094 - 0.01915\ln\gamma)}{0.230 + 0.0336\gamma - 0.185(1.9+\gamma)^{-1}}$, f_P is peak spectral frequency, γ is peak rise factor, the value is 3.3, σ is peak shape parameter, the value :

$$\sigma = \begin{cases} 0.07, & \omega \le \omega_0 \\ 0.09, & \omega > \omega_0 \end{cases}$$

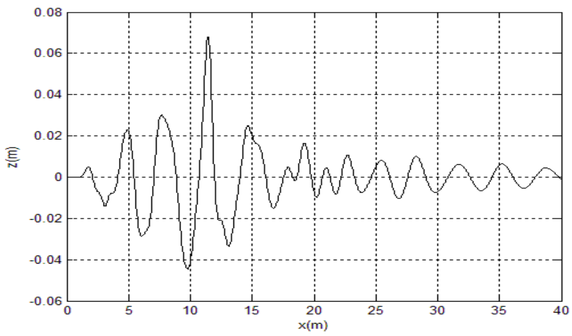
The numerical simulation conditions are as follows: frequency range f=[0.50Hz， 1.16Hz], H_s is 0.04m, peak spectral frequency is 0.625Hz, the water depth h=0.5m, the focus time in Test t=24s, wave focus at x=8m in front of Wavemaker. The sampling frequency is 50Hz, the sampling time is 60s.



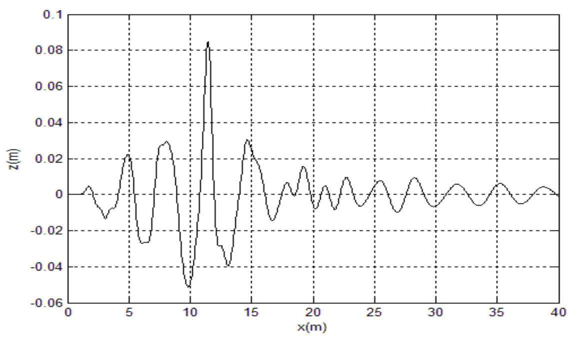
(a) Ep=0



(b) Ep =0.1



(c) Ep =0.2

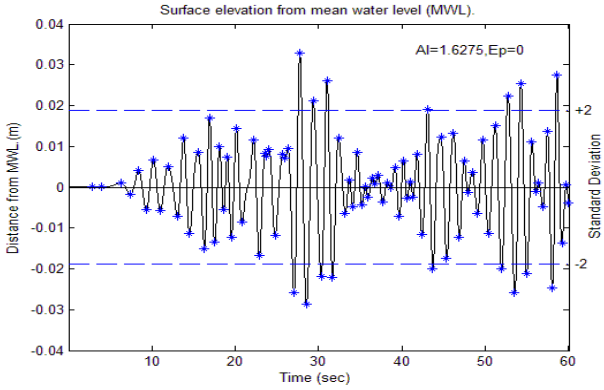


(d) Ep =0.3

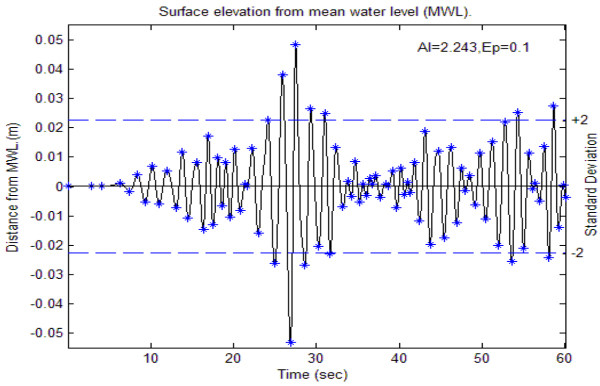
Figure 2. Wave surface fluctuation at wave focusing time

IOP Conf. Series: Earth and Environmental Science **310** (2019) 022031 doi:10.1088/1755-1315/310/2/022031

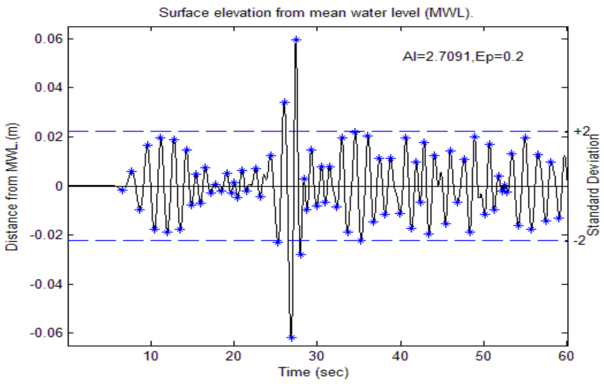
Figure 2 shows the wave surface fluctuation at the wave focusing time under different energy ratios. When Ep=0, the input focused wave train has zero energy and no freak wave appears. When Ep=0.1, freak wave occurs, focusing at 11.5 M. Because of the non-linear effect, they lag behind the set focusing position at 8 m, which accords with the Baldock (1996) experimental results. It shows that the non-linear effect of waves plays an important role in the development of wave focusing. When Ep = 0.2 and Ep = 0.3, the higher the wave height is, the more obvious the abnormal wave phenomenon is. Therefore, the model can be used to generate abnormal wave simulation, and abnormal wave satisfying different conditions can be generated through energy distribution.



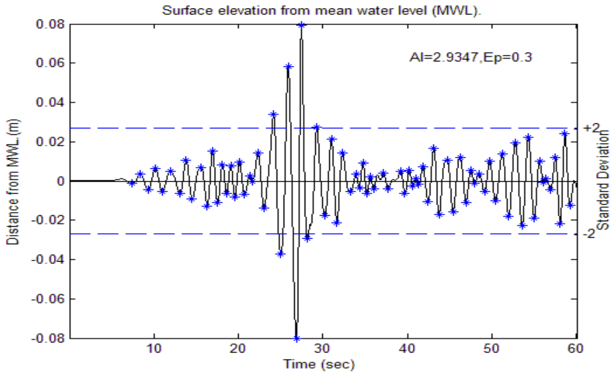
(a) Ep=0



(b) Ep =0.1



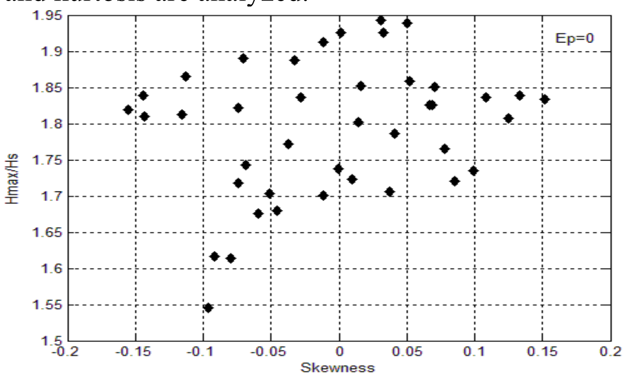
(c) Ep =0.2



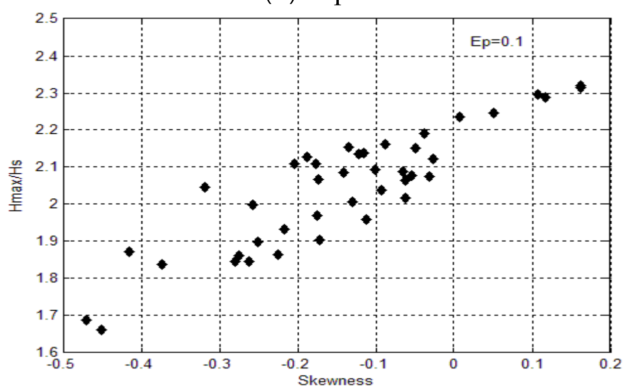
(d) Ep =0.3

Figure 3. Time series of focused wave

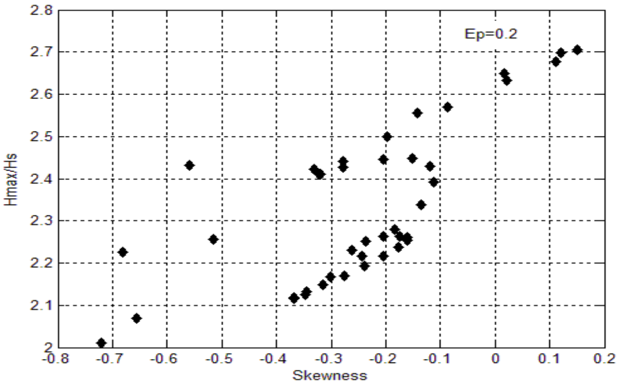
Fig. 3 shows the time series of focused waves with different energy ratios. AI represents wave deformity parameters. At Ep=0, i.e., the input focusing wave train energy is zero, AI=1.6275, there is no abnormal wave; at Ep=0.1, AI=2.243, i.e., there is abnormal wave, the focusing time is about 26 seconds. Because of the non-linear effect, it lags behind the set focusing time 24 seconds, which is in line with the results of Baldock(1996). It shows that the non-linear effect of wave delays the focusing time. At Ep = 0.2, AI = 2.7091; at Ep = 0.3, AI = 2.9347. With the increase of input focused wave energy, the higher the wave height, the more obvious the freak wave phenomenon. In the numerical simulation process, a wave train is output every 0.25m at 10-20m, and the effects of different energy ratios on wave skewness and kurtosis are analyzed.



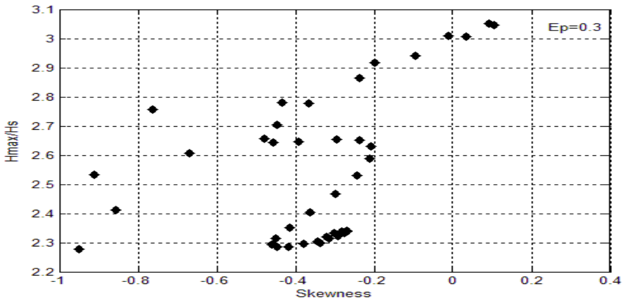
(a) Ep=0



(b) Ep =0.1



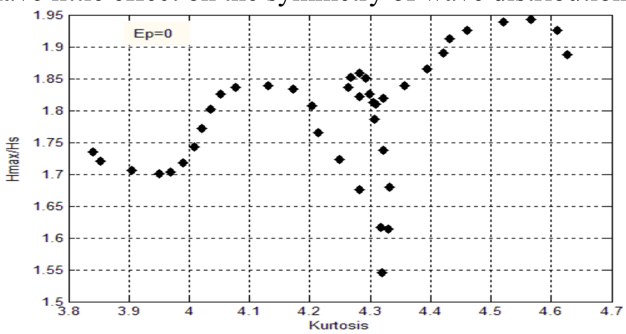
(c) Ep =0.2



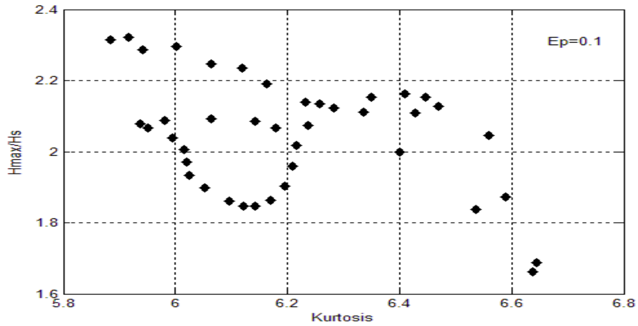
(d) Ep =0.3

Figure 4. Relationship between Hmax/Hs and skewness

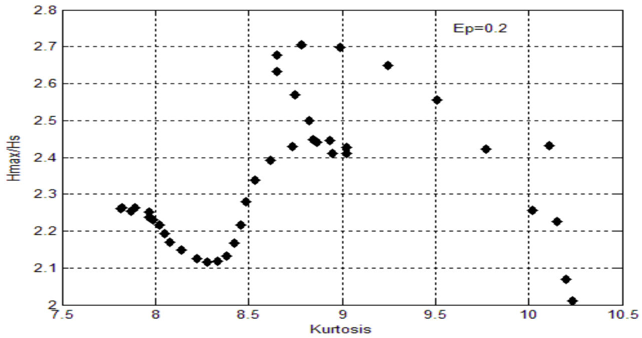
Fig. 4 shows the relationship between wave deformity parameter Hmax/Hs and skewness at different energy ratios. When Ep = 0, the input focused wave train energy is zero and the skewness value is uniformly distributed; when Ep = 0.1, the skewness value is between - 0.1 and - 0.3. With the increase of Ep value, the parameters of wave deformity increase, but the skewness value varies very little and still concentrates between - 0.1 and - 0.3, indicating that different energy ratios have little effect on Skewness and have little effect on the symmetry of wave distribution.



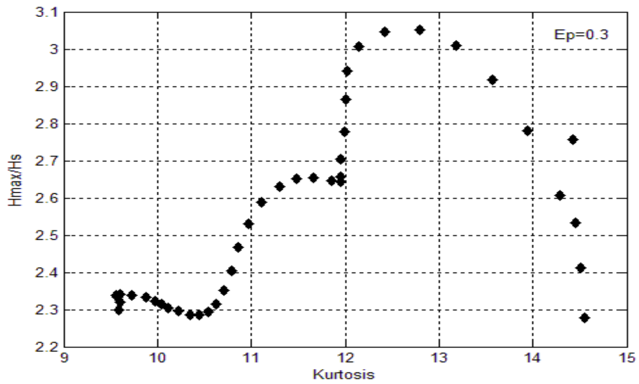
(a) Ep=0



(b) Ep =0.1



(c) Ep =0.2



(d) Ep =0.3

Figure 5. Relationship between Hmax/Hs and kurtosis

Fig. 5 shows the relationship between Hmax/Hs and kurtosis at different energy ratios. At Ep=0, that is, the input focused wave train energy is zero and the skewness value is near 4, which satisfies the Gauss distribution. With the increase of Ep value, the wave deformity parameter increases and the peak value changes. This shows that different energy ratios have a greater impact on the skewness, and the larger the energy of the deformed wave train, the steeper the wave distribution pattern.

5. Conclusion

Based on the layered Boussinesq equation, a wave model is established to simulate deformed waves, and the numerical generation of deformed waves is realized. Through the calculation results, the evolution process and the non-linear characteristic parameters of the abnormal wave are analyzed and discussed, and the following conclusions are drawn:

The effects of different energy distribution modes on the simulation of abnormal waves are analyzed. Focusing model of simulated abnormal wave based on layered Boussinesq equation can reproduce abnormal wave events on random wave surface when the energy input of abnormal wave train reaches 10%. The simulation efficiency is high, the influence on skewness is small, and the influence on kurtosis is large.

The influence of phase modulation on the simulation of abnormal waves is analyzed. Focusing model of simulated deformity wave based on layered Boussinesq equation can generate deformity wave when the phase angle distribution is smaller than that. The change of phase angle distribution has less influence on Skewness and more influence on kurtosis.

References

[1] Kharif, C. and Pelinovsky, E.: Physical mechanisms of the rogue wave phenomenon[J], Europ. J. Mech. B/Fluids.2003,22:603–634.
[2] Kriebel D.L. Effieient simulation of extreme waves in a random sea[J].Rogue waves 2000 Workshops, Brest, 2000:1-2.
[3] Jing-gang Huang. Simulation of Two-Dimensional Focused extreme Wave[D].Dalian:Dalian University of Technology,2005.
[4] Yu-guo Pei. Generation and Basic Characteristics of Abnormal Waves [D]. Dalian:Dalian University of Technology,2007.
[5] Xiao-xia Liu. Numerical simulation of freak waves in three-dimensional wave field [D]. Dalian:Dalian University of Technology,2008.
[6] Xi-zeng Zhao. Experimental Study and Numerical Simulation of Freak waves [D] . Dalian:Dalian University of Technology,2008.
[7] Lynett, P., Liu, P.L.-F., A two-dimensional, depth-integrated model for internal wave propagation[J]. Wave Motion in press. 2002, 36:221-240.
[8] Patrick Lynett. A multi-layer approach to modeling generation propagation and interaction of water waves [D]. USA: CornellUniversity, 2002.
[9] Longuest-Higgins, M. S. and R. W. Stewart. The changes in amplitude of short gravity waves on steady non-uniform current [J] .J.Fluid Mech., 1961,10:529-549