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Generalized Spatial Three-Stage Least Square (GS3SLS) for Unemployment Rate and Economic Growth Modelling in East Java

Agus Muslim^{1,2}, Anwar Fitrianto¹, I Made Sumertajaya¹ and Anik Djuraidah^{1*}

¹Department of Statistics, IPB University, Bogor 16680, Indonesia

²Statistics of Kepulauan Riau Province, Tanjungpinang, Indonesia

*E-mail: anikdjuraidah@gmail.com

Abstract. Spatial models have attracted considerable interest in recent economics both on an empirical and theoretical level. One of the most widely used spatial models is the single equation model. Spatial simultaneous equation model as an extension of the widely used spatial single equation models can use more than one equation. Generalized Spatial Three-Stage Least Square (GS3SLS) estimator is one of the spatial simultaneous models. This paper presents the assessment of spatial simultaneous equations with GS3SLS method including Queen, Rock and Customized Contiguity. GS3SLS with customized contiguity (distance inverse) can share the best model of other continuity. It can be concluded that there is a relationship between the unemployment rate and economic growth in East Java. Labour Force Participation Rate (LFPR) and economic growth significantly affect Unemployment Rate and Human Development Index (HDI) and unemployment rates significantly affect economic growth. By using the customized contiguity, both the unemployment rate and economic growth give simultaneous spatial effect.

1. Introduction

In the practice of regional sciences, by drawing on a wealth of theoretical formulations about human, spatial behavior can deal with issues faced by municipalities and regions. When the data presented use thematic maps, it appears that the data from adjacent areas almost contain identical values. Regional science is dependent on the premise that location and distance are important works in human geography and market activities [19].

The collection of techniques that deals with the uniqueness caused by space in the statistical analysis of regional science models is considered to be the domain of spatial econometrics. Spatial econometrics is the way in which spatial effects are taken into account [7]. Spatial econometrics is a subfield of applied econometrics concerned on complications in estimation and testing that may result from the presence of spatial dependence or spatial autocorrelation and spatial heterogeneity [22][8]. Spatial models have attracted considerable interest in the recent economics and econometrics literature, on both an empirical and theoretical level.

Cliff and Ord (1973, 1981) introduced the single equation model as one of the most widely used on spatial models. This model is any of variant models introduced by Whittle (1954) [10][11]. In many situations, such a one way or unidirectional cause and effect relationship is not meaningful. This occurs if Y is determined by the X's, and some of the X's are, in turn, determined by Y. In short, there is a two way or simultaneous, the relationship between Y and (some of) the X's, which makes the



distinction between dependent and explanatory variables of dubious value. It is better to lump together a set of variables that can be determined simultaneously by the remaining set of variables precisely what is done in simultaneous-equation models. In such models, there is more than one equation and unlike the single equation models, simultaneous equation models one may not estimate the parameters of a single equation without taking into account information provided by other equations in the system [13].

The spatial model presented a spatial weight matrix in this expression results in a quadratic form in the error terms. Consequently, Ordinary Least Square (OLS) estimator will be biased as well as inconsistent for the parameters of the spatial model [7] and assumptions of the method of OLS are the explanatory X variables either non-stochastic or, if stochastic (random) distributed independently of the stochastic disturbance term. If neither of these conditions is met, then, as shown later, the least-squares estimators are not only biased but also inconsistent [13].

Kelejian and Pruscha (2004) extended the Clift and Ord single equation model by using the estimator of a simultaneous system of cross-sectional equations with spatial dependencies and introduced two parameter estimation methods for the simultaneous spatial equation. Those are Generalized Spatial Two-Stage Least Square (GS2SLS) and Generalized Spatial Three-Stage Least Square (GS3SLS). GS3SLS is extended from GS2SLS, where the first step employs GS2SLS and the second step employs Seemingly Unrelated Regression (SUR). The GS3SLS method is the full assessment of information that considers the residual correlation among equations and can solve the problem of OLS estimator when it is employed in simultaneous spatial models [18].

Ansellin (1980, 1984a) argued that the structure of spatial dependence incorporated in the spatial weight matrix should be judiciously opted, and related to general concepts from spatial interaction theory, such as the notions of accessibility and potential. In line with a model-driven approach to spatial econometrics, the weight matrix should bear a direct relation to a theoretical conceptualization of dependence structure, and there are many weighted matrixes that can be used in a spatial model such as rook contiguity, queen contiguity matrix and customized contiguity matrix [2][3].

East Java contains the largest number of municipalities in Indonesia, approximately 38 municipalities. In 2015 the economic growth rate is 5.44 percent, showing slower growth than 2014 which was recorded at 5.86 percent. East Java is a province that experiences the slowest growth of the largest economy in Java. The pattern of economic growth in East Java province, when viewed in terms of territorial, indicates that advanced regions are still lagging behind. Gaps highest and lowest economic growth in East Java province is considerable, where the highest economic growth rate in Batu amounted to 6.69 percent and the lowest in Lumajang amounted to 4.62 percent. And the unemployment rate has increased from 4.19 percent in 2014 to 4.47 percent in 2015 and the unemployment rate extends the highest gap among municipalities (Statistics Indonesia, 2015).

This paper presents the assessment spatial of simultaneous equations with GS3SLS method using Queen, Rock and Customized Contiguity.

2. Data

Okun's law describes one of the most famous empirical relationships in macroeconomics. Proposed by economist Arthur Okun in 1962, it basically states that if GDP affects the unemployment rate declines rapidly and the growth is very low or negative, the unemployment rate will rise, and if growth potentially equals, the unemployment rate will remain unchanged. Factors affecting unemployment such as labor force participation rate (channel transmission) [14], then unemployment is also caused by poverty warning [1] and also amount and rate of population growth [12] [20]. There is a correlation among the number of labor wages to economic growth in a region [21] and human development cycle and economic growth, there are some correlated between economic growth and human development index [23].

Table 1. Selection variables.

Variables	Names of Variabel	Information
Endogen Variables		
Y_1	Unemployment Rate	Percentage of the number of unemployed to the labor force
Y_2	Economic Growth	Economic Growth Rates
Exogen Variables		
X_1	Labor Force Participation Rate (LFPR)	Percentage of total working population and unemployment (labor force) to Total Population age 15 and above (working age)
X_2	Percentage of poverty	percentage of population below the Poverty Line
X_3	Poverty Gaps Index	Poverty gap index
X_4	Population Growth Rate	Population growth rate 2010-2015
X_5	Labor wage	Wages received in the form of money or goods by workers who have the status of workers/employees
X_6	Human Development Index (HDI)	Human development index (HDI)

Source: Statistics of Indonesia

3. Method

3.1. Spatial Simultaneous Equation

An extension of widely used spatial single equation models can be regarded as the spatial simultaneous equation model [10]. In particular, the following system of spatially interrelated cross-sectional equations corresponds with n cross-sectional units:

$$\mathbf{Y}_n = \mathbf{Y}_n \mathbf{B} + \mathbf{X}_n \mathbf{C} + \mathbf{W}_n \mathbf{Y}_n \mathbf{A} + \mathbf{U}_n \quad (1)$$

With:

$$\mathbf{Y}_n = (\mathbf{y}_{1,n}, \dots, \mathbf{y}_{G,n}), \mathbf{X}_n = (\mathbf{x}_{1,n}, \dots, \mathbf{x}_{K,n}), \mathbf{U}_n = (\mathbf{u}_{1,n}, \dots, \mathbf{u}_{G,n}), \mathbf{U}_n = \bar{\mathbf{U}}_n \mathbf{R} + \mathbf{E}_n, \\ \mathbf{E}_n = (\varepsilon_{1,n}, \dots, \varepsilon_{G,n}), \mathbf{R} = \text{diag}_{j=1}^G(\rho_j)$$

Where $\mathbf{y}_{j,n}$ is the $n \times 1$ vector of cross-sectional observations on the dependent variable in the j th equation, $\mathbf{x}_{k,n}$ is the $n \times 1$ vector of cross-sectional observations on the k th exogenous variable, $\mathbf{u}_{j,n}$ is the $n \times 1$ disturbance vector in the j th equation, $\mathbf{W}_n$ is an $n \times n$ weights matrix of known constants, and $\mathbf{B}, \mathbf{C}$, and $\mathbf{A}$ are correspondingly defined as parameter matrices of dimension $G \times G$, $K \times G$ and $G \times G$. $\mathbf{W}_n \mathbf{Y}_n$ is the spatial lag for $\mathbf{Y}_n$ and $\mathbf{W}_n \mathbf{y}_{j,n}$ shows spatial lag $\mathbf{y}_{j,n}$. The i th elemen of $\mathbf{w}_n \mathbf{y}_{j,n}$ is given by :

$$\bar{y}_{ij,n} = \sum_{r=1}^n w_{ir,n} y_{rj,n} \text{ with } w_{ir} = \begin{cases} 1, & \text{for } i \text{ and } r \text{ are said to be neighbour} \\ 0, & \text{for any other} \end{cases}$$

The equation above shares three interpretations. First, showing how the relationship among variables in $\mathbf{Y}$ with its own self (spatial own lag), related to the value of other endogenous variables in the surrounding locations (cross spatial lag), related to the value of other endogenous variables in each location. Second, detecting the relationship between the independent variables $\mathbf{Y}$ and the exogenous variables $\mathbf{X}$ after all aspects of the spatial and endogenous variables are controlled. Third, the model can be interpreted as a reduced form. The reduced form is nonlinear and shows the expected value of the dependent variables at each location depends on not only the exogenous variables at each location but also on exogenous variables at all other locations. Using the sixth equation to get an additional solid form by removing the restriction on the model parameter will still contain matrix observation

endogenous variables, exogenous variables and spatial lag of the endogenous variables referring to j th equation y_j, x_j , and WY_j . So we will find the equation:

$$\begin{aligned} y_{j,n} &= Z_{j,n}\delta_j + u_{j,n} \\ u_{j,n} &= \rho_j W_n u_{j,n} + \varepsilon_{j,n} \end{aligned} \quad (2)$$

With $Z_{j,n} = (Y_{j,n}, X_{j,n}, W_n Y_{j,n})$; $\delta_j = (\beta'_j, \gamma'_j, \lambda'_j)'$. $j = 1, \dots, G$

3.2. Limited Information Estimation: GS2SLS

The proposed GS2SLS estimation procedure consists of three steps. First, two-step least squares (2SLS) using H_n as the instrument matrix to estimate the model parameter vector, δ_j . The first stage assessment will be obtained for disturbance $u_{j,n}$. disturbance estimator. Second, using the estimated disturbance $u_{j,n}$. to conduct parameter estimation ρ_j uses a generalized procedure moment. Third, the estimate for ρ_j is used to account for the spatial autocorrelation in the disturbance $u_{j,n}$. using a Cochran-Orcutt transformation. The GS2SLS estimator for δ_j is obtained by estimating the transformed model by 2SLS using H_n as the instrument matrix.

First Stage: Two Stage Least Square Estimation

The estimation process is to do two-step least squares (2SLS). The simultaneous spatial equation will be conducted to obtain $\hat{\delta}_j$. The first step of 2SLS is to get the value $\tilde{Z}_{j,n}\tilde{Z}_j$ using instrument variable matrix P_H , Values $\tilde{Z}_{j,n}\tilde{Z}_j$ is obtained by using the following formula:

$$\tilde{u}_{j,n} = y_{j,n} - Z_{j,n}\tilde{\delta}_{j,n} \quad (3)$$

Second Stage: Spatial Autoregressive Parameter Estimation

$$\tilde{Z}_{j,n} = P_H Z_{j,n}, \text{ with } P_H = H_n (H_n' H_n)^{-1} H_n'$$

The next step is to use $\tilde{Z}_{j,n}$ results for parameter estimation. The 2SLS estimator of $\hat{\delta}_j$ is then given by:

$$\tilde{\delta}_{j,n} = (\tilde{Z}_{j,n}' Z_{j,n})^{-1} \tilde{Z}_{j,n}' y_{j,n} \quad (4)$$

The second stage estimation process for spatial simultaneous equation uses generalized moments procedure to estimate the spatial autoregressive parameter of the disturbance process of each equation [16]. Steps carried out are to take the second equation which then obtains the equation.

$$u_{j,n} - \rho_j W_n u_{j,n} = \varepsilon_{j,n}, \quad j = 1, \dots, m \quad (5)$$

$$W_n u_{j,n} - \rho_j W_n^2 u_{j,n} = W_n \varepsilon_{j,n} \quad (6)$$

The following three equation systems are obtained from the relationship among equations (5) and equations (6)

$$\frac{E[u_{j,n}' W_n u_{j,n}]}{n} = \rho_j \frac{E[u_{j,n}' W_n^2 u_{j,n} + (W_n u_{j,n})' W_n u_{j,n}]}{n} - \rho_j^2 \frac{E[(W_n u_{j,n})' W_n^2 u_{j,n}]}{n} + \frac{E[\varepsilon_{j,n}' W_n \varepsilon_{j,n}]}{n}$$

Expected value of $\frac{1}{n} E[\varepsilon_{j,n}' \varepsilon_{j,n}]$, $\frac{1}{n} E[(W_n \varepsilon_{j,n})' W_n \varepsilon_{j,n}]$ and $\frac{1}{n} E[\varepsilon_{j,n}' W_n \varepsilon_{j,n}]$ are ,

$$\begin{aligned} \frac{1}{n} E[\varepsilon_{j,n}' \varepsilon_{j,n}] &= \frac{1}{n} E \left[\sum_{i=1}^n \varepsilon_{ij,n}^2 \right] = \frac{1}{n} E \sum_{i=1}^n [\varepsilon_{ij,n}^2] = \frac{1}{n} \sum_{i=1}^n E[\varepsilon_{jn}^2] = \frac{1}{n} \sum_{i=1}^n \sigma_j^2 = \frac{1}{n} n \sigma_j^2 = \sigma_j^2 \\ \frac{1}{n} E[(W_n \varepsilon_{j,n})' W_n \varepsilon_{j,n}] &= \frac{1}{n} Tr(W_n' W_n) \sigma_j^2 = \frac{1}{n} \sigma_j^2 Tr(W_n' W_n) \\ \frac{1}{n} E[\varepsilon_{j,n}' W_n \varepsilon_{j,n}] &= \frac{1}{n} Tr(W_n) \sigma_j^2 = \frac{1}{n} \sigma_j^2 Tr(W_n) = \frac{1}{n} \sigma_j^2 (0) = 0 \end{aligned}$$

In the form of a matrix, it can be written as

$$\begin{bmatrix} \frac{E[\mathbf{u}'_{j,n} \mathbf{u}_{j,n}]}{n} \\ \frac{E[(\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}]}{n} \\ \frac{E[\mathbf{u}'_{j,n} \mathbf{W}_n \mathbf{u}_{j,n}]}{n} \end{bmatrix} = \begin{bmatrix} \frac{2 \frac{E[\mathbf{u}'_{j,n} \mathbf{W}_n \mathbf{u}_{j,n}]}{n}}{n} & -\frac{E[(\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}]}{n} & 1 \\ \frac{2 \frac{E[(\mathbf{W}_n^2 \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}]}{n}}{n} & -\frac{E[(\mathbf{W}_n^2 \mathbf{u}_{j,n})' \mathbf{W}_n^2 \mathbf{u}_{j,n}]}{n} & \frac{1}{n} \text{Tr}(\mathbf{W}_n' \mathbf{W}_n) \\ \frac{E[\mathbf{u}'_{j,n} \mathbf{W}_n^2 \mathbf{u}_{j,n} + (\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}]}{n} & -\frac{E[(\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n^2 \mathbf{u}_{j,n}]}{n} & 0 \end{bmatrix} \begin{bmatrix} \rho_j \\ \rho_j^2 \\ \sigma_j^2 \end{bmatrix} \quad (7)$$

Form of Equation (7) together with the form of the moment formula $\gamma_{j,n} = \mathbf{\Gamma}_{j,n} \alpha_j$. With α_j is a parameter that contains $[\lambda_j \quad \lambda_j^2 \quad \sigma_j^2]$. Therefore, $\mathbf{\Gamma}_{j,n} \gamma_{j,n}$ is already considerable, so that the parameter values can be counted by $\alpha_j = \mathbf{\Gamma}_{j,n}^{-1} \gamma_{j,n}$. Suppose estimator for $\mathbf{\Gamma}_{j,n} \gamma_{j,n}$ is $\mathbf{G}_{j,n}$ and $\mathbf{g}_{j,n}$. Then the moment estimator are as follows

$$\mathbf{G}_{j,n} = \begin{bmatrix} \frac{2 \frac{\mathbf{u}'_{j,n} \mathbf{W}_n \mathbf{u}_{j,n}}{n}}{n} & -\frac{(\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}}{n} & 1 \\ \frac{2 \frac{(\mathbf{W}_n^2 \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}}{n}}{n} & -\frac{(\mathbf{W}_n^2 \mathbf{u}_{j,n})' \mathbf{W}_n^2 \mathbf{u}_{j,n}}{n} & \frac{1}{n} \text{Tr}(\mathbf{W}_n' \mathbf{W}_n) \\ \frac{\mathbf{u}'_{j,n} \mathbf{W}_n^2 \mathbf{u}_{j,n} + (\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}}{n} & -\frac{(\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n^2 \mathbf{u}_{j,n}}{n} & 0 \end{bmatrix} \quad (8)$$

$$\mathbf{g}_{j,n} = \begin{bmatrix} \frac{\mathbf{u}'_{j,n} \mathbf{u}_{j,n}}{n} & \frac{(\mathbf{W}_n \mathbf{u}_{j,n})' \mathbf{W}_n \mathbf{u}_{j,n}}{n} & \frac{\mathbf{u}'_{j,n} \mathbf{W}_n \mathbf{u}_{j,n}}{n} \end{bmatrix} \quad (9)$$

$\mathbf{u}_{j,n}$ is the 2SLS residual vector so that the form of the moment can be formulated as follows:

$$\mathbf{g}_{j,n} = \mathbf{G}_{j,n} \alpha_j + \zeta_{j,n} \quad (10)$$

$\zeta_{j,n}$ is the residual vector regression. To obtain the estimator $(\tilde{\alpha}_j, \tilde{\sigma}_j^2)$ with generalized moment estimation procedure is to minimize the squared residual $\zeta_{j,n}$ with the first derivative of the α_j . Thus obtained,

$$\begin{aligned} \zeta'_{j,n} \zeta_{j,n} &= [\mathbf{g}_{j,n} - \mathbf{G}_{j,n} \alpha_j]' [\mathbf{g}_{j,n} - \mathbf{G}_{j,n} \alpha_j] \\ \zeta'_{j,n} \zeta_{j,n} &= \mathbf{g}'_{j,n} \mathbf{g}_{j,n} - 2 \alpha'_j \mathbf{G}'_{j,n} \mathbf{g}_{j,n} + \alpha'_j \mathbf{G}'_{j,n} \mathbf{G}_{j,n} \alpha_j \\ \frac{\partial \zeta'_{j,n} \zeta_{j,n}}{\partial \alpha_j} &= -2 \alpha'_j \mathbf{G}'_{j,n} \mathbf{g}_{j,n} + \alpha'_j \mathbf{G}'_{j,n} \mathbf{G}_{j,n} \alpha_j = 0 \\ \mathbf{G}'_{j,n} \mathbf{g}_{j,n} &= \mathbf{G}'_{j,n} \mathbf{G}_{j,n} \alpha_j \end{aligned}$$

Then obtained for α_j estimator is:

$$\tilde{\alpha}_j = (\tilde{\lambda}_j, \tilde{\sigma}_j^2) = [\mathbf{G}'_{j,n} \mathbf{G}_{j,n}]^{-1} \mathbf{G}'_{j,n} \mathbf{g}_{j,n} \quad (11)$$

Third Stage: Generalized Spatial Two-Stage Least Square

In this stage, a Cochrane –Orcutt type transformation is applied to dependent, endogenous, and exogenous variables of the single equation spatial econometric model by using an estimated spatial autoregressive parameter to account for the spatial correlation.

$$\mathbf{y}^*_{j,n}(\rho) = \mathbf{y}_{j,n} - \rho_j \mathbf{W}_j \mathbf{y}_{j,n} \quad (12)$$

$$\mathbf{Z}^*_{j,n}(\rho) = \mathbf{Z}_{j,n} - \rho_j \mathbf{W}_n \mathbf{Z}_{j,n} \quad (13)$$

Then the equation becomes:

$$\mathbf{y}^*_{j,n}(\rho_j) = \mathbf{Z}^*_{j,n}(\rho_j) \delta_j + \varepsilon_{j,n} \quad (14)$$

The parameter (ρ_j) assumed known based on the valuation of the second stage. Estimator for δ_j is $\tilde{\delta}_j$ which is obtained from the procedure Generalized Spatial Two-Stage Least Square (GS2SLS) based on 2 SLS process

$$\hat{\delta}_j = [\hat{\mathbf{Z}}_{j,n}^*(\rho_j)' \mathbf{Z}_{j,n}^*(\rho_j)]^{-1} \hat{\mathbf{Z}}_{j,n}^*(\rho_j)' \mathbf{y}_{j,n}^*(\rho_j), \quad j = 1, \dots, G \quad (15)$$

With $\hat{\mathbf{Z}}_{j,n}^*(\rho_j) = \mathbf{P}_H \mathbf{Z}_{j,n}^*(\rho_j)$ and $\mathbf{P}_H = \mathbf{H}_n (\mathbf{H}_n' \mathbf{H}_n)^{-1} \mathbf{H}_n'$

3.3. Full Information Estimation: GS3SLS

3.3.1. Seemingly Unrelated Regression Zellner introduced a modeling multiple regression equation with the residual of the equation the other mutual autocorrelated [6]. The assumptions that must be met in the SUR model equations are $E(\varepsilon) = 0$ and $E(\varepsilon' \varepsilon) = \sigma_{ij} \mathbf{I}_T$. Zellner assumed that the structure of the variance-covariance matrix system SUR model equation can be expressed:

$$E(\varepsilon' \varepsilon) = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} [\varepsilon_1 \quad \varepsilon_2 \quad \dots \quad \varepsilon_n] \quad \text{become} \quad E(\varepsilon' \varepsilon) = \begin{bmatrix} E(\varepsilon_1 \varepsilon_1) & E(\varepsilon_1 \varepsilon_2) & \dots & E(\varepsilon_1 \varepsilon_n) \\ E(\varepsilon_2 \varepsilon_1) & E(\varepsilon_2 \varepsilon_2) & \dots & E(\varepsilon_2 \varepsilon_n) \\ \vdots & \vdots & \ddots & \vdots \\ E(\varepsilon_n \varepsilon_1) & E(\varepsilon_n \varepsilon_2) & \dots & E(\varepsilon_n \varepsilon_n) \end{bmatrix}$$

Because $E(\varepsilon' \varepsilon) = \sigma_{ij} \mathbf{I}_T$, So it can be written

$$E(\varepsilon' \varepsilon) = \begin{bmatrix} \sigma_{11} \mathbf{I}_T & \sigma_{12} \mathbf{I}_T & \dots & \sigma_{1n} \mathbf{I}_T \\ \sigma_{21} \mathbf{I}_T & \sigma_{22} \mathbf{I}_T & \dots & \sigma_{2n} \mathbf{I}_T \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} \mathbf{I}_T & \sigma_{n2} \mathbf{I}_T & \dots & \sigma_{nn} \mathbf{I}_T \end{bmatrix} \quad (16)$$

Equation (16) when described by Kronecker multiplication becomes:

$$E(\varepsilon' \varepsilon) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_{nn} \end{bmatrix} \otimes \mathbf{I}_n = \mathbf{\Sigma} \otimes \mathbf{I}_n = \mathbf{\Omega}$$

The matrix $\mathbf{\Sigma}$ is the variance-covariance matrix of error of size $n \times n$ and $\mathbf{I}$ is the identity matrix of size $T \times T$. SUR model parameter estimation requires the inverse of the variance-covariance matrix of residuals so that it becomes

$$\mathbf{\Omega}^{-1} = \mathbf{\Sigma}^{-1} \otimes \mathbf{I}_n \quad (17)$$

3.3.2. Generalized Spatial Three-Stage Least Square (GS3SLS) The GS2SLS estimator takes into account potential spatial correlation but is limited in the information, and GS3SLS estimator takes into account potential cross-equation correlation in the innovation vector ε_j . To utilize the full information it is helpful to stack the equation (14) as

$$\mathbf{y}_n^*(\rho) = \mathbf{Z}_n^*(\rho) \delta + \varepsilon_n \quad (18)$$

Where: $\mathbf{y}_n^*(\rho) = (\mathbf{y}_{1,n}^*(\rho_1)', \dots, \mathbf{y}_{G,n}^*(\rho_G)')'$; $\mathbf{Z}_n^*(\rho) = \text{diag}_{j=1}^G (\mathbf{Z}_{j,n}^*(\rho_j))$; $\rho = (\rho_1, \dots, \rho_G)'$ and $\delta = (\delta_1', \dots, \delta_G')'$. If $E(\varepsilon_n) = 0$ and $E(\varepsilon_n \varepsilon_n') = \mathbf{\Sigma} \otimes \mathbf{I}_n$

Parameter estimator for spatial simultaneous equation with GS3SLS after entering the cross-equation correlation (parameter SUR model) is:

$$\tilde{\delta} = [\hat{\mathbf{Z}}_n^*(\rho) \mathbf{\Omega}^{-1} \hat{\mathbf{Z}}_n^*(\rho)]^{-1} \hat{\mathbf{Z}}_n^*(\rho) \mathbf{\Omega}^{-1} \mathbf{y}_n^*(\rho)$$

If ρ and $\mathbf{\Sigma}$ are known, a natural system of instrumental variables estimator of δ would be:

$$\tilde{\delta} = \left[[\hat{\mathbf{Z}}_n^*(\rho) (\mathbf{\Sigma}^{-1} \otimes \mathbf{I}_n) \hat{\mathbf{Z}}_n^*(\rho)]^{-1} \hat{\mathbf{Z}}_n^*(\rho) (\mathbf{\Sigma}^{-1} \otimes \mathbf{I}_n) \mathbf{y}_n^*(\rho) \right] \quad (19)$$

Where $\hat{\mathbf{Z}}_n^*(\rho) = \text{diag}_{j=1}^G (\hat{\mathbf{Z}}_{j,n}^*(\rho_j))$ and $\hat{\mathbf{Z}}_n^*(\rho) = \mathbf{P}_H \mathbf{Z}_{j,n}^*(\rho_j)$ [18].

To obtain spatial simultaneous equation model of the unemployment rate and economic growth in East Java Province use Generalized Spatial Three-Stage Least Square (GS3SLS) with the following stages:

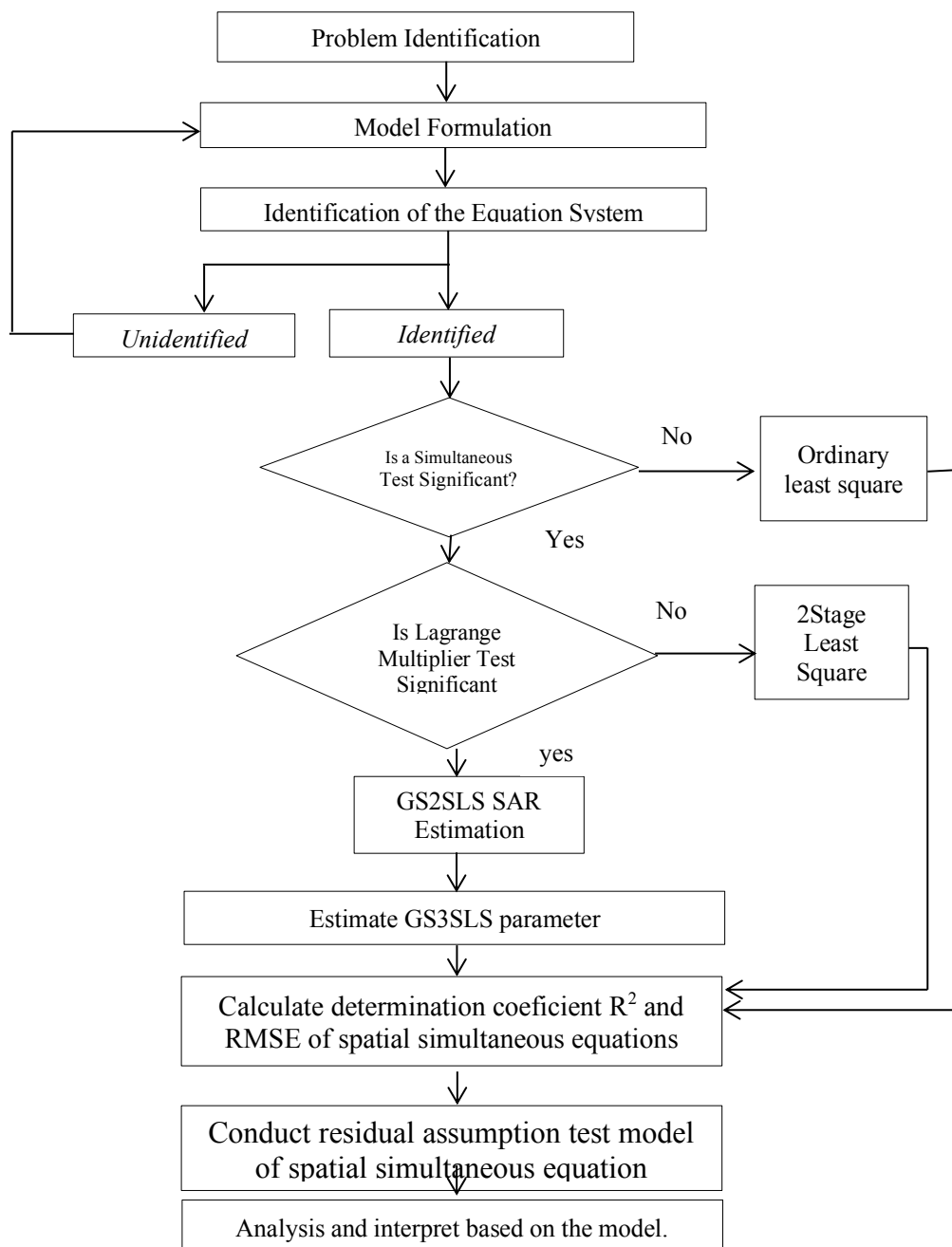


Figure 1. Flow of Analysis Methode.

4. Results and Discussion

4.1. Unemployment rate and economic growth in East Java

Figure 2.a and 2.b show that the unemployment rate and economic growth in East Java, for adjacent areas have the same value. The highest value for the unemployment rate in Kediri at 8.46 and the lowest in Pacitan at 0.87 while for economic growth, the highest in Batu at 6.69 and lowest in Lumajang at 4.62

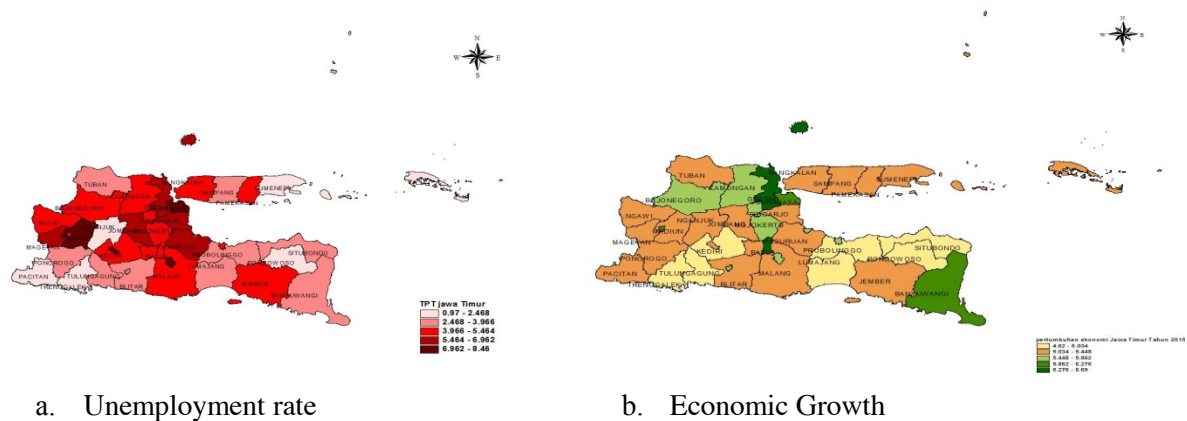


Figure 2. Unemployment Rate and Economic Growth in East Java

4.2. GS3SLS estimation

For the unemployment rate and economic growth modeling in this study involves two equations that have a relationship of mutual influence. In both these equations are suspected simultaneous relationships. There are independent variables in the equation acts as dependent variables in the other equation. A system of simultaneous equations is a set of equations in which the dependent variable in one or more equation is also an independent variable in some other equations. Thus a variable can have two roles at the same time as the independent variable and the dependent variable. To estimate the parameters in the model system of simultaneous equations is preceded by a test of simultaneity. Simultaneous testing can be done through identification of order and rank condition as well as Hausmant empirical test.

4.2.1. Identification order condition. The results of the examination order condition from equations in the model system of simultaneous equations show that the expression is categorized as an over-identified equation because the whole equation is identified overidentified it can be done with GS3SLS parameter estimation.

Table 2. Inspection of the order condition equation system.

Equation	K-k	g-l	Status
Unemployment rate	8-5	2-1	Overidentified
Economic Growth	8-3	2-1	Overidentified

4.2.2. The simultaneity Haustman test. With residual significant at $\alpha = 0.05$, it can be said that there is a simultaneous effect between the equations in the model of the unemployment rate and economic growth in East Java. It means that all the equations contain elements related to cross each other. The equation for the unemployment rate and economic growth indicate an element of simultaneity between endogenous variables that are on the right side with the endogenous variables on the left side. Simultaneity test showed that both equations contain simultaneous effects. Therefore, it can be done simultaneously with the parameter estimation equation GS3SLS.

Table 3. The simultaneity test results of equation system unemployment rate and economic growth model in East Java.

Variables	F-Statistic	prob	Information
RES Unemployment Rate	6.51	0,000	There Simultaneous Effects
RES economic Growth	5.18	0,000	There Simultaneous Effects

4.2.3. *Spatial dependencies test.* Initial identification to see the spatial effect on the regression model is done by using Lagrange Multiplier test. The results of this test can also identify the type of spatial regression model that will be formed, whether autoregressive spatial model (SAR), spatial error (SEM), or spatial autoregressive model with autoregressive disturbances (SARMA).

This identification is done to see the inter-regional linkages or the effect of autoregressive spatial effects. Lagrange Multiplier test with a rook, queen and customized contiguity (raw normalized matrix from Euclidean distance and distance inverse) shows that there are spatial lag dependencies in both equations at a significance level of 0.10. These results detect spatial dependency lag is more dominant than the spatial dependency error, so that simultaneous spatial modeling is best formed GS3SLS SAR [18].

4.2.4. *Assessment of parameters of simultaneous spatial equations GS3SLS.* GS3SLS estimation procedure consists of two equations with endogenous variables Unemployment Rate and Economic Growth. Spatial weights used are Rook Contiguity, Queen Contiguity, and Customized Contiguity. While GS3SLS procedure is applied using Spatial Autoregressive (SAR) simultaneous equation system since the results of spatial lag dependency test with a rook, queen and customized contiguity illustrate the spatial dependency lag effect is more dominant compared to the dependency error.

According to the simultaneous equations model using a SAR system with Rook Contiguity, as follows:

Unemployment Rate

$$\begin{aligned}
 &= 3.96 + 5.61 \text{ Economic Growth}_i - 1.96 \text{ Labour Force Participation Rate}_i \\
 &- 0.05 \text{ Percentage of Poverty}_i + 0.12 \text{ Poverty Gap Index}_i \\
 &+ 0.00008 \text{ Population Growth Rate}_i \\
 &+ 0.37 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Unemployment Rate}_i
 \end{aligned}$$

Economic Growth

$$\begin{aligned}
 &= -0.37 + 0.13 \text{ Unemployment Rate}_i + 0.02 \text{ labor Wage}_i \\
 &+ 0.19 \text{ Human Development Index}_i + 0.53 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Economic Growth}_i
 \end{aligned}
 \tag{20}$$

Queen contiguity, as follows:

Unemployment Rate

$$\begin{aligned}
 &= 5.35 + 4.35 \text{ Economic Growth}_i - 2.73 \text{ Labour Force Participation Rate}_i \\
 &- 0.15 \text{ Percentage of Poverty}_i + 0.16 \text{ Poverty Gap Index}_i \\
 &- 0.01 \text{ Population Growth Rate}_i \\
 &+ 0.20 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Unemployment Rate}_i
 \end{aligned}$$

Economic Growth

$$\begin{aligned}
 &= -0.45 + 0.12 \text{ Unemployment Rate}_i + 0.03 \text{ Wage Workers}_i \\
 &+ 0.24 \text{ Human Development Index}_i + 0.37 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Economic Growth}_i
 \end{aligned}
 \tag{21}$$

Customized contiguity (Euclidean distance), as follows:

$$\begin{aligned}
 \widehat{\text{Unemployment Rate}} &= 6.61 + 3.43 \text{ Economic Growth}_i - 2.85 \text{ Labour Force Participation Rate}_i \\
 &+ 0.04 \text{ Percentage of Poverty}_i + 0.07 \text{ Poverty Gap Index}_i \\
 &+ 0.005 \text{ Population Growth Rate}_i \\
 &- 6.38 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Unemployment Rate}_i \\
 \widehat{\text{Economic Growth}} &= -0.82 + 0.10 \text{ Unemployment Rate}_i + 0.08 \text{ labor Wage}_i \\
 &+ 0.32 \text{ Human Development Index}_i \\
 &- 1.61 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Economic Growth}_i
 \end{aligned} \tag{22}$$

Customized contiguity (distance inverse), as follows:

$$\begin{aligned}
 \widehat{\text{Unemployment Rate}} &= -58.88 + 4.02 \text{ Economic Growth}_i - 2.99 \text{ Labour Force Participation Rate}_i \\
 &+ 0.03 \text{ Percentage of Poverty}_i + 0.08 \text{ Poverty Gap Index}_i \\
 &+ 0.03 \text{ Population Growth Rate}_i - 6.65 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Unemployment Rate}_i \\
 \widehat{\text{Economic Growth}} &= 17.18 + 0.08 \text{ Unemployment Rate}_i + 0.04 \text{ labor Wage}_i \\
 &+ 0.37 \text{ Human Development Index}_i \\
 &- 11.77 \sum_{i=1}^n \sum_{j=1}^m w_{ij} \text{ Economic Growth}_i
 \end{aligned} \tag{23}$$

All weight matrix/contiguity on GS3SLS indicate that on the equations of the unemployment rate, economic growth significantly affect the unemployment rate and on the equations of economic growth as a response variable, indicates unemployment rate as endogen variable affect the economic growth. It means there is a mutual relationship and it can be concluded that there is a simultaneous relationship between the unemployment rate and Economic Growth.

On the equations of the unemployment rate, Labor Force Participation Rate (LFPR) is significantly affected Unemployment Rate at the level $\alpha = 15\%$, While exogenous variable, Percentage of poverty, Poverty depth index (P1) and the population growth rate did not significantly affect unemployment rate at any level of α for all weight contiguity. However, on the equation of economic growth, when we use rook and queen contiguity, the only unemployment rate that affect economic growth but in customized weight contiguity (raw normalized matrix from euclidean distance and distance inverse), human development index also affect economic growth. Customized contiguity can lead to the addition of significant numbers of variables into the model.

The determination coefficients of the spatial simultaneous equation with rook contiguity is 7.62% for the unemployment rate and 4.41 % for economic growth as the response variable, for queen contiguity is 11.88 % for the unemployment rate and 7.14 % for economic growth as a response variable. Meanwhile. the customized contiguity (raw normalized matrix) is 32.17 % for the unemployment rate and 42.9 % for economic growth as the response variable and customized contiguity (distance inverse) is 50.98 % for the unemployment rate as a response variable and 44.94 % for economic growth as the response variable. Customized contiguity can increase the value of the determination coefficient.

Determination coefficient can indicate the percentage of contribution of independent variables to variable response, for example, determination coefficients of the simultaneous spatial equation with rook contiguity is 7.62% for the unemployment rate as response variable. This indicates that the percentage of contribution of variables of economic growth, LFPR, percentage of poverty, poverty gap

index and population growth rate to increase unemployment rate of 7.62% or in other words variation of predictor variable used in model (economic growth, LFPR ,percentage of poverty, poverty gap index and population growth rate) were able to explain 7.62% variation of unemployment rate. While the rest of 92.38% influenced or explained by other variables that are not included in the model of the unemployment rate. While the coefficient of determination on the equation of economic growth response variable with rook contiguity is 4.41%. This shows that the contribution of variables of Unemployment Rate, Labor Wage and HDI to the acceleration of economic growth rate is 4.41%, while the rest of 95.59% is explained by other variables not included in the model.

Based on the value and p-value coefficient parameter spatially lagged dependent variable, it can be concluded that there is no linkage between locations (no spatial effect) when using rook and queen contiguity, but when we use customized weight contiguity (raw normalized matrix from Euclidean distance and distance inverse) there is a spatial effect. From the description, it can be concluded that by using customized weight matrix/ contiguity (raw normalized matrix from Euclidean distance and inverse distance), between the variables of the unemployment rate and economic growth have a simultaneous and spatial effect.

4.2.5. Best model criteria. Based on the four alternative, unemployment rate and economic growth model will be determined the model to be analyzed further. Determination of the model is based on three criteria: The coefficient of determination (R^2), Akaike Information Criterion (AIC), and RMSE values. Unemployment rate and economic growth model with customized contiguity using the distance inverse is relatively better than the estimation results with Rook, Queen, and Customized by Euclidean distance with the highest R^2 , smallest RMSE and AIC.

4.3. Testing Assumptions Residual Spatial Simultaneous Equations

Spatial simultaneous equations models formed spatial residual assumption is necessary to test the modeling results. The test includes normality and homogeneity test. Assumption residual showed that GS3SLS Spatial Autoregressive (SAR) model with Rook, Queen, and Customized Contiguity show p-value above 0.05 by Anderson Darling normality test, and p-value above 0.05 by Hall Pagan LM test, the assumptions of normality and heterogeneity residuals fulfilled.

5. Conclusion

GS3SLS with customized contiguity with distance inverse can provide a better estimate than the other continuity. Not all cases can use GS3SLS, and further research is needed to determine what kind of cases can be done by using GS3SLS or in a subsequent analysis it is necessary to establish an appropriate GS3SLS estimation method for all conditions of the various varieties. In this study involves simultaneous spatial equation using two equations only, need to develop GS3SLS which more than two equations.

Labor Force Participation Rate (LFPR) is significantly affecting the Unemployment Rate, and Human Development Index (HDI) is significantly affect economic growth. By using customized weight matrix/customized contiguity (Inverse Distance). Between the variables of the unemployment rate and economic growth have a simultaneous and spatial effect.

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