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Development of a Practical Uniaxial Cyclic Constitutive Model for Concrete

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Abstract. The uniaxial stress-strain curves of concrete suggested in Chinese Concrete Standard (GB50010-2010) are combined with the modified Yassin hysteretic rule. The integrated cyclic constitutive model for concrete is aimed at the nonlinear analysis of the reinforced concrete (RC) structures under strong earthquake. The model is implemented into general-purpose FEA software ABAQUS through the user material subroutine VUMAT. Then, some typical performances of the proposed model are verified. Furthermore, a seismic damage analysis of a multi-story RC frame structure is conducted using the cyclic model. The results show that the constitutive model is applicable to the nonlinear seismic analysis of RC structures.

1. Introduction

With the increase of high-rise building projects and the rapid development of computer software and hardware, the elastoplastic analysis of building structures under strong earthquakes has attracted extensive attention of structural designers and researchers. The uniaxial cyclic constitutive model for the concrete is important to the elastoplastic analysis of RC frame structures. Over the past several years, practical and simplified models, such as the Park-Kent model [1], the Blakeley-Park model [2] and the Yassin model [3] were proposed, while more complicated models which can represent more characteristics of the concrete are developed, such as the Chang-Mander model, Bahn-Hsu model, Sima-Roca- Molins model, among others [4-8]. However, the computational cost of those models is comparatively high.

In the Chinese Standard: *Code for Design of Concrete Structures* (GB50010-2010) [9], the more accurate and practical stress-strain curves of concrete under compression and tension are suggested, and the expressions of tensile and compressive damage parameters are adopted in the model, which is convenient for damage evaluation of structures. However, the unloading and reloading path in compression is a single line, which seems too simplified for the seismic or cyclic analysis of the RC structures. By comparison, the hysteretic rule given in Yassin model is relatively refined, consisting of multiple straight lines [3]. However, many studies indicate that the unloading path under compression of concrete is a curve-shaped path. Therefore, the modification for the hysteretic rule in Yassin model is proposed based on the Bezier curve. And then, the combination of the skeleton curve in the Chinese Standard and the modified Yassin hysteretic rule is introduced to form a practical cyclic constitutive model with more advantages. The present constitutive model can describe many typical behaviors of concrete subjected to the cyclic loading, such as cracking under tension, plasticity under compression, stiffness degradation under cyclic loading and hysteretic effect under compression. Based on ABAQUS/Explicit, an explicit solver of ABAQUS, the user material subroutine VUMAT is developed according to the proposed model. Then, the equivalent simulation method of the fiber model [10] is

used to test and verify the model under various load cases. Furthermore, the elastoplastic response and damage evolution of a four-story RC frame under earthquake is analyzed.

2. Uniaxial cyclic constitutive model for concrete

2.1 Tensile and compressive skeleton curves based on Chinese Standard
The stress-strain curves of concrete under uniaxial tension and compression given in the new edition of *Code for Design of Concrete Structures* (GB50010-2010) adopt tensile and compressive damage parameters and provide a list of model parameters on the basis of a large number of experiments. This is very convenient for designers to use and the curves agree well with the experiments. The experiment results show that the stress-strain curve under uniaxial monotonic loading can be used as the skeleton curve of the cyclic loading model [11].

The stress-strain curve of concrete under uniaxial tension is determined by the following expressions:

$$\sigma = (1-d_t)E_c\varepsilon \qquad (1)$$

$$d_t = \begin{cases} 1-\rho_t\left[1.2-0.2x_t^5\right] & x_t \leq 1 \\ 1-\dfrac{\rho_t}{\alpha_t\left(x_t-1\right)^{1.7}+x_t} & x_t > 1 \end{cases} \qquad (2)$$

$$x_t = \frac{\varepsilon}{\varepsilon_{t,r}} \qquad (3)$$

$$\rho_t = \frac{f_{t,r}}{E_c\varepsilon_{t,r}} \qquad (4)$$

where, E_c is the elastic modulus of concrete; σ and ε are the tensile stress and strain, respectively; d_t is the tensile damage parameter, the damage evolution is determined by Equation (2); $f_{t,r}$ is the tensile strength; $\varepsilon_{t,r}$ is the strain corresponding to the $f_{t,r}$.

The stress-strain curve of concrete under uniaxial compression is determined by the following expressions:

$$\sigma = (1-d_c)E_c\varepsilon \qquad (5)$$

$$d_c = \begin{cases} 1-\dfrac{\rho_c n}{n-1+x_c^{\,n}} & x_c \leq 1 \\ 1-\dfrac{\rho_c}{\alpha_c\left(x_c-1\right)^2+x_c} & x_c > 1 \end{cases} \qquad (6)$$

$$\rho_c = \frac{f_{c,r}}{E_c\varepsilon_{c,r}} \qquad (7)$$

$$n = \frac{E_c\varepsilon_{c,r}}{E_c\varepsilon_{c,r}-f_{c,r}} \qquad (8)$$

$$x_c = \frac{\varepsilon}{\varepsilon_{c,r}} \qquad (9)$$

where, d_c is the compressive damage parameter, the damage evolution is determined by the Equation(6), $f_{c,r}$ is the compressive strength; $\varepsilon_{c,r}$ is the strain correspond ding to $f_{c,r}$.

Moreover, a supplementary equation is necessary to be introduced for ensuring that the elastic modulus has the same value under the tension and compression, as follows:

$$E_c = 1.2\frac{f_{t,r}}{\varepsilon_{t,r}} \qquad (10)$$

2.2 Modified Yassin hysteretic rule

The hysteretic rule of Yassin model [3] uses a series of straight lines to represent the hysteretic behavior of concrete under cyclic loading, which is similar to the model of Blakeley and Park [2]. That type of rule is relatively concise and practical. Especially, it can describe the hysteretic behavior of concrete under compression in a unified manner. However, the bilinear unloading path in the Yassin model does not consistent with the experiment. Thus, the modification to the unloading path is made through a second-order Bezier curve, in view of the good qualities of the curve.

The compressive hysteretic behavior of the modified rule is shown in Figure 1. In the compressive part of the model, with the increase of strain, the stiffness of concrete degrades continually during unloading and reloading. The degenerated stiffness is determined by the so-called focus model. The focus is determined by the tangent at origin and the reloading stiffness line at the residual strength. The coordinates of the focus are as follows:

$$\varepsilon_r = \frac{0.2 f_{c,r} - E_{20}\varepsilon_{20}}{E_c - E_{20}} \qquad (11)$$

$$\sigma_r = E_c \varepsilon_r \qquad (12)$$

where, E_{20} is the reloading stiffness corresponding to $0.2 f_{c,r}$, which can be calibrated by the experiment; ε_{20} is the strain corresponding to $0.2 f_{c,r}$.

In the original model, the response is controlled by two line segments in the unloading path from the compressive skeleton to the zero stress. Some studies show that the unloading stiffness of concrete varies continuously during the cyclic loading process. And the second-order Bezier curve is able to describe the transition from initial unloading stiffness to the final unloading stiffness smoothly. Thus, it is introduced in the present study for the description of the unloading behavior in the compression stage.

Firstly, a reference point is required. The coordinates of the reference point are as follows:

$$\varepsilon_a = \frac{E_i\varepsilon_m - E_p\varepsilon_p - \sigma_m}{E_i - E_p} \qquad (13)$$

$$\sigma_a = E_p\left(\varepsilon_a - \varepsilon_p\right) \qquad (14)$$

where,

$$\varepsilon_p = \varepsilon_m - \frac{\sigma_m}{E_r} \qquad (15)$$

$$E_r = \frac{\sigma_m - \sigma_r}{\varepsilon_m - \varepsilon_r} \qquad (16)$$

E_i is the initial unloading stiffness, which equals to E_c herein; E_p is the final unloading stiffness, which equals to $0.5 E_r$ herein; E_r is the reloading stiffness; σ_m and ε_m are the unloading stress and strain, respectively in the compressive skeleton curve; ε_p is the plastic strain due to compressive unloading.

Thus, the Bezier-curve unloading path is given by the following expression:

$$\sigma_{\max} = \left(\sigma_m - 2y_a\right)t^2 + 2\left(y_a - \sigma_m\right)t + \sigma_m \qquad (17)$$

where,

$$t = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}}{2a_2} \in [0,1] \qquad (18)$$

$$a_0 = \varepsilon_m - \varepsilon \qquad (19)$$

$$a_1 = 2(x_a - \varepsilon_m) \qquad (20)$$

$$a_2 = \varepsilon_m - 2x_a + \varepsilon_p \qquad (21)$$

According to the original model, the linear reloading path is defined by the expression:

$$\sigma_{\min} = \sigma_m + E_r(\varepsilon - \varepsilon_m) \qquad (22)$$

For the partial unloading and reloading between two small envelopes, i.e., the full unloading and reloading paths, it is assumed that the model respond linearly with the slope E_c.
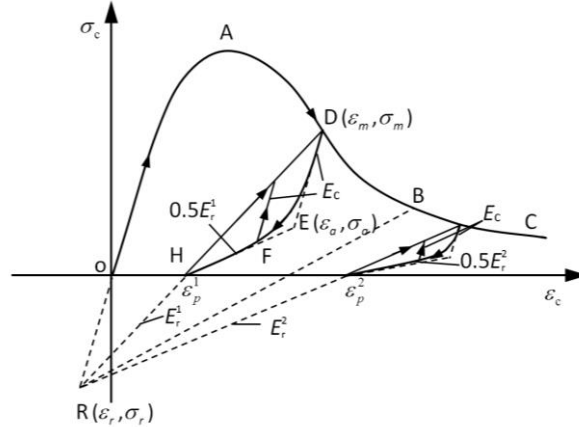

Figure 1. Modified Yassin hysteretic rule in the compression stage

Figure 2 shows two consecutive tensile hysteresis loops. The model assumes that the tensile stress can be generated at any position along the strain axis whether it is caused by initial tensile loading or unloading under compression. The unloading and reloading paths of the model are straight lines in the original model, but the hysteretic rule does not account for the influence of the compressive damage on the degradation of the tensile stiffness. In the present study, this effect is taken into account. Therefore, the tensile stress expression is modified as follows:

$$\sigma = E_{tr}\varepsilon \qquad (23)$$

$$E_{tr} = (1 - d_t)(1 - sd_c)E_c \qquad (24)$$

where, $s \in [0,1]$ is a parameter calibrated through the experiment.
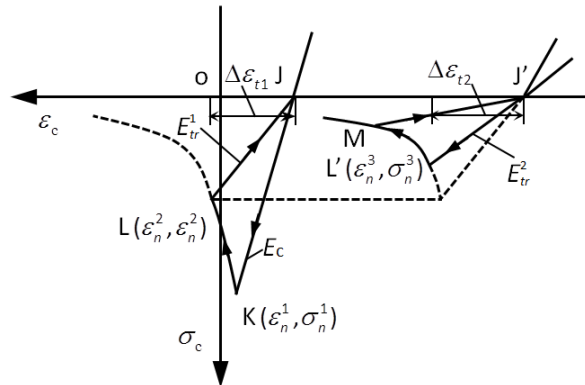
Figure 2. Modified Yassin hysteretic rule in the tension stage

2.3 Implementation of uniaxial cyclic constitutive model in ABAQUS
The general purpose finite element software ABAQUS has powerful and convenient pre and post processors and implicit and explicit solvers. Based on ABAQUS, a material subroutine VUMAT is

developed in FORTRAN for the proposed model of concrete. The subroutine is embedded in ABAQUS to realize the nonlinear seismic response analysis of RC structures using the equivalent modelling strategy of the fiber beam-column elements.

3. Basic behaviors of the model

The response of the proposed uniaxial cyclic constitutive model in six typical load cases is simulated by using a single beam element B31 in ABAQUS to check the validity. The six typical load cases are monotonic tensile loading, monotonic compressive loading, repeated tensile loading, repeated compressive loading, cyclic loading started from tension and partial unloading and reloading in compression. As shown in Figures 3-8.
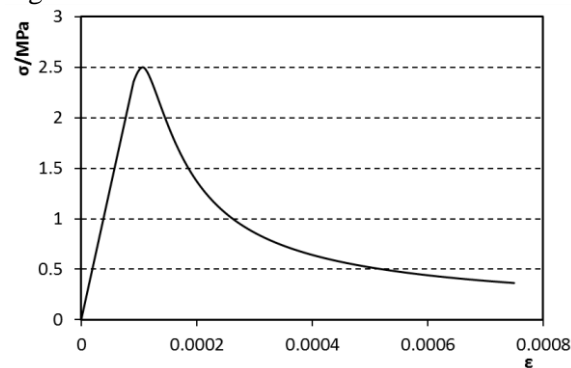
Figure 3. Stress-strain curve under the monotonic tension
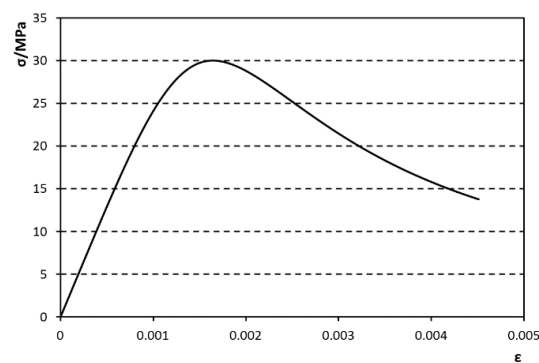
Figure 4. Stress-strain curve under the monotonic compression
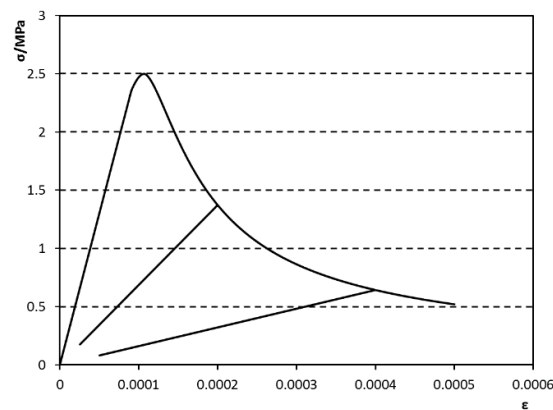
Figure 5. Stress-strain curve under repeated tensile loading
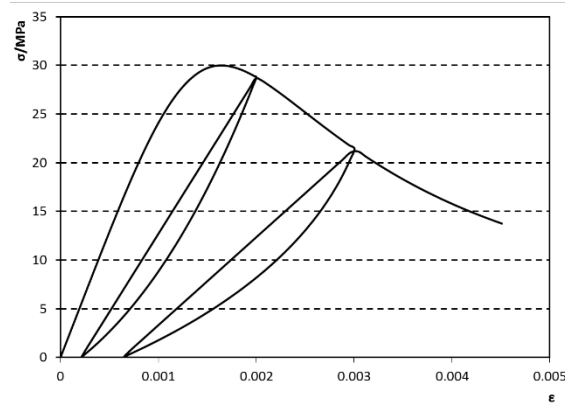
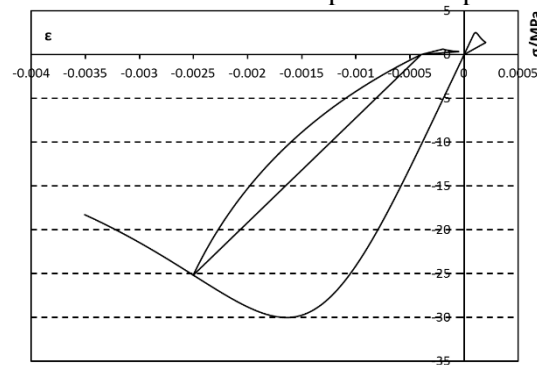Figure 6. Stress-strain curve under repeated compressive loading

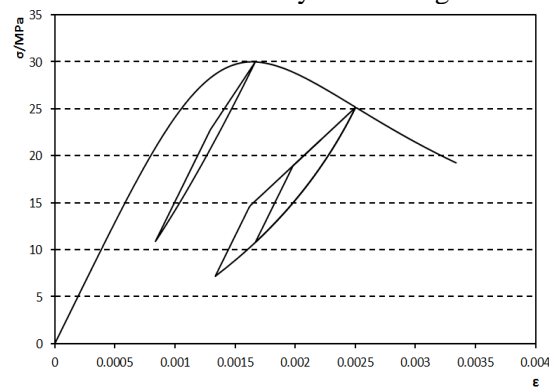Figure 7. Stress-strain curve under cyclic loading started from tension

Figure 8. Stress-strain curve under partial unloading and reloading in compression

The simulation results of the above six typical load cases show that the model can describe the basic typical characteristics of uniaxial response of concrete under cyclic loading properly, and meet the needs of nonlinear analysis of concrete structures under random cyclic loading such as earthquake.

4. Case study

4.1 Analytical model
A RC planar frame structure is taken as an example. The frame has four stories and two bays. The bay is 6 m in width, the first floor is 4 m in height, and the other floors are 3.5 m in height. The section of the column is 600 mm × 600 mm, the section of the beam is 300 mm × 500 mm, and the reinforcement ratios of the beam and the column are both 2%. The finite element model of the frame structure is

shown in Figure 9, and the section of beam and column in the model is given in Figure 10. The strength grade of the concrete is C30 and the strength grade of the steel bar is HRB400. The proposed cyclic constitutive model is adopted for concrete, and the simplified elastic-perfectly plastic model is used for steel bars. However, the confinement effect of the concrete is not considered in this example. Both concrete and steel bars are simulated by B31 beam elements. The explicit solver ABAQUS/Explicit is used.
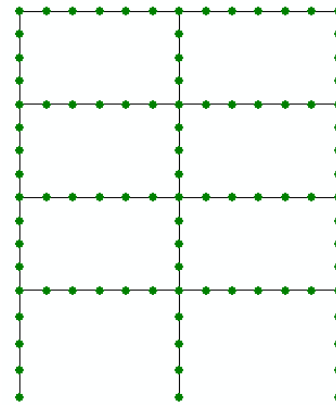

Figure 9. Finite element model of the frame

RC beam section   Concrete beam section   Rebar beam section

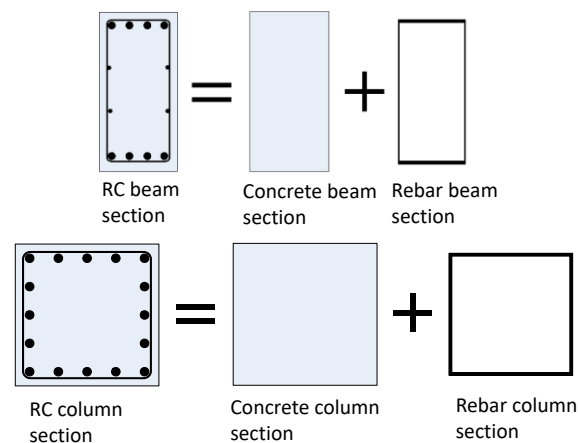RC column section   Concrete column section   Rebar column section

Figure 10. Discretization of beam and column sections

The whole process of seismic damage evolution of the frame structure under Kobe earthquake is simulated. The acceleration amplitude of the earthquake recorder is about 0.35g. The waveform is shown in Figure 11.
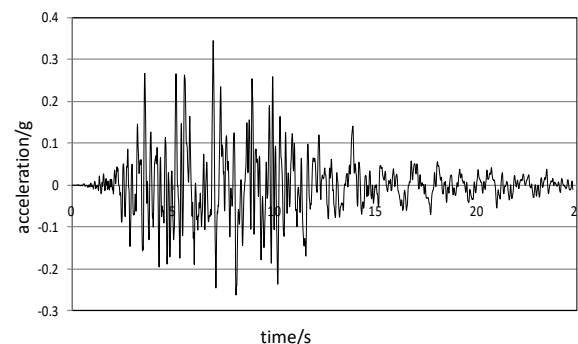

Figure 11. Waveform of Kobe earthquake recorder

4.2 Results and discussion

Damage value of each concrete fiber can be obtained by this analysis. Damage contour of typical concrete fiber is selected to exhibit the damage evolution process of the whole frame. As shown in Figure 12(a), the damage of the frame structure initially appeared at 3.0 s under earthquake. In Figure 12(b), the damage occurred at the bottom of the column in the first floor and the beam ends in the second and third floor. The damage of the middle column was relatively serious, and the damage of the beam ends in the secondary and third floors was more serious than that of the fourth floor beam ends, as shown in Figure 12(b). After 6.0 s, the existing damage area developed rapidly, and the damage occurred at the beam and column ends mostly. At 8.0 s, as shown in Figure 12(c), the damage area expanded and the damage degree increased. From 8.0 s till the end of the excitation, as shown in Figures 12(d)-(e), the development of the damage area tended to be stable.
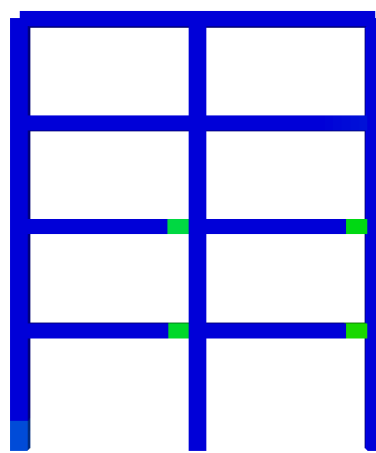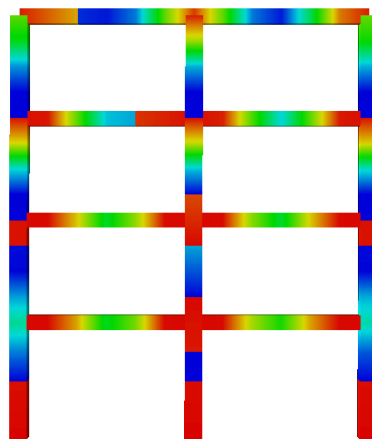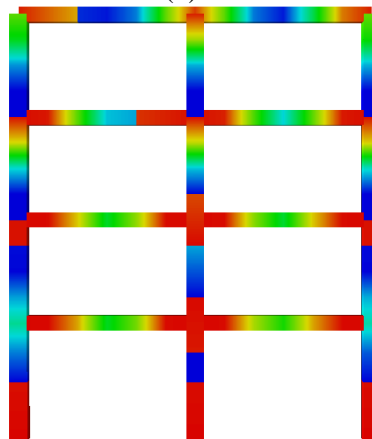

(a) 3.0s
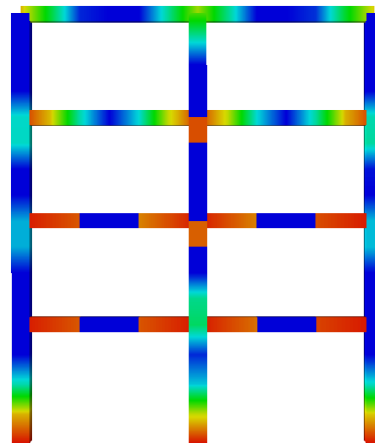
(b) 6.0s
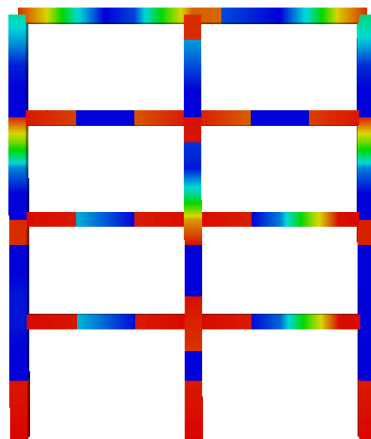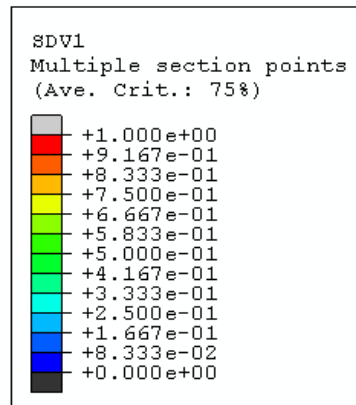
(c) 8.0s

(d) 14.5s

(e) 25.0s

Figure 12. Tensile damage contour under Kobe earthquake

5. Conclusions

Based on *Code for Design of Concrete Structures* (GB50010-2010) and the modified Yassin hysteretic rule, a practical uniaxial cyclic constitutive model of concrete is established. In addition, the corresponding material subroutine module is developed in ABAQUS. The stability and applicability of the model are verified under several typical load cases. Furthermore, the seismic damage evolution process of a RC frame structure is simulated numerically. According to the seismic damage evolution contour, the seismic damage of RC multi-story frame structure mainly concentrates at the bottoms of the columns in the first floor and the ends of the beams and columns, which is in agreement with the actual seismic damage phenomenon of the frame structure.

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