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## Interaction Analysis of Multiple Cracks Based on the Maximum Shear Stress Criterion

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# Interaction Analysis of Multiple Cracks Based on the Maximum Shear Stress Criterion

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**Abstract:** In order to analyse the interaction of multiple cracks in the process of the cracks propagation and penetration, the overlap triple-cracks penetration test was simulated according to the maximum shear stress criterion. The stress intensity factors of the crack tips were calculated by means of element-free method (EFM). The results show that the maximum shear stress criterion can simulate the crack propagation process well. With comparison with the single crack model, the formation mechanism of the wing cracks and the shear cracks, and the interaction between the cracks and its influence on the multi-cracks propagation path were also explained.

## 1. Introduction

Due to the interaction between cracks in rock mass, the expansion mode of the multiple cracks is more complicated than that of the single crack, and the penetration of multiple cracks will cause more serious safety problems. Therefore, the study of the interaction and penetration mechanism of multiple cracks has been the hot spot in the field of rock mechanics. The research [1] proposed a simplified pseudo-tension method, Kachanov method, to calculate the interaction of multiple cracks. The research [2] improved the Kachanov method to increase its accuracy. Zhou Xiaoping used the dislocation model method to study the influence of multi-crack interaction of complex shapes on the stress intensity factor, which can analyze the influence of interaction between bending cracks and periodic arrangement cracks on the stress intensity factor [3].

In this paper, the overlap triple-cracks penetration test [4-5] was simulated according to the maximum shear stress criterion of multi-cracks penetration [6], and the stress intensity factors of the crack tips were calculated in the whole process of expansion until penetration occurs by means of the element-free method (EFM). With comparison with the single crack model, the formation mechanism of the wing cracks and the shear cracks, and the interaction between the cracks and its influence on the multi-cracks propagation path were explained.

## 2. Maximum shear stress criterion for multi-cracks penetration

In researches [4-5], the uniaxial and biaxial loading tests were carried out on prefabricated multi-cracks cement specimens under the servo-controlled dual-axis loading system, and the stress intensity factors of the crack tips in the specimens were calculated by finite element method. It was found that the cracks in the test specimens are gradually penetrated due to shear slip. Based on the test



results, the author proposed the maximum shear stress criterion for multi-cracks penetration [6]. That is,

(1) The shear crack extends in the direction where the absolute value of the shear stress  $\tau_{\gamma\theta}$  reaches maximum.

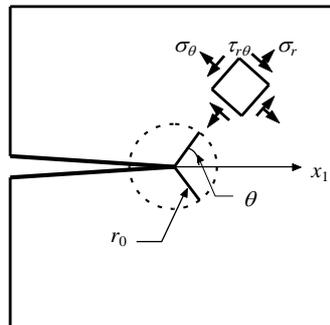


Figure 1. Determination of crack angle

Since the shear stress at the tip of the crack tends to infinity, approximate calculation can be done at each point on a tiny circumference around the crack tip, as shown in Figure 1. When the extreme value of the shear stress is found, the crack angle  $\theta_0$  can be determined.

(2) When the maximum value of the shear stress  $\tau_{\gamma\theta}$  near the tip of the multi-cracks reaches the critical value, the crack will undergo shear expansion, that is

$$(\tau_{\gamma\theta})_{\max} = \tau_{\gamma\theta\text{cri}} \quad (1)$$

Where,

$$(\tau_{r\theta})_{\max} = \frac{1}{2\sqrt{2\pi r_0}} \cos \frac{\theta_0}{2} [K_I \sin \theta_0 + K_{II} (3 \cos \theta_0 - 1)] \quad (2)$$

$\tau_{\gamma\theta\text{cri}}$  can be determined by the fracture toughness  $K_{IIC}$  of type II crack, and the cracking criterion is

$$K_{r\theta} = \frac{1}{2} \cos \frac{\theta_0}{2} [K_I \sin \theta_0 + K_{II} (3 \cos \theta_0 - 1)] = K_{IIC} \quad (3)$$

In the formula,  $K_{\gamma\theta}$  is called the shear type effective stress intensity factor.

### 3. Analysis of the connection mechanism of overlapping three cracks

According to the overlap triple-cracks test in the literature [5], the EFM model of the triple-cracks expansion was established, as shown in Figure 2. The loading process in Figure 3 was simulated until the crack started expanding. 50 dense nodes were arranged at the crack tip. The material fracture toughness  $K_{IC} = 0.025 \text{MPa} \cdot \text{m}^{1/2}$ ,  $K_{IIC} = 0.027 \text{MPa} \cdot \text{m}^{1/2}$ . In order to make comparison, the expansion of the single crack was also simulated, that is, the model only including crack CD shown in Figure 2.

The variation of stress intensity factors during the cracking process are shown in Figure 4. Considering the symmetry, only the results of crack tips A, B and C are selected. The stress intensity factors of the crack tip C in the single crack model (only crack CD) is shown in Figure 5. The cracking path of the overlapping three cracks from the initial cracking to the expansion and the penetration failure is shown in Figure 6. Figure 7 shows the propagation paths of triple-cracks specimens. The crack propagation path of the corresponding single crack model crack is shown in Figure 8.

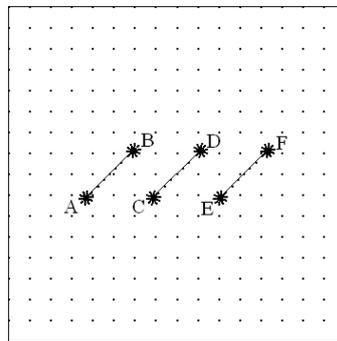


Figure 2. Distribution of triple-cracks nodes

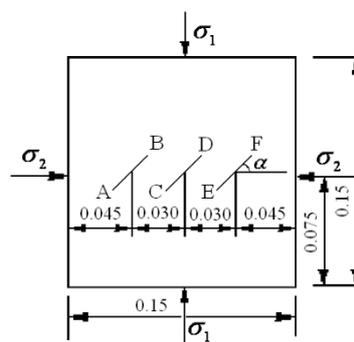
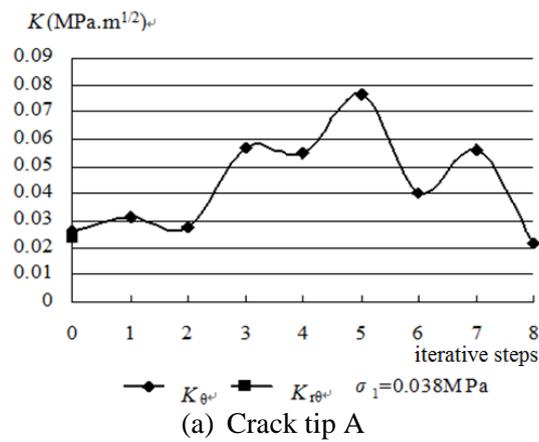
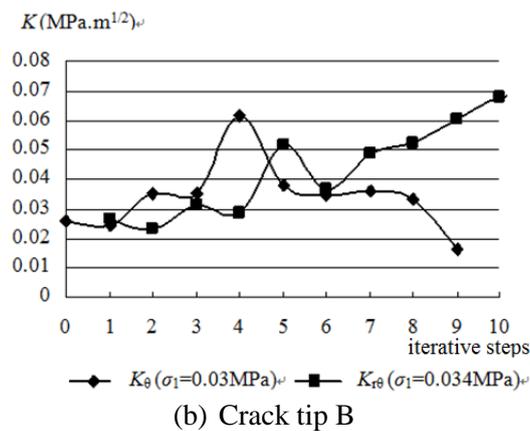


Figure 3. Triple-cracks specimens (unit: m)



(a) Crack tip A



(b) Crack tip B

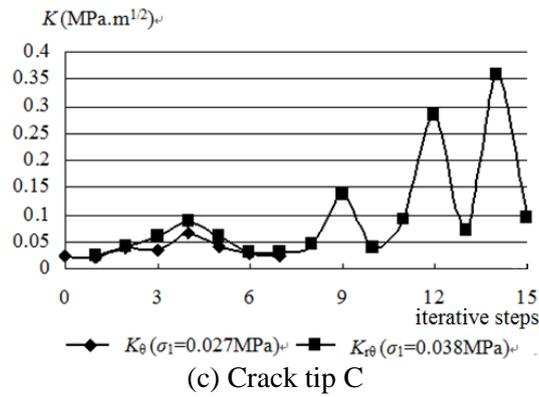


Figure 4. Variation of stress intensity factors of the crack tips of triple-cracks model

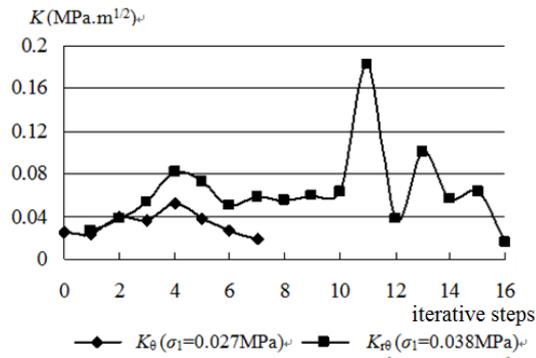


Figure 5. Variation of stress intensity factor of crack tip C of single-crack model

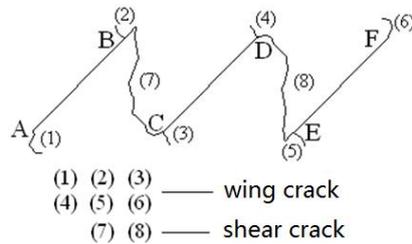


Figure 6. Calculation expansion path of three cracks

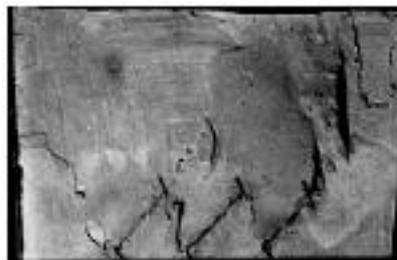


Figure 7. Propagation paths of triple-cracks specimens

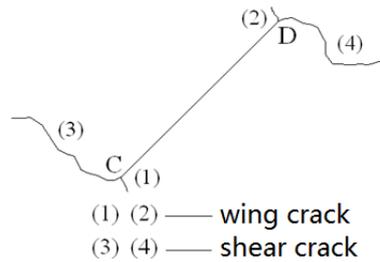


Figure 8. Calculation expansion path of single crack

It can be seen from Figure 4 that the  $K_{\theta}$  of the crack tip A is reduced to below  $K_{IC}$  in the 8th iterative step after the crack is expanded and the crack stops cracking, corresponding to the wing crack (1) in Figure 6. And  $K_{\gamma\theta}$  of the crack tip A is still less than  $K_{IIC}$  under the extreme load ( $\sigma_1=0.038\text{MPa}$ ), that is, no shear crack will occur at crack tip A. The change of  $K_{\theta}$  of the crack tip B is similar to that of the crack tip A, and the crack stops at the 9th iteration step, corresponding to the wing crack (2) in Figure 6. On the other hand,  $K_{\gamma\theta}$  of the crack tip B is larger than  $K_{IIC}$  when  $\sigma_1=0.034\text{MPa}$ , forming shear crack. With the increase of  $\sigma_1$ ,  $K_{\gamma\theta}$  of the crack B gradually increases and the shear crack further expands. When  $\sigma_1=0.038\text{MPa}$ , the shear crack from the crack tip B connects with that propagated from the crack tip C, forming the crack (7) in Figure 6. The change of  $K_{\theta}$  of the crack tip C is similar to that of the crack tip A and B and at the end of the seventh iteration the crack stops, corresponding to the wing crack (3) in Figure 4. Similar to that of the crack tip B,  $K_{\gamma\theta}$  of the crack tip C is larger than  $K_{IIC}$  when  $\sigma_1=0.038\text{MPa}$ , forming shear crack. And then  $K_{\gamma\theta}$  of the crack tip C keeps increasing until the 15th step of the iteration, when the crack tip B and the crack tip C are connected, as shown in the crack (7) in Figure 6.

The calculated expansion path of the overlapped three cracks in Figure 6 is basically consistent with the test results as shown in Figure 7, which shows that the proposed maximum shear stress criterion can reflect the penetration mechanism of multiple cracks fairly well.

As to the single-fracture model, the variation of  $K_{\theta}$  of the crack tip C in Figure 5 is similar with that of the triple-cracks model and the wing crack is formed shown in the crack (1) in Figure 8. But the variation law of  $K_{\gamma\theta}$  of the crack tip C is quite different with that of the triple-cracks model. Especially in the second half of the shear crack propagation,  $K_{\gamma\theta}$  of the crack tip C decreases to be less than  $K_{IIC}$ , forming the shear crack (3) as shown in Figure 8. There is a significant deviation in the direction of the shear crack (3) in Figure 8 with the connection direction of the shear crack (7) in the Figure 6.

#### 4. Conclusion

According to the maximum shear stress criterion of multi-cracks penetration, the expansion and penetration process of overlapping three cracks were simulated by means of element-free method. The interaction mechanism of multiple cracks is analyzed in depth by comparison with single-crack model. The results show that the maximum shear stress criterion can simulate the crack propagation process well. The multi-crack propagation mechanism is more complicated than that of the single crack. It is also possible to form a penetration crack when the shear crack is fully expanded. At the same time, the interaction between the cracks will affect the difficulty of cracking, and this interaction will be enhanced during the crack propagation process.

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