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Study on two - dimensional linear harmonic oscillator characteristics based on MATLAB software

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Abstract. Based on the theory of quantum mechanics, this paper systematically analyzes the basic characteristics of n-dimensional linear harmonic oscillator in quantum mechanics, focuses on the eigenfunction and probability density of one-dimensional harmonic oscillator, and simulates the eigenfunction and probability density of some energy levels with MATLAB software. Finally, MATLAB software was used to compare the probability distribution of linear harmonic oscillator in classical mechanics and quantum mechanics. The results indicate that the number of points of intersection between a wave function and a $\varphi=0$ line is n; The probability distribution satisfies the normalization condition; Taking different values of n, the probability distribution function of harmonic oscillator in quantum mechanics has n different nodes, and the amplitude of harmonic oscillator in classical mechanics also changes accordingly. φ_0 by the ground state probability distribution of quantum mechanics and classical mechanics distribution probability of the simulation image can be seen that the shape of the two distribution curve is the opposite, but when n is large, the probability density of quantum mechanics $|\phi_n(\xi)|^2$ local average and classical probability distribution $P(\xi)$, that is, classical mechanics and quantum mechanics ZhongZhen gradually increase the probability distribution of similarity. These results are reflected in the image, and the characteristics shown in the image are consistent with the theoretical results.

1. Introduction

Harmonic oscillator model is the most common and precise model in classical and quantum mechanics [1, 2]. It reflects the cyclical phenomenon of the nature, and makes it is an ideal candidate to understand the physical basis of a series of complex phenomenon. The small vibration of any system which the nearby the equilibrium position can be generally regarded as simple harmonic motion, namely the harmonic oscillation. And the harmonic oscillation are reflected in many physical problem [3-12], such as, the molecular vibration [4], the lattice vibrations [5,6], the nuclear surface vibration [7-9] and the vibration of the radiation field [10-12], etc. For many actual harmonic oscillation, they can be decomposed into several independent one-dimensional harmonic vibration, starting from the simplified physical model of the linear harmonic oscillator. Therefore, the linear harmonic oscillator is an important issue in theoretical physics, and it plays an important role in mechanics, electricity dynamics, electronic circuit, atomic physics, molecular physics, solid-state physics, quantum field theory and quantum optics [13-16].

In the field of quantum mechanics, it has more special significance [17,18]. On the one hand, many complex movement can be simplified as resonance movement by harmonic oscillator model. On the other hand, it has a special role in methodology [19]: Firstly, its exact solution can be used as the



application examples of schrödinger equation, and it is essential to study quantum mechanics; Secondly, the ‘generation’ and ‘annihilation’ operators, which are obtained by solving the harmonic oscillator problem, play an important role in the establishment of the quadratic quantization problem and the analysis of the zero point energy problem of the so-called electromagnetic vacuum [20,21]. Therefore, it is necessary for us to study it systematically.

In this paper, taking one- and two-dimensional linear harmonic oscillator as an example, we study the characteristics of linear harmonic oscillator by theoretical analysis and numerical simulation in classical mechanics and quantum mechanics, respectively.

2. Research methods

2.1 One-dimensional linear harmonic oscillator in quantum mechanics

When the potential energy function of a linear harmonic oscillator is $U=\omega^2 x^2/2$, the corresponding stationary schrödinger equation is [17]

$$\frac{\hbar^2}{2m} \frac{d^2 \varphi}{dx^2} + \left(E - \frac{1}{2} m \omega^2 x^2 \right) \varphi = 0 \quad (1)$$

So, the eigenfunction of the linear harmonic oscillator, which is normalized, is expressed as

$$\varphi_n(\xi) = N_n e^{-\frac{\xi^2}{2}} H_n(\xi) \quad (2)$$

where

$$N_n = \left(\frac{\alpha}{\sqrt{\pi} 2^n n!} \right)^{\frac{1}{2}},$$

$$\alpha = \sqrt{\frac{m\omega}{\hbar}},$$

$$\xi = \alpha x,$$

and $H_n(\xi)$ is the Hermitian polynomial. The previous Hermite polynomials are given, respectively

$$H_0 = 1, H_1 = 2\xi, H_2 = 4\xi^2 - 2, H_3 = 8\xi^3 - 12\xi,$$

$$H_4 = 16\xi^4 - 48\xi^2 + 12, H_5 = 32\xi^5 - 160\xi^3 + 120\xi$$

2.2 Two-dimensional linear harmonic oscillator in quantum mechanics

The two-dimensional linear harmonic oscillator theory is discussed in rectangular coordinates and polar coordinates, respectively.

2.2.1 Eigenfunctions of two-dimensional linear harmonic oscillators in rectangular coordinates

The steady state schrödinger equation of the system is expressed by

$$-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} \right) \psi + \frac{1}{2} \mu \omega^2 (x^2 + y^2) \psi = E \psi \quad (3)$$

The natural unit $\hbar=\mu=\omega=1$ and $\psi(x, y)=\varphi(x)\varphi(y)$ are selected. By take advantage of the research method of one-dimensional harmonic oscillator, we find that the two-dimensional harmonic oscillator is independent of each other in the x and y directions. Finally, the wave function of the two-dimensional harmonic oscillator is obtained

$$\psi(x, y) = N_{nx} N_{ny} e^{-\frac{1}{2}(x^2+y^2)} H_{nx}(x) H_{ny}(y) \quad (4)$$

where, the energy of the system is

$$E = n_x + n_y + 1 = N + 1 \quad (5)$$

the normalized coefficient is

$$N_{nx} = \left(\frac{\alpha}{\pi^{\frac{1}{2}} 2^{nx} nx!} \right)^{\frac{1}{2}}, \quad N_{ny} = \left(\frac{\alpha}{\pi^{\frac{1}{2}} 2^{ny} ny!} \right)^{\frac{1}{2}} \quad (6)$$

and $H_{nx}(x)$, $H_{ny}(y)$ are the Hermitian polynomial, and it is expressed for $H_{nx}(x)$ by

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2} = \sum_{k=0}^{\left[\frac{n}{2}\right]} (-1)^k \frac{n!}{k!(n-2k)!} (2x)^{n-2k} \quad (7)$$

It can be clearly seen from the above wave function and energy formula that the degeneracy of the harmonic oscillator is $n+1$. When $n=0$, the corresponding wave function has no degeneracy. Similarly, when $n=1$, the wave function degeneracy is 2.

2.2.2 Eigenfunctions of two-dimensional linear harmonic oscillators in polar coordinates

Using rectangular coordinate system, we firstly study the image distribution of wave function, then we will further study it in polar coordinates. The steady state schrodinger equation of the system is

$$\left[-\frac{\hbar^2}{2\mu} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \right) + \frac{1}{2} \mu \omega^2 r^2 \right] \psi = E \psi \quad (8)$$

The natural unit $\hbar=\mu=\omega=1$ is selected. Make

$$\psi(r, \varphi) = \frac{1}{\sqrt{2\pi}} R(r) [\sin(m\varphi) + \cos(m\varphi)] \quad (9)$$

Therefore,

$$\left[\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2} + (2E - r^2) \right] R(r) = 0 \quad (10)$$

In order to solve the equation (11), the two special cases of $r \rightarrow 0$ and $r \rightarrow \infty$ are studied by the analysis, we finally obtain

$$R(r) = r^{|m|} e^{-\frac{r^2}{2}} u(r) \quad (11)$$

According to Eqs. (11) and (12), it obtain

$$\frac{d^2 u}{dr^2} + \left(\frac{2|m|+1}{r} - 2r \right) \frac{du}{dr} + [2E - 2(|m|+1)] u = 0 \quad (12)$$

Make $\xi=r$, we obtain

$$\xi \frac{d^2 u}{d\xi^2} + (|m|+1-\xi) \frac{du}{d\xi} - \left[\frac{|m|+1}{2} - \frac{E}{2} \right] u = 0 \quad (13)$$

By Eq. (9), it can be obtained as

$$u(r) = F(-n_r, |m|+1, r^2) \quad (14)$$

According to Eqs. (12) (15), the wave function can be expressed as

$$\psi(r, \varphi) = \frac{1}{\sqrt{2\pi}} N_{n_r, m} r^{|m|} e^{-\frac{r^2}{2}} F(-n_r, |m|+1, r^2) [\sin(m\varphi) + \cos(m\varphi)] \quad (15)$$

3. The calculation results and discussion

3.1 The simulation of eigenfunctions for one-dimensional linear harmonic oscillator

Based on the eigenfunction of one-dimensional linear harmonic oscillator, using MATLAB software, six eigenfunction graphs of one-dimensional linear harmonic oscillator is obtained, as shown in Figure 1.

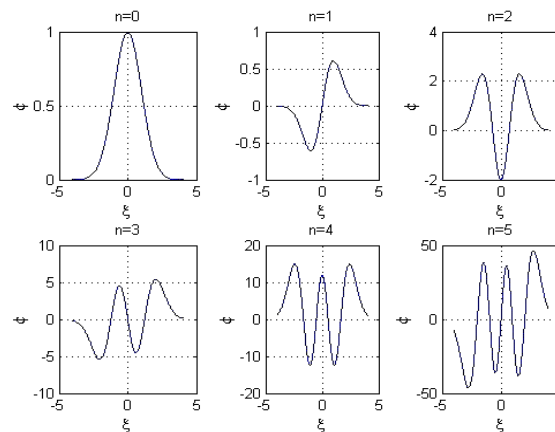


Figure 1 Eigenfunctions of one-dimensional harmonic oscillators at different energy levels.

As can be seen from figure 1, we find that the number of intersection points between the wave function and the line with $\psi=0$ is n , and the characteristics of the images are consistent with the theoretical results.

In addition, we obtain the six probability density graphs of the linear harmonic oscillator by MATLAB software, as shown in Figure 2.

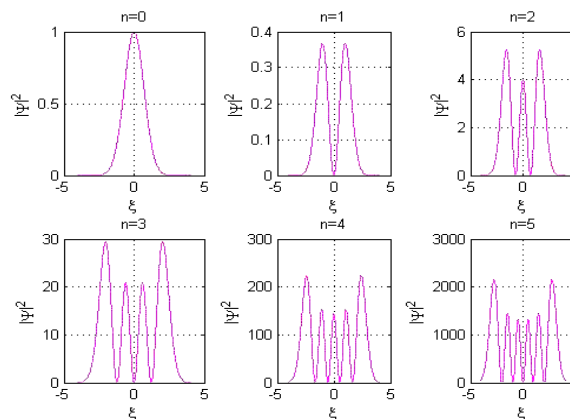


Figure 2 probability density of one-dimensional harmonic oscillator at different energy levels.

As can be seen from Figure 2, we find that the probability distribution satisfies the normalization condition. That is to say, the sum of the contained areas is 1, and the characteristics of the image are consistent with the theoretical results. With the help of the image, a complete and clear understanding of the physical significance of the probability density of one-dimensional harmonic oscillator in quantum mechanics is further obtained.

3.2 The simulation of two-dimensional linear harmonic oscillator

Based on the amplitude of two-dimensional linear harmonic oscillator, we obtain amplitude images of the linear harmonic oscillator at different energy levels using MATLAB software, as shown in Figures 3-5.

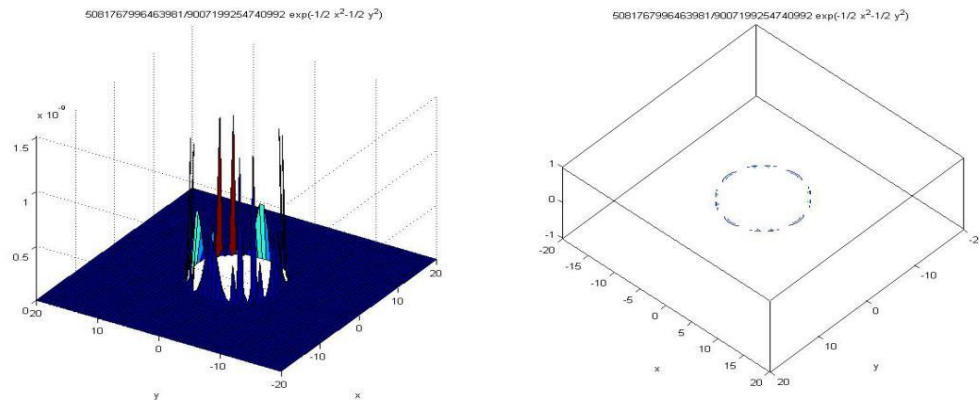


Figure 3 The amplitude and the distribution of contour at $nx=0$ and $ny=0$

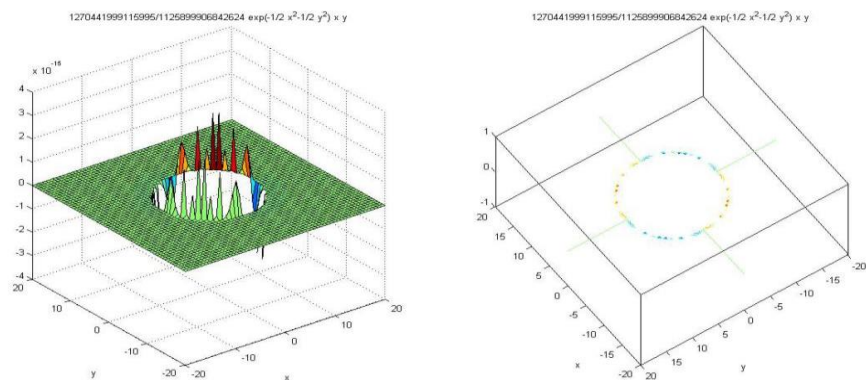


Figure 4 The amplitude and the distribution of contour at $nx=1$ and $ny=1$

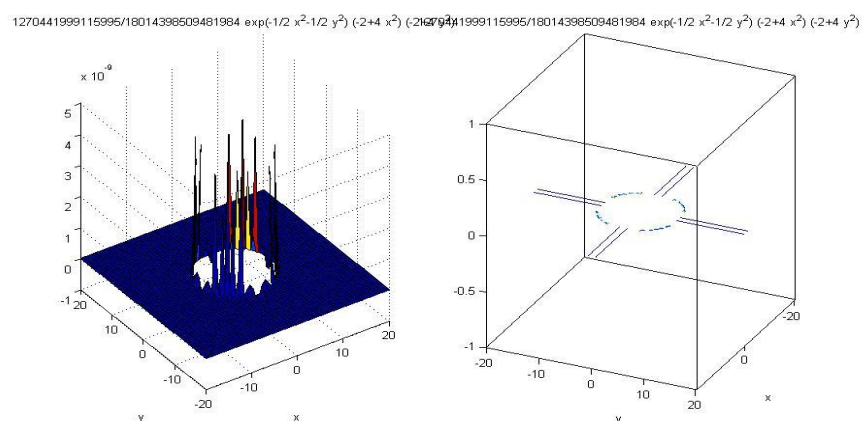


Figure 5 The amplitude and the distribution of contour at $nx=2$ and $ny=2$

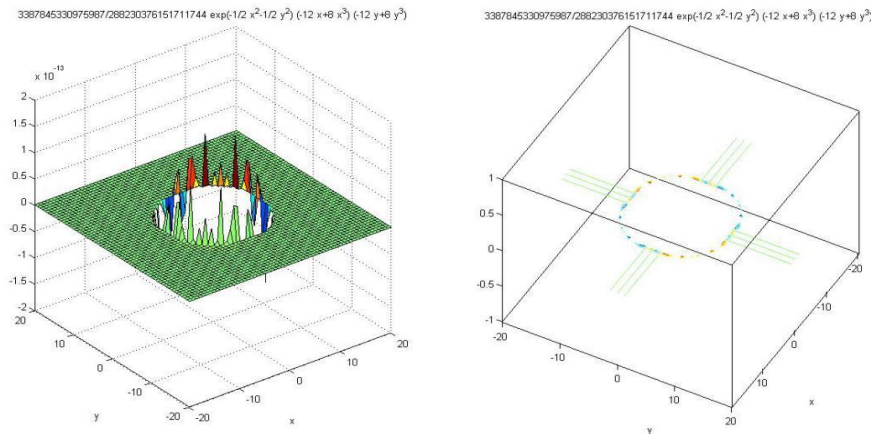


Figure 6 The amplitude and the distribution of contour at $n_x=3$ and $n_y=3$

It can be seen that from the analysis of the above four figures:

(1) When $n=0$, the corresponding amplitude and contour distribution are not degenerate, and the characteristics of amplitude and contour distribution are shown in Figure 4.

(2) When $n=1$, $n=2$ and $n=3$, the corresponding amplitude and contour distribution are not degenerate, and the characteristics of the amplitude and contour distribution are shown in Figures 5, 6 and 7.

(3) The number of intersecting lines between the contour distribution and the plane is n ; The number of amplitude from the harmonic oscillator is $n+1$.

4. Conclusion

In this paper, various characteristics of one- and two -dimensional harmonic oscillator in classical mechanics and quantum mechanics are systematically analyzed. In quantum mechanics, the eigenfunctions and probability density of one-dimensional harmonic oscillator are analyzed, and some images of the eigenfunctions and probability density of energy levels are drawn by MATLAB software. The probability distribution of one-dimensional linear harmonic oscillator in classical mechanics and quantum mechanics is compared by MATLAB. The results indicate that the number of intersection points between the wave function and a line with $\varphi=0$ is n ; The probability distribution satisfies the normalization condition that the sum of the contained areas is 1; Taking different values of n , the probability distribution function of harmonic oscillator in quantum mechanics has n different nodes, and the amplitude of harmonic oscillator in classical mechanics also changes accordingly, a comparison of the distribution probability of quantum mechanics with that of classical mechanics with φ_0 in the ground state, the two distribution curves have the opposite shape, but when n is large, probability density of quantum mechanics $|\phi_n(\xi)|^2$ local average and classical probability distribution $P(\xi)$ converge. In other words, the similarity of probability distribution of oscillators in classical mechanics and quantum mechanics increases gradually. These results are reflected in the image, and the characteristics shown in the image are consistent with the theoretical results.

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