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Analysis of Eco-epidemiological SIS Model under Sparse Effect

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Abstract. In this paper we formulated and analyzed the eco-epidemiological SIS model under sparse effect，and the sufficient condition of the local asymptotical stability of the equilibrium was studied with the method of latent root，analyze the global asymptotical stability of the boundary equilibriums, and also discuss the local asymptotical stability of the positive equilibrium，and established the sufficient condition of their stability .

1. Introduction

A large number of epidemiological dynamics models only involve the epidemic of a single population. However, in nature, the people cannot exist alone, and then they are usually competing with other species for food resources, external environment or prey on other species. Therefore, it is necessary to consider the impact of disease on the basis of population dynamics model, or the interaction of populations based on epidemic model, that is, we combine the epidemic dynamics model with the population dynamics model, so as to make the established model more realistic than the individual epidemic model or the individual population dynamics model. In [1, 2, 3],the authors discuss modeling and analysis of a predator-prey model with disease in the prey. In [4, 5, 6],the authors research eco—epidemiological stochastic model of predator with epidemic.

We discuss eco-epidemiological SIS model disease only in the spread between infected predator is death and predators, and the infected predator does not feed on prey.

We discuss the model as follows.

$$\begin{cases} \dfrac{dX}{dt} = aX^2\left(r-X\right)-bXS \\ \dfrac{dS}{dt} = eXS-d_1S-\beta SI+\delta I \\ \dfrac{dI}{dt} = cI+\beta SI-d_2I-\delta I \end{cases} \tag{0.1}$$

Where X denote the densities of the prey，S denote the densities of susceptible predator and I is the infective predator. The function $f(X)=aX^2(r-X)$ is the density - dependent function with sparasing effect, b denote the growth rate of predator for predating prey, $e=lb(0<l\le 1)$，l is the transmission coefficient,，c is the infective predator with sparssing effect，d_1 is the death rate of susceptible predator and d_2 denote the infective predator, Note that $d_1<d_2$, thinking about the

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death of illness, β is the adequate rate. This model supposed the infectious disease but with no recovery.

Theorem1: $R_3^{\ +}$ are positively invariant with respect to system.

2. The analysis of equilibrium point

The nonnegative equilibrium point of model (0.1) as follows:

$$E_0=(0,0,0), E_1=(r,0,0), E_2=\left(\frac{d_1}{e},\frac{ad_1(er-d_1)}{e^2b},0\right),$$

$$E_3=\left(\frac{r-\sqrt{r^2-4h}}{2},\frac{d_2+\delta-c}{\beta},\frac{\left[e\left(r-\sqrt{r^2-4h}\right)-2d_1\right](d_2+\delta-c)}{2\beta(d_2-c)}\right)\triangleq\left(X_1^*,S_1^*,I_1^*\right),$$

$$E_4=\left(\frac{r+\sqrt{r^2-4h}}{2},\frac{d_2+\delta-c}{\beta},\frac{\left[e\left(r+\sqrt{r^2-4h}\right)-2d_1\right](d_2+\delta-c)}{2\beta(d_2-c)}\right)\triangleq\left(X_2^*,S_2^*,I_2^*\right)$$

where $h=\dfrac{b(d_2+\delta-c)}{a\beta}$. let $d_2>c$.

when $r>\dfrac{d_1}{e}$, then E_2 exist; when $r>\dfrac{2d_1}{e}$, $2\sqrt{h}<r<\dfrac{he^2+d_1^2}{ed_1}\triangleq r^*$, then E_3 exist, when $r>r^*$, then E_4 exist.

Because when $h>\left(\dfrac{d_1}{e}\right)^2$, $r>\dfrac{2d_1}{e}$, $2\sqrt{h}<r<\dfrac{he^2+d_1^2}{ed_1}$, when E_3, E_4 exist, E_2 exist.

(2) The Conditions of Existence for equilibriums:

Where, with no conditions E_0, E_1 exist, while $r>\dfrac{d_1}{e}$, then boundary equilibrium point E_2 exist; when $r=\dfrac{d_1}{e}$, then $E_2=E_1$, when $r>\dfrac{2d_1}{e}$, $2\sqrt{h}<r<r^*$, then equilibrium point E_3 exist, when $r>r^*$, then endemic equilibrium E_4 exist. when $r=2\sqrt{h}$, then $E_3=E_4$.

(3) The the equilibrium point for local asymptotical analysis in the system .

According to the analysis of the characteristic equation about the Jacobian matrix of system of (0.1) , we have the result .

Theorem2 when $\sigma<1$, boundary equilibrium point E_1 is locally asymptotically stable, when $1<\sigma<2$, equilibrium point E_2 is locally asymptotically stable, when $\sigma>2$, $R_0<1$, equilibrium point E_3 is instable, when $R_0>1$, $\beta>\dfrac{d_2-c}{d_2+\delta-c}$, endemic equilibrium E_4 is locally asymptotically stable. Where $\sigma=\frac{eb}{d_1}$, $R_0=\frac{r}{r^*}=\frac{red_1}{he^2+d_1^2}$.

3. Global Stability

Theorem 3 In R_+^3, when $\sigma\le 1$,we have the global stability of boundary equilibrium $E_1=(k,0,0)$.

Proof In order to prove the result of system (0.1) is non dimensional change

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$$\begin{cases} \dfrac{dx}{dt} = x\left[ar^2x(1-x)-bs\right] \\ \dfrac{ds}{dt} = erxs - d_1s - \beta si + \delta i \\ \dfrac{di}{dt} = ci + \beta si - d_2i - \delta i \end{cases} \tag{3.1}$$

Constructing a Liapunov function V : $R^3_{+x} \to R$

$$V(t) = x - 1 - \ln x + s + i$$

Find the derivative along the solution of system (3.1)

$$V'(t) = x' - \frac{x'}{x} + s' + i' = -(1-x)\left[ar^2x(1-x)-cs\right] + erxs - d_1s - (d_2-c)i$$
$$\le -(1-x)\left[ar^2x(1-x)-cs\right] + (erx-d_1)s$$

Obviously, in Ω , On the right hand side of the first term must be nonpositive, Otherwise $ar^2x(1-x)-cs<0$, then it is contradiction that E_0 is "saddle - node". In Ω , $(erx-d_1)s \le (er-d_1)s$. when $\sigma<1$, for the second term , on the right hand side of it is negative. When $\sigma<1$, $V'(t)\le 0$, when $\sigma=1$, $V'(t)\le -(1-x)\left[ar^2x(1-x)-cs\right]\le 0$, on the same $V'(t)=0$, If and only if $x=1$, $s=0$. therefore, we have the result that $\lim\limits_{t\to+\infty} s(t)=0$, $\lim\limits_{t\to+\infty} x(t)=1$, then we have the result from the third equations of system(9), when $t\to\infty$, $s(t)\to 0$, $i(t)\to 0$ is satisfied . Therefore , E_1 is globally stable in Ω .

Theorem 4 when $R_0>1$, the disease becomes endemic disease, and E_4 is globally stable.

Proof When $R_0>1$, then E_4 is exist, we construct a Liapunov function

$$V(t) = w_1\left(X - X_2^* - X_2^*\ln\frac{X}{X_2^*}\right) + w_2\left(S - S_2^* - S_2^*\ln\frac{S}{S_2^*}\right) + w_3\left(I - I_2^* - I_2^*\ln\frac{I}{I_2^*}\right)$$

Where $w_i>0,(i=1,2)$, obviously we have the conclusion $V(t)\ge 0$.

We have the derivative along the solution of system (0.1) as following

$$V'(t) = -aw_1\left(X+X_2^*-r\right)\left(X-X_2^*\right)^2 + \left(w_2e-w_1b\right)\left(X-X_2^*\right)\left(S-S_2^*\right) +$$
$$\left(w_3-w_2\right)\beta\left(S-S_2^*\right)\left(I-I_2^*\right) + \delta\left[\left(I+I^*\right)-\left(S+S^*\right)\right]$$

(where $q_1 = X - X_2^*, q_2 = S - S_2^*, q_3 = I - I_2^*$)

let $w_1 = A$, $w_2 = B$, $w_3 = C$, then

$$V'(t) = -aA\left(X+X_2^*-r\right)q_1^2 + \left(Be-Ab\right)q_1q_2 + \left(C-B\right)\beta q_2q_3 +$$
$$\delta\left[\left(I+I^*\right)-\left(S+S^*\right)\right]$$

Choose the appropriate positive number A , B , C , so that $Be-Ab=0, C-B=0$, then

$$V'(t) = -aA\left(X+X_2^*-r\right)q_1^2 + \delta\left[\left(I+I^*\right)-\left(S+S^*\right)\right]$$

Existence knowledge with E_4 : $X+X_2^*-r\ge 0$, because of $\left(I+I^*\right)-\left(S+S^*\right)<0$. So $V'(t)<0$. E_4 is globally stable in R_+^3.

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