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Instability Criterion for Surrounding Rock of Water-Rich Roadway in Sandy-Gravel Stratum Based on Catastrophe Theory

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Abstract. In order to solve the problem of instability discrimination for water-rich roadway in sandy-gravel stratum, taking the displacement function and damaged area function as potential functions of the roadway surrounding rock system based on catastrophe theory, the instability criterions of the roadway in sandy-gravel stratum under seepage were established, i.e. the criterion for extremal modulus of displacement and criterion for the damaged area. Then the specific process for stability discrimination was further proposed. A PFC^{2D} numerical model for a shaft bottom roadway in a large coal mine in Xinjiang was established. The calculation results show that as seepage pressure increases, modulus of displacement and damaged area in surrounding rock present irregular fluctuations and similar change rule. In the latter stage of the increase of seepage pressure, the deformation and damage of the roadway all have experienced a sharp sudden raise. And the conclusion of instability discrimination obtained from the calculation results of catastrophic eigenvalues for modulus of displacement and damaged area are consistent with damage evolution and instability process of the surrounding rock, which verifies the feasibility and applicability of the instability criterions.

Keywords: Cusp catastrophe model; Roadway in sandy-gravel stratum; Seepage; Damaged zone; Instability criterion

1. Introduction

The Mesozoic sandy-gravel stratum is widely distributed in western China. Most of them with poor cementation are rich in water or under the confined aquifers [1] (Figure 1). Under high seepage pressure and external disturbances, the rock-water mixture inrush or water inrush into the excavation site are easily produced [2], which seriously threatens mine safety [3]. Therefore, it is of great significance to establish the instability criteria for the surrounding rock to predict and control mine disaster.

Figure 1. Water-rich surrounding rock in sandy-gravel stratum

Due to seepage effect and uncertain disturbances, the deformation and failure of roadway in sandy-gravel stratum obviously behaves nonlinear characteristics. Thus catastrophe theory will be useful for its stability prediction. Huang [4] introduced a general method for evaluating the stability of slopes and underground chambers using the cusp catastrophe theory and its application in prediction of landslide hazard. Pan and Xue [5,6] analyzed the physical process for instability of chamber caused by rock burst and the mechanism for water inrush from water-bearing faults in subsea tunnel by the cusp catastrophe model. Qin et al. proposed a sufficient and necessary condition for slope instability based on the cusp catastrophe model for landslide instability [7]. Fu et al. [8] established energy dissipation criteria, yield area criteria, etc. for surrounding rock instability in the underground engineering by using the cusp catastrophe theory.

At present, there are few reports on the instability criteria for the surrounding rock composed of discrete media under unstable seepage. In this work, the instability criterions for the water-rich roadway in sandy-gravel stratum were established based on the cusp catastrophe theory.

2. Instability Criterions for the Water-Rich Roadway in Sandy-Gravel Stratum

2.1. The Criterion for Extremal Modulus of Displacement

2.1.1. Expression of the criterion and its derivation. Under external disturbances, the seepage field of the water-rich roadway in sandy-gravel stratum will experience complex changes, and the roadway stability will be affected significantly. For theoretical analysis, it's assumed that the initial stress of the surrounding rock keeps constant, while the seepage pressure P_s and deformation changes with time t. so there is a certain mapping relation between the displacement H of each point at the roadway surface and the seepage pressure P_s. i.e. $H=f(P_s)$, which is also the displacement potential function of the roadway system.

The function $H=f(P_s)$ is expanded at $P_s=0$ using Taylor series taking to the 4th order term, one gets:

$$H=f(P_s)=\frac{1}{4!}\cdot\frac{\partial^4 H}{\partial P_s^4}\cdot P_s^4+\frac{1}{3!}\cdot\frac{\partial^3 H}{\partial P_s^3}\cdot P_s^3+\frac{1}{2!}\cdot\frac{\partial^2 H}{\partial P_s^2}\cdot P_s^2+\frac{\partial H}{\partial P_s}\cdot P_s+f(0)=a_4 P_s^4+a_3 P_s^3+a_2 P_s^2+a_1 P_s+a_0 \quad (1)$$

The above formula is marked as

$$H=a_0+\sum_{i=1}^{4}a_i P_s^i \quad (2)$$

where $a_0=f(0)$, $a_i=\frac{1}{i!}\cdot\frac{\partial^i H}{\partial P_s^i}\bigg|_{P_s=0}$.

For the formula (1), setting $P_s = x - \xi$, $\xi = a_3 / (4a_4)$, it can be rewritten as:

$$H = c_4 x^4 + c_2 x^2 + c_1 x \tag{3}$$

where $c_4 = a_4$, $c_2 = a_2 - 3a_3\xi + 6a_4\xi^2$, $c_1 = a_1 - 2a_2\xi + 3a_3\xi^2 - 4a_4\xi^3$.

Further making $H = c_4 V$, $\mu = c_2 / c_4$, $v = c_1 / c_4$, thus the formula (3) is transformed into the standard form of the potential function for cusp catastrophe model, which is

$$V(x) = x^4 + \mu x^2 + vx \tag{4}$$

where x is a state variable; μ and v are control variables. The equation for equilibrium surface is

$$V'(x) = dV / dx = 4x^3 + 2\mu x + v = 0 \tag{5}$$

And the equation for singularity set of the equilibrium surface is

$$V''(x) = d^2V / dx^2 = 12x^2 + 2\mu = 0 \tag{6}$$

Then the equation for bifurcation set can be obtained from equations (5) and (6):

$$8\mu^3 + 27v^2 = 0 \tag{7}$$

It is the critical position where the catastrophe phenomenon of the system occurs. Therefore, catastrophic eigenvalue $\Delta = 8\mu^3 + 27v^2$ can be used as a instability criterion for the surrounding rock system. When $\Delta \leq 0$, the system state may cross the bifurcation set resulting in catastrophe phenomenon[9,10]. The roadway is in an unstable state.

2.1.2. Procedure for instability discrimination. For engineering application, firstly the displacement values corresponding to k different seepage pressures are gained ,which constitutes a time series, i.e. the extremal modulus series of displacement $\{H(1), H(2), H(3), H(4), ...H(k)\}$. $H(k)$ is the maximum displacement modulus on the roadway surface corresponding to the k-th seepage pressure step, as follows:.

$$H(k) = \max\{y_1, y_2, y_3, y_4, ...y_i\} \tag{8}$$

where $y_i = \sqrt{u_i^2 + v_i^2}$ is the displacement modulus of the i-th key position; u_i and v_i are the horizontal and vertical displacements respectively.

Then a fourth-order power function that is displacement potential function for the surrounding rock system similar to formula (1) can be obtained using the least square method for polynomial fitting of $\{H(1), H(2), H(3), H(4), ...H(k)\}$. c_i will be further determined by the coefficient a_i and the catastrophic eigenvalues Δ also be calculated. Finally, a judgment on stability of the surrounding rock system can be made. This evaluation process is shown in Figure 2.
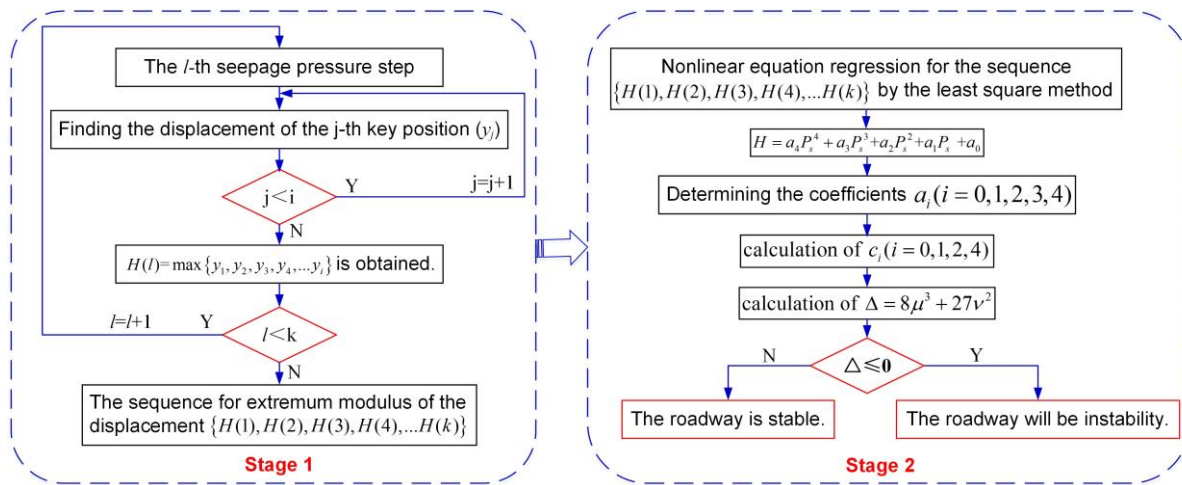
Figure 2. The evaluation process of surrounding rock stability

2.2. The Criterion for Area of Damaged Zone

2.2.1. Expression of the criterion and its derivation. The seepage field of water-rich roadway in sandy-gravel stratum changes significantly caused by external disturbances, which will not only increase the surrounding rock deformation, but also cause extension of the damaged zone. It means that nonlinear increase for area of the damaged zone may cause catastrophic instability of the roadway. Thus the stability of the roadway system can be characterized intuitively and visually by the area of damaged zone (D).

Consequently the relationship between the damaged area D and seepage pressure P_s can be described by a function $D = f(P_s)$, which also is the damaged potential function of the surrounding rock system.

Similar to formula (1), the function $D = f(P_s)$ can be written approximately as:

$$D = f(P_s) = b_4 P_s^4 + b_3 P_s^3 + b_2 P_s^2 + b_1 P_s + b_0 \tag{9}$$

Through the same transformation method, formula (9) is transformed into standard form of the potential function for cusp catastrophe model, i.e.

$$V(z) = z^4 + \mu_1 z^2 + \nu_1 z \tag{10}$$

where $\mu_1 = e_2 / e_4, \nu_1 = e_1 / e_4, e_4 = b_4, e_2 = b_2 - 3b_3\zeta + 6b_4\zeta^2, e_1 = b_1 - 2b_2\zeta + 3b_3\zeta^2 - 4b_4\zeta^3, \zeta = b_3 / (4b_4)$.

In this case, the catastrophic eigenvalue $\Delta_1 = 8\mu_1^3 + 27\nu_1^2$ can also be used as the criterion for instability of the roadway surrounding rock.

2.2.2. Procedure for instability discrimination. According to the damaged field of surrounding rock when the seepage pressure P_s changes to k different levels, the damaged area D in surrounding rock under each state can be obtained, which form the damaged area sequence $\{D(1), D(2), D(3), D(4), ...D(k)\}$.

Then the nonlinear regression of sequence $\{D(1), D(2), D(3), D(4), ...D(k)\}$ is performed using least-squares method. So formula (9) is obtained. After standardization to equation (10), the catastrophic eigenvalues Δ_1 in different states are also gained, and the stability of surrounding rock can be determined.

3. A Numerical Example and Verification

3.1. The Numerical Model and Parameters for Calculation

A coal mine is located in the northwest coalfield of Xinjiang, China, where the coal-bearing strata are mainly Mesozoic Jurassic strata. The shaft bottom roadway with buried depth of 250 m for mining level I is located in sandy-gravel stratum, and the size of this semicircular arch roadway is 5 m $\times$ 4m. The sandy-gravel stratum is 20 m in thickness, with low strength and poor cementation. The overlying stratum is water rich sandstone. The water inflow of the heading face is 25 m^3/h, which seriously affects the roadway stability.

A plane strain model of the roadway (30 m$\times$30 m) is established by PFC2D (Figure 6), in which the top, bottom, left and right boundaries are fixed. The excavation face is a permeable boundary. Horizontal and vertical ground stresses are 3.5 MPa, 5 MPa respectively. After repeated debugging the mesoscopic parameters of the model are determined in Table 1. The distribution characteristics of the deformation and damaged zone in surrounding rock were calculated under different seepage pressures.
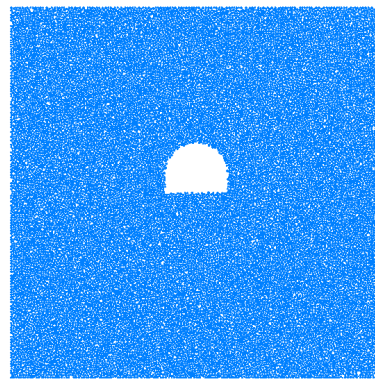

Figure 3. Numerical model

Table 1. Mesoscopic parameters for the PFC model

Minimum radius of particles (mm)	Particle size ratio	Particle density (kg.m^{-3})	Particle contact modulus (GPa)	Particle stiffness ratio	Particle friction coefficient	Parallel bond modulus (GPa)	Parallel bond stiffness ratio	Normal bond strength (MPa)	Tangential bond strength (MPa)
80	1.5	2500	4.0	1.3	0.3	4.0	1.3	1.2±10	2.4±10

3.2. Results and Discussion

After image recognition for the numerical results by image analysis system, the extremal modulus of displacement and damaged area of the roadway surrounding rock under different seepage pressures were extracted, as shown in Figure 4. Both the modulus of displacement and damaged area show irregular fluctuations with similar change rule. As the seepage pressure increases, they experience a short increase and then decrease rapidly. When P_s = 0.7 MPa, they all drop to the minimum. Then the modulus of displacement and damaged area climb sharply and peak (64mm, 129.44 m^2). After exceeding peak value, they decline to a lower value. The curves show that in the later stage of the increase of seepage pressure, the deformation and damage of the roadway surrounding rock have a sharp sudden increase, which may lead to instability and failure of the roadway.
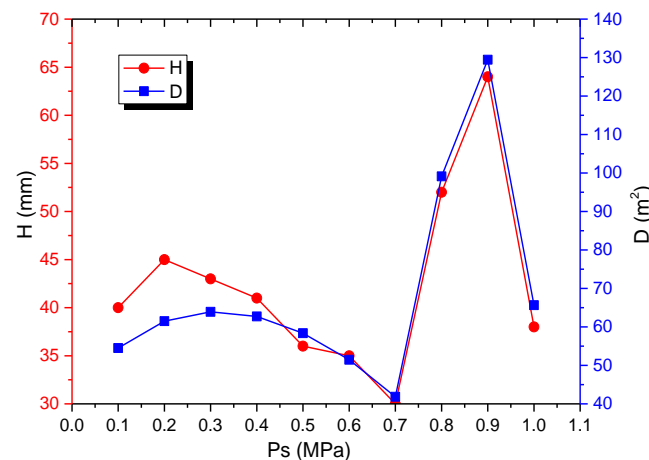
Figure 4. Change for the modulus of displacement and damaged area

Applying the method in Section 2 to process and calculate the above data for modulus of displacement and damaged area, the parameters in cusp catastrophe model were obtained, as shown in Table 2. When $P_s < 0.9$, the catastrophe eigenvalues of the modulus of displacement and damaged area are both greater than 0, which indicates that the roadway is stable. While $P_s = 0.9$, $\Delta < 0$ and $\Delta_1 < 0$, it can be judged that the roadway is in instability. At the moment of instability, the damaged area of surrounding rock reaches 129.44 m², which is 7.48 times of the sectional area of the roadway.

Table 2. Parameters in the cusp catastrophe model

P_s	0.5	0.6	0.7	0.8	0.9	1.0
c_4	-4.17e+3	0.000	-9.47e+2	1.21e+3	-1.52e+2	-1.63e+3
c_2	51.667	4.12e+16	44.896	-1.02e+2	5.89e+2	3.70e+2
c_1	-21.400	3.55e+30	-29.871	26.567	5.82e+2	9.630
μ	-0.012	-2.50e+28	-0.047	-0.084	-3.885	-0.227
v	0.005	-2.15e+42	0.032	0.022	-3.842	-0.006
e_4	-1.76e+2	-56.458	-1.13e+2	4.50e+3	3.42e+2	-3.67e+03
e_2	-1.52e+2	-93.343	-1.29e+2	-3.09e+2	-8.75e+2	8.27e+02
e_1	-47.671	-1.24e+2	-77.229	-52.460	6.92e+2	47.859
μ_1	0.863	1.653	1.139	-0.069	-2.558	-0.225
v_1	0.270	2.203	0.681	-0.012	2.021	-0.013
Δ	0.001	0.000	0.026	0.008	-70.719	-0.093
Δ_1	7.113	1.67e+2	24.315	0.001	-23.564	-0.087
Status	Stable	Stable	Stable	Stable	Instability	Instability

When P_s increases to 0.9 MPa, instability occurs in the roadway, and the area of damaged zone also has a step raise (Figure 5). Meanwhile, the damaged area is about 3.09 times of that at $P_s = 0.7$ MPa before instability, especially the damage and failure in roof and floor of the roadway are serious, which proves that the instability of surrounding rock has indeed occurred at that moment. Therefore, this is consistent with the discriminating results obtained from the instability criterions that are established based on cusp catastrophe model.
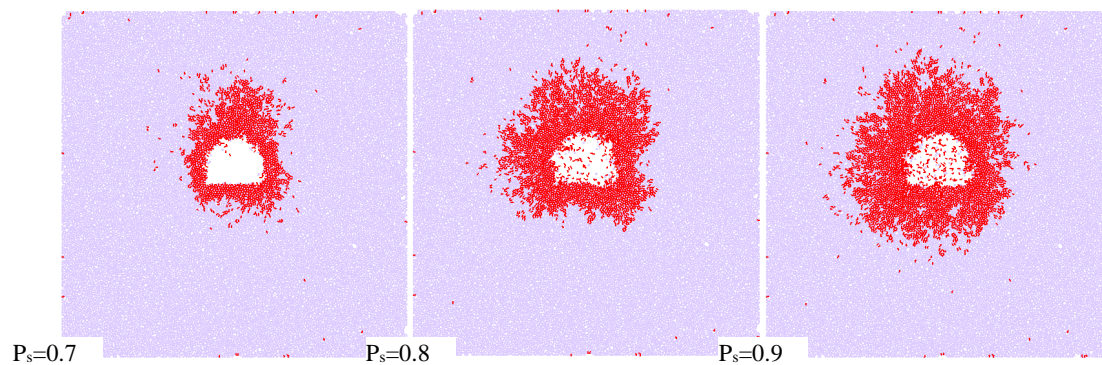
P$_s$=0.7   P$_s$=0.8   P$_s$=0.9

Figure 5. Distribution of damaged zone before instability of the surrounding rock

4. Conclusions

(1) The instability criterions of the water-rich roadway in sandy-gravel stratum under seepage were established, i.e. the criterion for extremal modulus of displacement and criterion for the damaged area. A method and its process by detection data or numerical results for stability discrimination of water-rich roadway in sandy-gravel stratum were further proposed.

(2) A PFC^{2D} numerical model for a shaft bottom roadway was established, then the modulus of displacement and damaged area of surrounding rock were obtained. As seepage pressure increases, they present irregular fluctuations and similar change rule. In the latter stage, the deformation and damage of the roadway surrounding rock all have experienced a sharp sudden increase.

(3) The calculation results show that when P$_s$ = 0.9, $\triangle < 0$ and $\triangle_1 < 0$. According to the criterions established, instability of the roadway can be determined. Thus the state of surrounding rock changed abruptly. This is consistent with damaged zone evolution and simulation of instability process, so the feasibility and applicability of the criterions were verified.

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