On Dynamic Constitutive Model of Granite Under Impact Loading: Effect of Damage on Dynamic Strength

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Abstract. Dynamic mechanical properties of granite were tested by a split Hopkinson pressure bar system at constant strain rates. The experimental results show that granite behaves linearly elastic at the initial loading stage, followed by a significant plastic deformation after yielding, and yield strength, ultimate strength and elastic modulus are all rate dependent. On this basis, the simplified Zhu-Wang-Tang (ZWT) constitutive model is applied in this paper, assuming elastic and high frequency viscoelastic elements only. The damage evaluation equation is introduced to the simplified ZWT model to investigate the effect of damage on the dynamic strength of granite due to impact load. Finally, the proposed model is applied to fit the experimental data. It is known that the fitted stress-strain curves at different strain rates are in agreement with the experimental ones. Compared with experimental data, the error of yield stress, peak stress and corresponding strain obtained by fitting method are all not larger than 10%. This demonstrates that the proposed dynamic constitutive model can accurately describe the dynamic mechanical properties of granite under impact loading.
Keywords: granite; SHPB; dynamic mechanical properties; ZWT model

1. Introduction

Dynamic loadings, such as blasting and impact, lead to a different rock deformation mechanism, compared to hydrostatic loadings. In the past sixty years, dynamic mechanical properties of rocks have drawn much attention and a large number of experiments have been performed to investigate rock behavior under impact loadings [1-7]. Due to the complexity of rock properties and the transience of dynamic loads that were employed in experiments, the stress-strain curves obtained from the experiments generally demonstrate diverse mechanical behavior, which may not be representative and therefore not be useful for engineering. In order to better understand the mechanical properties and failure mechanism of rocks under dynamic loadings, it is of great practical significance to construct a constitutive model which can describe the dynamic properties of rock materials reasonably and succinctly.

At present, many achievements have been made in the dynamic constitutive theory of rocks. Based on the results dynamic mechanical properties, Kinoshita Shigenori et al. [8] suggested that Bingham model (i.e. overstress model) can be used to describe the dynamic mechanical properties of rock. On this basis, Yu et al. [1] proposed a modified overstress model, and determined the constitutive parameters for different rocks, which have clear physical meanings. However, the modified overstress model could not describe the characteristic variation of elastic modulus with the loading rates. Zheng et al. [9] introduced the damage variable into the viscoelastic constitutive model. In this model, the rate dependent strength and elastic modulus can be described by the use of parallel Maxwell models at

different relaxation time. However, the model may be too complex for engineering practice, as many parameters in the model need many experiments to determine. Based on analysis of measured dynamic constitutive curves, Shan et al. [10] proposed a time dependent damage model for rock properties under dynamic loading, involving a statistic damage model and a viscoelastic model. Zhao et al. [11-12] modified the Zhu-Wang-Tang model by replacing the elastic component with damage model to establish a damage viscoelastic dynamic constitutive model.

In this paper, we report experimental data on stress-strain behavior of granite under impact load, employing strain rates 19.1/s-190.5/s. The experiments were performed using SHPB system at constant strain rates. Based on the experimental data, we simplify the ZWT model and construct a damage dynamic constitutive model that considers the effect of damage on the rock failure under compact loadings.

2. Split Hopkinson Pressure Bar Test

The granite samples used in this study were collected from a quarry located at Changping district, Beijing. The cylindrical specimens were made from the granite blocks using a diamond drill. The samples used in the experiments are 50mm in diameter and 40mm in length. Both end surfaces were finely polished to ensure that the roughness did not exceed 0.02mm.

The experiments are performed at constant strain rates using a special designed Hopkinson system in the SHPB laboratory, China University of Mining and Technology (Beijing) [14]. Compared to the conventional split Hopkinson pressure bar, a cone-cylinder striker is designed in our system to ensure a better control of constant loading [13,14]. The striker is made of 7075 aluminum-magnesium alloy whose wave impedance is close to that of the granite specimens used in the experiments. The diameters of two end surfaces of the striker are 35mm, 50mm, respectively, and the length is 400mm. The diameters of incident bar and transmission bar are both 50mm and the length are 2000mm. The schematic diagram of the SHPB system used in this paper is shown in Fig. 1.

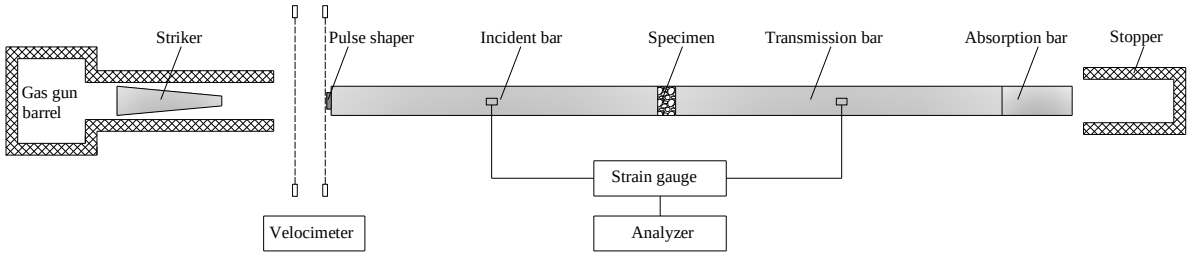


Figure 1. Schematic diagram of SHPB experimental system

3. Dynamic Mechanical Properties of Granite

3.1. Dynamic Mechanical Analysis

In order to study the mechanical response of granite at different strain rates, the uniaxial impact tests were conducted on granite samples at 26 impact velocities, which demonstrate 130 experimental data sets in final. The experimental data covers the strain rates from 19.1/s to 190.5/s. Fig.2 is the stress-strain curves of granite specimens tested at constant strain rates of 39.1/s, 67.6/s, 100.1/s and 131.9/s. As shown in Figure 2, the granite samples show linearly elastic behavior in the initial stage that strain is lower than 0.0005. It also shows, in the early (elastic) stage, the higher the stain rates, the better the linearity and the larger the dynamic elasticity modulus, in general. As strain rate is higher than 100/s, the stress fluctuates several times after the onset of the plastic deformation of the sample. This finding is consistent with the conclusion drawn by Shan et al. [4]. In the strain range of 60/s-190/s, the sample behaves as the ideal plasticity. The yield stress increases with an increase of strain rate, indicating that the granite exhibits obvious work hardening behavior.

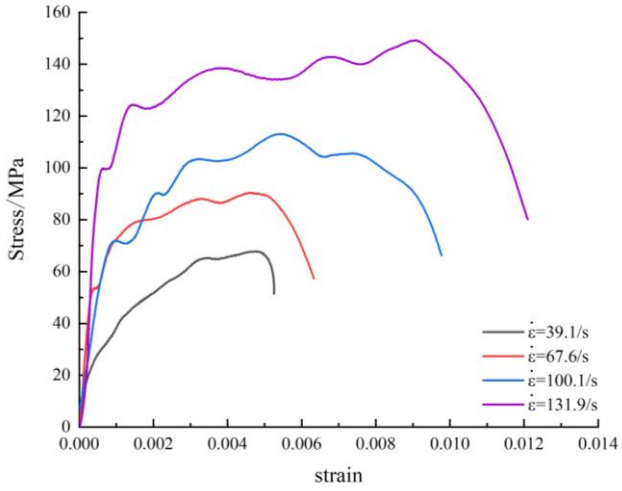


Figure 2. Stress-strain curve of granite specimens

Dynamic peak stress of granite against strain rates is shows in Fig. 3. The peak stress of the samples increase with the increase of strain rate, showing a significant rate dependence. It is found that, in the range of strain rate tested (19.1/s-190.5/s), peak stress may be well fitted to a power law of strain rates (see Figure 3). The fitted relation yields:

$$\sigma = -2547 + 2385\dot{\varepsilon}^{0.024} \tag{1}$$

Figure 4 shows the peak strain measured at different strain rates in the experiments. The peak strain increases with the increase of the strain rate, yielding a linear relation that can be expressed as

$$\varepsilon = (10.7 + 0.082\dot{\varepsilon}) \times 10^{-3} \tag{2}$$

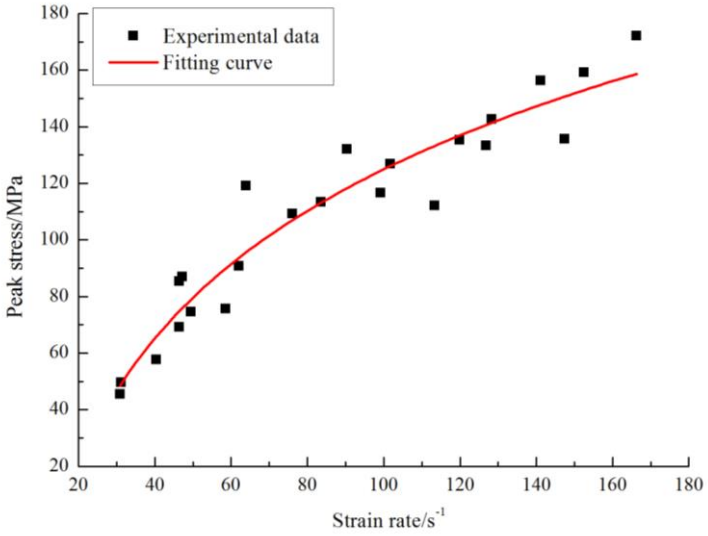


Figure 3. Peak stress - Strain rate curve

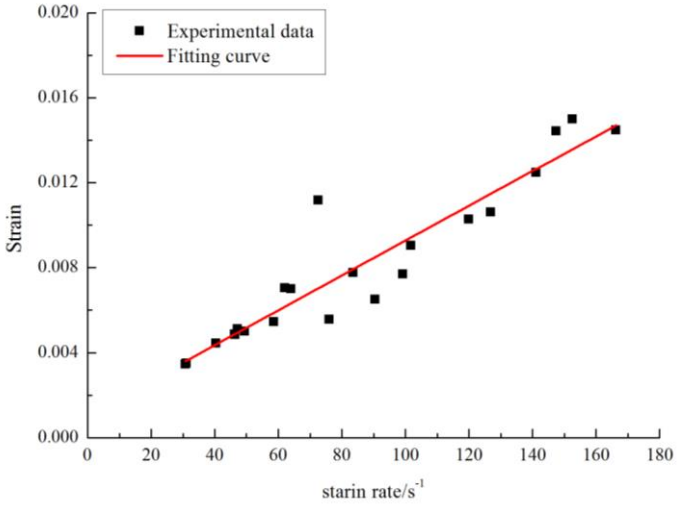


Figure 4. Strain rate- strain relationship

3.2. Damage Evaluation of Granite

Defects in rock materials may lead to the damage softening effect, and therefore the internal damage and its evolution may play a role in mechanical properties of rocks under impact loads. Recently, many variables have been proposed to describe the extent of damage. For engineering application, damage variable is generally defined at a macro level, that is, the macroscopic parameters of rocks are measured to evaluate the extent of damage. These macroscopic mechanical parameters include elasticity modulus, acoustic velocity, and density, etc. In this paper, we use the reduction of acoustic velocity of rock to characterize damage, yielding

$$D = 1 - \frac{\tilde{V}_p^2}{V_p^2} \tag{3}$$

Where V_p is acoustic velocity of intact rock materials, and $\tilde{V}_p$ is the acoustic velocity of damaged rock materials.

The acoustic velocity of intact granite samples and damaged granite samples were measured, respectively, by means of a RSM-SY5 instrument. Inserting the obtained values into equation (3), we obtained the values of the damage variable (D) for the samples at different strain rates. The damage values versus different peak stress are plotted in Fig. 5. It is shown that the damage value increases with the increase of peak stress, maybe yielding an exponential function expressed as

$$D = 0.144 \exp\left(\frac{\sigma}{60.19}\right) - 0.332 \tag{4}$$

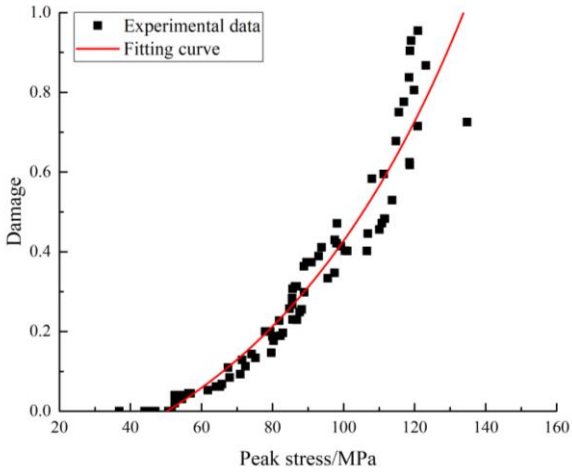


Figure 5. Relationship between granite damage and peak stress

It can be seen from equation (4), that σ=50.2MPa, as D=0, indicating that if the peak stress caused by impact load is less than 50.2MPa, the granite sample will not be damaged. Similarly, if the peak stress exceeds 133.9MPa, i.e. D equals to 1, the granite sample will be fractured completely. On this basis, we define a peak stress threshold for damage as σ_0=50.2Mpa. Following the same approach and using equations (3) and (4), we also define the strain rate and strain thresholds at which points the sample starts damaging. The strain rate and strain thresholds yield 31.7/s, 0.37%, respectively.

4. Dynamic Constitutive Model of Granite

The stress-strain curves of granite under impact loading show significant work hardening and plastic flow characteristic, which can be accurately described by the Zhu-Wang-Tang (ZWT) model. However, rock is a heterogeneous material, and its interior contains a large number of defects, such as pores, micro cracks, other weak materials, etc. The rheological process of rock is frequently accompanied by the expansion of internal defects. Therefore, under impact loading, the impact of damage evaluation on rock failure could not be neglected and the ZTW model should be improved to establish a dynamic constitutive model for granite.

4.1. Simplification of ZWT Constitutive Model

Zhu et al. [16] proposed a viscoelastic constitutive equation with two viscous models to study dynamic mechanical behavior of epoxy resin under one-dimensional stress state. This equation is called ZWT constitutive model, and its expression is as follow:

$$\sigma = f_e(\varepsilon) + E_1\int_0^t \dot{\varepsilon}(t)\exp(-\frac{t-\tau}{\varphi_1})\mathrm{d}\tau + E_2\int_0^t \dot{\varepsilon}(\tau)\exp\left(-\frac{t-\tau}{\varphi_2}\right)\mathrm{d}\tau \tag{5}$$

Where $f_e(\varepsilon) = E_0\varepsilon + \alpha\varepsilon^2 + \beta\varepsilon^2$, here,$\alpha$ and β are elastic constants.

The ZWT model consists of two Maxwell fluid models and a nonlinear elastic model in parallel, as shown in Fig. 6. Maxwell fluid model *C* describes the viscoelastic response of materials at low stain rates, while *B* for high strain rates. Generally, the relaxation time of φ_1 is from 10^0 to 10^2s^{-1}, and the relaxation time of φ_2 is from 10^{-6} to 10^{-4}s^{-1}[17]. However, the time scale of SHPB experiments are in the range of 10^{-6} to 10^{-4}/s so that the low frequency Maxwell model cannot describe the mechanical behavior of the samples observed in the experiments. As a result, the low frequency Maxwell model can be simplified as a spring with an elastic constant, E_1, and the equation (5) can be rewritten as

$$\sigma = f_e(\varepsilon) + E_1\varepsilon + E_2\int_0^t \dot{\varepsilon}(\tau)\exp\left(-\frac{t-\tau}{\varphi_2}\right)\mathrm{d}\tau \tag{6}$$

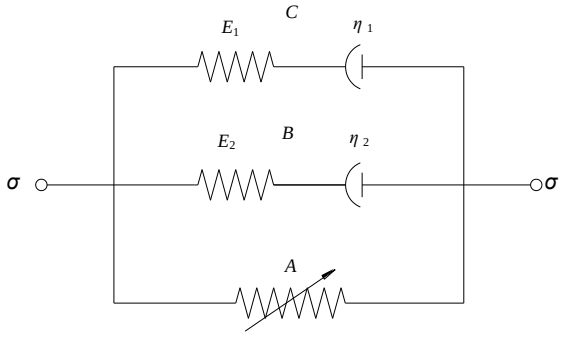


Figure 6. Z-W-T model

The quasi-static compression tests indicate that granite behaves near-linearly elastic in the early stage, and the magnitude of maximum strain is about 10^{-2}. So the term $f_e(\varepsilon)$ can be simplified as $f_e(\varepsilon)=E_0\varepsilon$. Assuming $E_a=E_0+E_1$, equation (6) becomes

$$\sigma=E_a\varepsilon+E_2\int_0^t\dot{\varepsilon}(\tau)\exp\left(-\frac{t-\tau}{\varphi_2}\right)\mathrm{d}\tau \tag{7}$$

Furthermore, the strain rate employed in the experiments is constant, i.e. $\dot{\varepsilon}(\tau)$=constant, so that equation (7) can be rewritten as

$$\sigma=E_a\varepsilon+E_2\varphi_2\dot{\varepsilon}[1-\exp(\frac{-\varepsilon}{\dot{\varepsilon}\varphi_2})] \tag{8}$$

4.2. Establishment of a Damaged ZWT Constitutive Model

Development of internal damage such as micro-cracks often leads to rock failure/damage. Following the strain equivalence principle proposed by Lemaitre, et al. [18], we may establish a damage constitutive equation of granite written as

$$\sigma_a=(1-D)\sigma_r \tag{9}$$

Here σ_a is effective stress, σ_r is mean stress, and D is damage variable.

Combining equations (8) and (9), we obtain a constitutive model that involves both viscoelastic and damage effects of granite, which yields,

$$\sigma=(1-D)\{E_a\varepsilon+E_2\varphi_2\dot{\varepsilon}[1-\exp(\frac{-\varepsilon}{\dot{\varepsilon}\varphi_2})]\} \tag{10}$$

The SHPB experimental results show that the number of cracks in specimen increases with the increase of strain rate. In other words, the damage evaluation is closely related to the strain rate. Assuming the damage evolution of granite follows a promoting thermal activation process [19], the damage variable D may be represented by the rate evaluation law as follow

$$D=K_D\int_0^t\dot{\varepsilon}^\delta dt \tag{11}$$

In the case of a constant loading rate, we assume a strain rate threshold that results in the occurrence of damage. Then, integrating equation (11) may represent the damage evaluation process of granite

$$D=\begin{cases}0 & \varepsilon\le\varepsilon_0\\ K_D\dot{\varepsilon}^{\delta-1}(\varepsilon-\varepsilon_0) & \varepsilon>\varepsilon_0\end{cases} \tag{12}$$

Where, K_D and δ are the dynamic response parameters of rocks. According to the second contents, the strain rate threshold of granite in this paper is ε_0 =3670$\mu\varepsilon$.

Substituting equation (12) into (10) gives

$$\sigma = [1 - K_D \dot{\varepsilon}^{\delta-1}(\varepsilon - \varepsilon_0)]\{E_a \varepsilon + E_2 \varphi_2 \dot{\varepsilon}[1 - \exp(\frac{-\varepsilon}{\dot{\varepsilon}\varphi_2})]\} \tag{13}$$

Equation (13) may represent the dynamic constitutive model of granite. This model contains five parameters of K_D, δ, E_a, E_2 and φ_2, which are concise and have clear physical meaning. Therefore this model would be easy to apply to engineering practice.

5. Application of Dynamic Constitutive Model

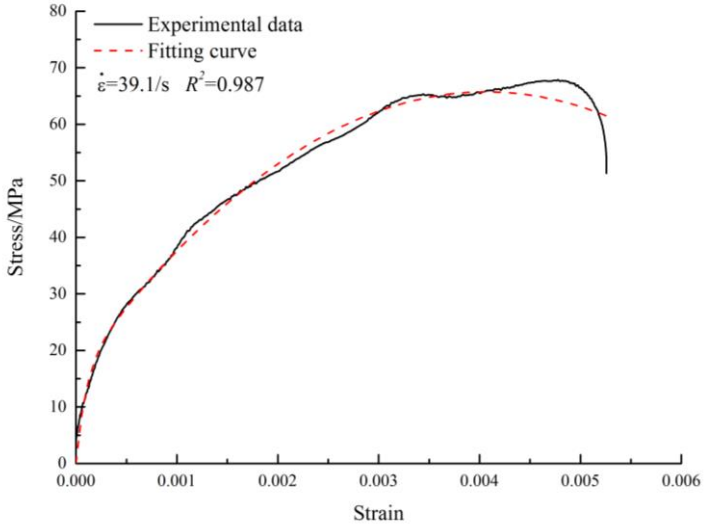
We now use equation (13) to fit the stress-strain curves of granite measured at four different strain rates, by means of the least square methods. The best fitting is shown in Fig. 7 and the fitting parameters are listed in Table 1.

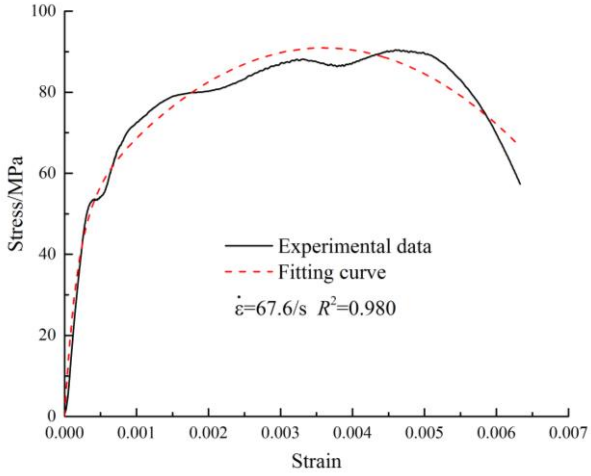
Table 1. Constitutive model parameters

$\dot{\varepsilon}$/s^{-1}	K_D	α	E_a/GPa	E_2/GPa	φ_2/μs
39.1	90.6	0.211	15.2	93.0	2.6
67.6	70.2	0.239	17.0	161.7	2.6
100.1	50.2	0.165	18.9	133.8	2.6
139.1	60.1	0.057	19.0	198.4	2.6

Fig. 7 shows that the fitting curves are in good agreement with the curve obtained from experiments, especially before rock sample yielding. The correlation coefficients (R^2) of all fitting curves are greater than 0.95. It is known that the parameter E_a which characterizes the effective elastic response increases with the increase of strain rate. We compared the relative errors of the yield stress and the peak stress obtained from fitting method and experimental data, neither of which exceeded 10%.

In summary, the proposed dynamic constitutive model can well describe the linear elastic properties of granite before yielding and accurately reflect the rate dependence of elasticity modulus and dynamic strength. Note that the model in this paper cannot accurately describe the fluctuation of stress-strain curves after yielding under high-speed impact, which may be related to less parameters in the model.


Experimental data
Fitting curve
$\dot{\varepsilon}$=39.1/s R^2=0.987
Stress/MPa
Strain
0
10
20
30
40
50
60
70
80
0.000
0.001
0.002
0.003
0.004
0.005
0.006


Experimental data
Fitting curve
$\dot{\varepsilon}$=67.6/s R^2=0.980
Stress/MPa
Strain
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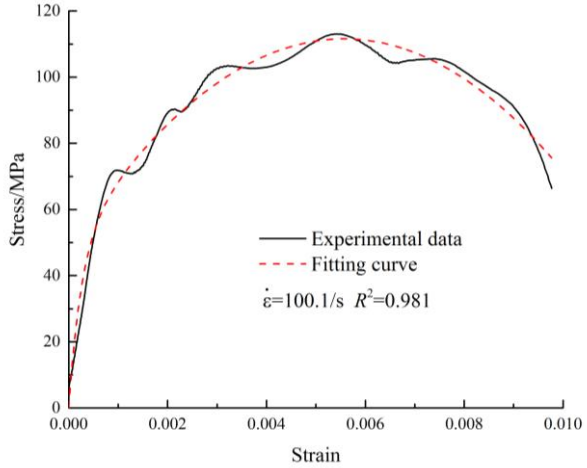
0.007


Experimental data
Fitting curve
$\dot{\varepsilon}$=100.1/s R^2=0.981
Stress/MPa
Strain
0
20
40
60
80
100
120
0.000
0.002
0.004
0.006
0.008
0.010

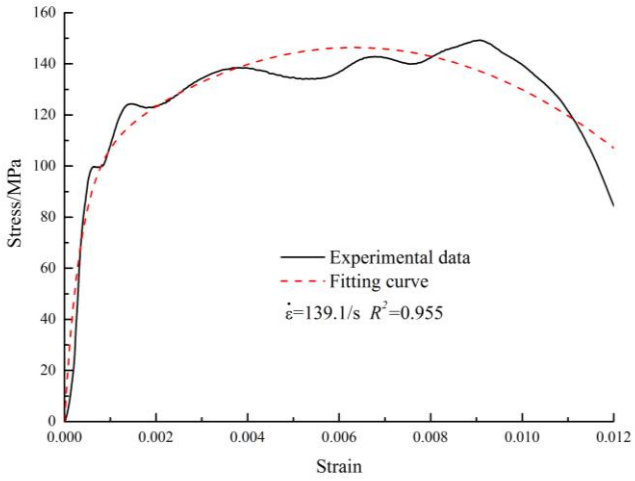


Figure 7. Stress-strain curve of granite at different strain rates

6. Conclusions

We performed experiments on granite under impact loading at constant strain rates using a constant strain rate SHPB system. Dynamic mechanical properties of granite at different strain rates were investigated. The experimental results illustrate that the granite samples show linear elastic behavior at the initial loading stage followed by significant strain hardening behavior. All dynamic parameters (such as elastic modulus, dynamic strength) are strain rate dependence. Based on the experimental findings, we improved the ZWT nonlinear viscoelastic constitutive model to describe the dynamic mechanical behavior of granite. The proposed model is concise in form and its parameters are easy to obtain. More importantly, the proposed model involves the effect of damage evaluation on dynamic strength of granite, and can well explain our experimental data.

References

[1] Yu Yalun, Jin Kexue. Study on the dynamic behavior of magnetites and granite under high strain rates [J]. Explosion and Shock wave, 1990,10(3):266-271.
[2] Dong Zhaoxing, Shan Renliang. Study on constitutive properties of rocks under high strain rates [J]. Engineering Blasting, 1999,5(2):5-9.
[3] Xi Daoying, Zheng Yonglai, Zhao Tao. The experimental research in dynamic constitutive laws of marble and shale [J]. Explosion and Shock wave, 1995,15(3):259-266.
[4] Shan Renliang, Chen Shilin, Li Baoqiang. Experimental study of granite constitutive properties under uniaxial impact [J]. Explosion and Shock wave, 2000,20(1):32-38.
[5] Xin Litian, He Xiang, Su Min. The mechanical properties of rocks under strong impact loading [J]. Chinese Journal of Geotechnical Engineering, 1996,18(6):61–68.
[6] Yang Jun, Gao Wenxue, Jin Qiankun. Experiment on dynamic damage property of rock and new damage model for rock fragmentation by blasting [J]. Chinese Journal of Rock Mechanics and Engineering, 2001,20(3):320-323.
[7] Wang Xuebin. Effect of loading rate on entire deformational characteristics of rock specimen [J]. Rock and Soil Mechanics, 2008, 29(2):353-358.
[8] Kinoshita Shigenori, Sato Kazuhiko, Kawakita Minoru. On the Mechanical Behaviour of Rocks under Impulsive Loading [J]. Bulletin of the Faculty of Engineering Hokkaido university, 1977,83:51-62.
[9] Zheng Yonglai, Xia Songyou. Viscoelastic damage constitutive model for rock [J]. Chinese Journal of Rock Mechanics and Engineering, 1996,15(s1):428-432.
[10] Shan Renliang, Xue Yousong, Zhang Qian. Time dependent damage model of rock under dynam-

ic loading [J]. Chinese Journal of Rock Mechanics and Engineering, 2003,22(10):1771-1776.
[11] Zhao Guangming, Xie Lixiang, Meng Xiangrui. A constitutive model for soft rock under impact load [J]. Explosion and Shock wave, 2013,33(2):126-132.
[12] Xie Lixiang, Zhao Guangming, Meng Xiangrui. Research on damage viscoelastic dynamic constitutive model of soft rock and concrete materials [J]. Chinese Journal of Rock Mechanics and Engineering, 2013,32(4):857-864.
[13] Tao Junlin, Tian Changjin, Chen Yuze, et al. Investigation of experimental method to obtain constant strain rate of specimen in SHPB [J]. Explosion and Shock wave, 2004,24(5):413-418.
[14] Liang Shufeng. The study on damage evolution and constitutive model of the granite under constant strain rate impact [D]. Beijing: China University of Mining and Technology (Beijing), 2016.
[15] Wang Lili, Shi Shaoqiu, Chen Jiangying. Research and application of ZWT nonlinear thermal viscoelastic constitutive model [J]. Journal of Ningbo University (NSEE), 2000(B12):141-149.
[16] Zhu Zhaoxiang, Xu Daben, Wang Lili. Thermoviscoelastic constitutive equation and time-temperature equivalence of epoxy resin at high strain rates [J]. Journal of Ningbo University, 1988(1):67-77.
[17] Chen Jiangying, Wang Lili. Rate-dependent constitutive equation of cement mortar [J]. Journal of Ningbo University, 2000,13(2):1-5.
[18] Lemaiter J, Sermage J P, Desmorat R. A two scale damage concept applied to fatigue [J]. International Journal of Fracture, 1999,97(1):67-81.