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Mathematical Model for Mangrove Protections Toward Nearshore Process

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Abstract. This research developed a mathematical model to evaluate positive effects of mangroves to nearshore process. Particular purpose of the study was to conduct the analytical calculation for the steady condition of wave energy density and wave heights distributions due to the presence of mangroves. The method used was an expansion of the equations for nearshore quantities from the general condition to the one with the existence of mangroves. First, the wave energy density was calculated analytically by solving the wave energy conservation equation. The analytic solution for energy quantity is a function of position variable, involving longshore currents component and other physical constant parameters. The next step was to insert the dissipation of wave energy flux due to wave breaking into another energy balance equation involving mangroves effects. The mangroves contribution is represented by the dissipation due to the existence of the plants in the water. Other quantities included in the model are the wave group velocity and the dissipation due to bottom friction flux. Then the wave heights distribution concerning the mangroves effect was derived from the wave energy density function governed.

Keywords: mangroves, wave heights, nearshore, mathematical model, analytic solution

1. Introduction

Beach vegetations such as mangroves have been well known providing the natural ability to protect the coastal areas and their communities by reducing the nearshore wave heights and lessen the sediment movements. However, the quantitative approach for investigations to determine the advantages of mangroves and muds in their growing areas towards the nearshore protection and other benefits in a broader context, has rarely been conducted. To fulfil the lack of investigations in this field, this research develop a mathematical model to evaluate the positive effects of mangroves to the nearshore process.

The mathematical modelling approach is an efective way in the effort to know the amount of protection provided by the mangrove plants by reducing coastal process caused by waves, which in turn will become input for the solution to the coastal hazards leading to coastal disaster. Particular purpose of the study was to conduct an analytical calculation of wave energy density and wave heights distributions due to the presence of mangroves. These wave quantity functions may also help as evaluation tools to the results of numerical methods as the commonly selected methods applied for solving problems. The method used was an expansion of the nearshore problem from the general condition to the one with existence of mangroves. The development of mathematical model was



conducted, the problem was solved analytically by implementing the mathematical integration methods to solve the differential equation problems. Here the steady case was solved.

2. The model formulation and methods

In this research the wave quantities in the direction parallel to the shore line was assumed to be uniform. The domain of the mathematical model developed is an area in the two dimensional coordinates (x,z) , which are the off-onshore with a sloping bottom configuration and vertical directions. The origin of the coordinates was selected at a certain offshore position of the still water surface. Water depth distribution $h(x)$ along the interval was given as the input for the model. First, the wave quantities were governed without the involvement of mangroves. The wave energy density was calculated analytically by solving the wave energy conservation equation. The equation was accompanied by some initial conditions obtained at the origin position. The second step was to insert the dissipation of the wave energy flux due to wave breaking into another energy balance equation involving mangroves effects. The mangroves contribution is represented by the dissipation flux due to the existence of the plants in the water. Other quantities included in the model are the wave group velocity, and the dissipation due to bottom friction flux. Then the wave heights concerning the mangroves effect were derived from the wave energy function governed by using the famous Snel's law for sloping bottom, and also by considering the wave breaking criterion.

2.1. The domain and variables

Consider two dimensions horizontal xz -plane. x refers the on-offshore axis and z be the distance measured upwards. Let $\eta = \eta(x)$ be the mean water level elevation measured above the still water level and $h = h(x)$ is defined as the depth of the water measured from the reference axis to the bottom. If 0 is a reference position selected in the sea and a is the position where bottom slope s starts, let $*$ symbolize the sloping related notation such that for the initial constant, the water depth is written as $h_0^* = h_0 + sa$. Let $\theta = \theta(x)$ represents the angle between the incident wave with the positive x -axis, $E = E(x)$ stands for the energy with c is the wave phase velocity, c_{gx} is the group velocity component in x -direction of the group velocity c_g , $H = H(x)$ is the wave height, g is the acceleration of gravity, and ρ is the fluid density. Moreover, let ω be the angular frequency of incident wave, and k be the wave number. The depth averaged flow velocity components in along shore direction is represented by $v = v(x)$.

2.2. The change of wave field due to mangroves

The change of wave field was calculated by using the wave energy conservation equation. Here, the total energy in mangrove plantation area is denoted by E_w with the subscript w stands for waves, the dissipation of wave energy flux due to wave breaking is D_b , the dissipation due to bottom friction flux is D_f , and the dissipation flux due to the plants in water is D_v . The model from [1] for the energy balance is given as

$$d_x(E_w c_g) = -D_b - D_f - D_v. \quad (1)$$

In this research, we proposed the analytical forms of the right hand side terms of equation (1), namely the dissipation terms D_b , D_f and D_v . From the calculation for energy density function, the wave heights quantity could be determined by using the relations formulated in linear wave theory as

$$E = 1/8 \rho g H^2,$$

and the breaking criterion for the breaking constant γ which is defined as

$$H = \gamma(h + \eta).$$

3. The analytic solution

The analytic solution is found by solving the equation (1) analytically by using an integration method to solve the ordinary differential equation.

3.1. The analytic energy density function

The analytic functions of nearshore energy quantity for steady and unsteady flows in the direction perpendicular to the shore line on a sloping bottom up to the wave breaking position have been derived [2]. For steady problem, the one-dimensional energy conservation equation reads

$$d_x(Ec_{gx}) + S_{yx}d_xv = 0, \quad (2)$$

where

$$n = 1/2[1 + 2kh(\sinh 2kh)^{-1}], \quad c = (gh)^{1/2},$$

$$c_{gx} = nc \cos \theta, \quad S_{yx} = 1/2 nE \sin 2\theta.$$

S_{yx} is the radiation stress components described in [3]. Recall for a very shallow water, $kh \ll 1$ such that $n = 1$ and $c_g = c$. For this assumption, we write the equation (2) as a homogeneous first order linear ordinary differential equation:

$$d_x(Ec \cos \theta) + E \sin \theta \cos \theta d_xv = 0. \quad (3)$$

By Snell's law for a constant K and the subscript 0 refers to offshore values of x -axis, the sloping bottom yields the relation

$$(\sin \theta)/c = (\sin \theta_0)/c_0 = K. \quad (4)$$

Let x_b represents the position of breaking. In this paper, the dissipation of wave energy flux D_b is approximated by the second term of equation (2) at the position of breaking, namely

$$D_b = S_{yx}d_xv, \text{ for } x = x_b. \quad (5)$$

To get the contribution of D_b in the solution of the main equation (1), we utilized equation (5) into equation (2) to be integrated as

$$Ec \cos \theta = - \int D_b dx \quad (6)$$

For the initial constant $E_0^* = E_0 c_0 \cos \theta_0$, the analytic steady solution of equation (2) for the energy between the area of sloping bottom until the breaking wave position x_b is obtained as already given in [2],

$$E(x) = (c \cos \theta)^{-1} E_0^* \exp(-Kv) \quad \text{for } a < x < x_b \quad (7)$$

The analytic steady energy solution of equation (7) taken at the breaking position then becomes

$$E(x_b) = (c \cos \theta(x_b))^{-1} E_0^* \exp(-Kv(x_b)). \quad (8)$$

3.2. The bottom friction energy function

The bottom friction force of Chezy law with the bottom friction coefficient C_f is $\mathbf{F} = \rho C_f \mathbf{u} |\mathbf{u}|$. It is interpreted as the force acts in the direction of flow velocity, with the magnitude proportional to the

square of the velocity. Following [4], assuming the net current \mathbf{u} is small compared to the orbital component, the quadratic form could be approximated by the linear term for the mean flow, namely

$$F = \rho C_f u (u_{\text{orbital}}^2 + v_{\text{orbital}}^2)^{1/2} \approx \rho C_f u u_{\text{orbital}} \quad (9)$$

Since $u_{\text{orbital}} = (\gamma/\pi)(g(h+\eta))^{1/2}$, the dissipation of bottom friction flux D_f is then defined as

$$D_f = Fu = (\rho C_f \gamma u^2 / \pi) (g(h+\eta))^{1/2}. \quad (10)$$

D_f is the transfer of wave energy flux to the turbulent bottom boundary layer, in the absence of vegetation [1]. The cross shore velocity component u in equation (10) is assumed here to be constant and small compared to the longshore component. By defining the surface slope s' as [5]

$$s' = -d_x(h+\eta), \quad (11)$$

the integration result of the bottom friction dissipation term contributes to the main solution of equation (1) is formulated as

$$\int D_f dx = -[2\rho C_f \gamma (h+\eta)] [g(h+\eta)]^{1/2} u^2 / (3\pi s') + C. \quad (12)$$

3.2.1. The analytic longshore currents component

The analytic steady energy solution (8) involves longshore currents component v above the sloping bottom area which has been derived analytically in [6] adapted from [7]. It was derived by considering forcing terms included in the y -momentum equation for along shore direction. Radiation stress must be balanced by the bottom friction and momentum exchange forces. By taking an empirical constant N as in [5], the longshore currents at the breaking position is written in the form of

$$v(x_b) = A v^* \left(\frac{P_1 - 1}{P_1 - P_2} \right), \quad (13)$$

where

$$v^* = 5\pi\gamma(16C_f)^{-1} [g(h+\eta)_b]^{1/2} s' \sin \theta(x_b);$$

$$P = N\pi s' (\gamma C_f)^{-1}; P_1, P_2 = -3/4 \pm (9/16 + 1/P)^{1/2}; A = P(1 - 5P/2)^{-1}.$$

3.3. The vegetation dissipation function

The drag force due to the presence of vegetation is defined with the drag coefficient C_d and the area of the plant A_v in the form of $F_d = 1/2 \rho C_d A_v u |u|$. Here the time averaged energy dissipation due to the plants in water D_v is referred to [8] and [9], with the drag coefficient of vegetation is denoted by C_d , the diameter of cylinders of plants is d , area per unit length of each vegetation stand b , and the elevation of the top of the plant relative to the bottom is s_v .

The dissipation of wave energy due to the plants D_v is then defined as [8]

$$D_v = 16\rho C_d d (3\pi k)^{-1} b^{-2} (gk/\omega)^3 (3k \cosh^3 kh)^{-1} (\sinh^3 ks_v + 3 \sinh ks_v) H^3. \quad (14)$$

The contribution of D_v in the solution of the main equation (1) can be written as

$$\int D_v dx = 16\rho C_d d (3\pi k)^{-1} b^{-2} (\sinh^3 ks_v + 3 \sinh ks_v) \int (gk/\omega)^3 (3k \cosh^3 kh)^{-1} H^3 dx. \quad (15)$$

Recall that the angular frequency ω is defined as

$$\omega^2 = gk \tanh kh$$

For $kh \ll 1$, then it is approximated that $\tanh kh \approx kh$ and $\cosh kh \approx 1$ so then $\omega^2 = gk^2 h$. By implementing these relations, and defining the water depth on the sloping bottom as $h = h_0 + sa - sx$, the result of equation (15) is obtained, so the wave energy density due to mangroves is formulated as

$$\int D_v dx = 16\rho C_d dg^{3/2} (9\pi ks)^{-1} b^{-2} (\sinh^3 ks_v + 3 \sinh ks_v) H^3 \int h^{-3/2} dh \quad (16)$$

or

$$\int D_v dx = -32\rho C_d dg^{3/2} (9\pi ks)^{-1} b^{-2} h^{-1/2} (\sinh^3 ks_v + 3 \sinh ks_v) H^3 + C. \quad (17)$$

3.4. The analytic energy density of mangrove contribution

By combining the analytic forms of equation (8) for the energy of breaking waves, equation (12) for the turbulent energy of bottom friction, and (17) for the energy loss due to the plants, the final result of analytic energy density due to the presence of mangrove is obtained:

$$E_w = -1/c \{ E_0^* \exp[-Kv(x_b)] - [2\rho C_f \gamma(h+\eta)] [g(h+\eta)]^{1/2} u^2 / (3\pi s') \} \\ + 1/c \left[32\rho C_d dg^{3/2} (9\pi ks)^{-1} b^{-2} h^{-1/2} (\sinh^3 ks_v + 3 \sinh ks_v) H^3 \right] + C, \quad (18)$$

with the form for $v(x_b)$ is given in equation (13). The constant C can be determined by giving some respective quantities at the wave breaking position.

4. Results and Discussion

The analytic function for energy density governed in equation (18) will help the investigations of the mangroves' contribution in nearshore process. The energy contains the contributions of the energy due to breaking waves, bottom frictions, and the plants. The result also shows the role of three kinds of slopes which contribute toward the energy density, namely the bottom slope, the waves surface slope, and the top of mangroves slope.

To utilize the result, the initial conditions and data are needed as the input. Wave celerity, wave angle, wave height, and water depth at a selected initial position offshore are needed to determine the portion of energy at the wave breaking. Water depth distribution and flow velocity component along the cross shore section contribute to the energy due to the bottom friction. Quantities of the plant, particularly the diameter of the trunk, the area per unit length of each vegetation, and the slope of the canopy are needed to determine energy due to the plants.

Further work might be needed to see the variability of value combinations of the slopes for the distribution of wave energy and heights, as well as the contribution of the longshore currents. The wave height distribution due to the presence of mangroves could also be directly calculated from the energy density by using the relation given in the Subsection 2.2. The wave energy density governed from the model can also be utilized for other calculations such as the water level distribution at the shore line.

5. Conclusion

The analytic forms should then help as the tool for the numerical as well as the experimental works in the subject which also could evaluate the performance of the functions obtained at the same time.

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