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To the Rods Bending Calculating Problem with Allowance for the Transverse Shear Deformation

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Abstract. A redetermined approach to bar systems calculation is considered in this work taking into account edge termination self-balanced components influence where the required components of stress condition must not violate the equilibrium condition of any body elementary volume. This approach provides transverse shear deformation consideration with the accuracy necessary for vessels bar constructions reliability assessment.

1. Introduction

Due to the considerable mathematical difficulties while elasticity theory three-dimensional problems solving two groups of problems are distinguished in the calculation practice. They enable to change elasticity theory equations system for the systems of approximate equations containing less independent variables than initial equations. These are the problems of rod theory and problems of plates and shells stress-strain behavior study. Kirchhoff-Love hypothesis for plates and shells and Kirchhoff-Clebsch hypothesis in rod theory were used as a basis of the classical approach. They enabled to change stress tensor for force and moment system.

Such an approximate approach used for practical problems solution has its restrictions determined by geometric characteristics of the examined constructions as well as by physical properties of the material.

Nowadays the progress in the field of composite materials contributes to a wide distribution of multilayered constructions in building, ship building, aircraft construction. They mostly have a low shear rigidity whereas the influence of such a parameter as geometric characteristics is sharply manifested while calculating the constructions from the materials with a small quantity of shear modulus. Therefore, there appears a necessity to specify the classic algorithms of bar structures calculation and their elements by means of the correction from transverse shear deformation with the help of the hypotheses.

S.P. Timoshenko's and S.A. Ambartsumyan's theories created by hypothesis method [1-4] enable to take into account transverse shear deformations by the introduction of deflection function, which depends on shear. That leads to the simplest structure of basic equations and clarity of geometric representations about tangential displacements distribution by the plates, shells or bar cross-section height. However, if for beams made of the materials with a great transverse modulus value calculations with the help of these methods do not lead to any significant differences in deflections values, than the difference in case of using the materials with the reduced shear modulus will be



significant. This can be explained by the fact, that the values of additional deflections from transverse shear deformations, determined according to different hypotheses, differ from each other greatly.

In all the considered theories the components of stress condition are determined directly from Hook's law equations, which leads to transverse shearing stresses values contradicting the equilibrium conditions of rods, plates and shells elementary volumes. The equilibrium is provided only according to the thickness of each layer or section in general depending on the hypotheses character and a theory construction method. The theories variants built on the basis of rods right sections hypothesis are not capable to reflect a complicated character of stress condition in the marginal zone because they do not take into account edge termination self-balanced components.

In connection with that we would like to mention V.V. Pikul's works [5], related to the considered class of theories. Having applied the principle of elasticity theory equilibrium equations maximal compliance, he built the theory of rods, thin plates and shells, the peculiarity of which is the independence of equations system solution order from the number of layers, the capability of edge termination self-balanced components influence and the complete satisfying equilibrium equations of any elementary volume.

Consequently, the necessity to develop a general principle of computational algorithms development appears, taking into account the influence of edge termination self-balanced components influence. However, the sought components of stress condition must not violate equilibrium conditions of any body elementary volume.

2. Problem statement

S.P. Timoshenko's method of putting elasticity theory equilibrium equations to deflection technical theory equations is practically the only one, in which the analysis of bars deflection shear influence is carried out. As long as it is based on the hypothesis about bars elements pure shear, than actually the analysis comes down to the assessment of shear influence on bars deflections. Determining stress components it should be taken into account that the only components, which obey equilibrium equations of elasticity theory, are normal stresses. All other components are either determined from Cauchy equations (transverse shearing stresses) or by reason of assumed smallness are not determined at all. For example, if we use Cauchy equation

$$\gamma_{yz} = w_{,y} + w_{,z}, \quad (1)$$

where $w_{,y}, w_{,z}$ – are derivatives of tangential displacement of w on y, z respectively, and suppose that an angle of shear is $\gamma_{zy} = f(y) \cdot \varphi$, than transverse shearing stresses are determined as

$$\tau_{zy} = Gf(y) \cdot \varphi, \quad (2)$$

where φ – is a shift function.

These stresses in general case do not satisfy equilibrium equations of elasticity theory. Such approach is quite appropriate if lumping is applied. However, in the recent years a structural design calculation method of finite elements has been used. In this case it is supposed that the received solutions converge to proximate when the number of elements is increased. Consequently, we should assume the same convergence in case of one-dimensional elements, which does not exist in the reality while determining tangential and transverse normal stresses. It should be noted that transverse normal stresses are ignored in the technical theory of bar bending though in disturbed zones they are comparable to principal stresses.

In spite of the relative simplicity of original assumptions in bars theory, in the computation practice there is no single opinion about an accounting degree of transverse shear influence, even in relation to homogeneous bars shear. The calculation of complex structures adjusted for transverse shear deformations influence can be encountered very rarely. That is why in the present article a

specified approach to bar system calculation is being considered. It enables to take into account the influence of transverse shear deformations.

In the technical variant of bending theory after determining displacement components and mean shear function there are found the components of stress, in particular transverse shearing stresses $\tau_{zy} = f(y) \cdot \varphi$. This last operation leads to stresses values τ_{zy} , which do not satisfy equilibrium equations of elasticity theory.

Presumably, reasoning will be more logical if the ratio (1) is straightly written as

$$w_{,y} = \gamma_{zy} - v_{,z}. \quad (3)$$

Then from (3) follows, that tangential displacements will be equal to

$$w(y) = -yv_{,z} + \int_0^y (w_{,y} + v_{,z}) dy. \quad (4)$$

The first summand determines tangential displacements of cross section points according to the theory of pure bending? The second one – takes into account the influence of shear.

Having introduced the notion of average transverse shear deformation throughout the height of the section - φ_{cp} we designate the approximate law of tangential displacements because of shear.

$$w(y, \gamma_{zy}) = \left(\int_0^y f(y) dy \right) \cdot \varphi_{cp} = f_1(y) \cdot \varphi_{cp}.$$

The equation (4) is as follows

$$w(y) = -yv_{,z} + \left(\int_0^y f(y) dy \right) \cdot \varphi_{cp} = -yv_{,z} + f_1(y) \cdot \varphi_{cp}.$$

In this case a shear deformation or shearing stress throughout the depth of section changes consistency is not obviously imposed on.

Average shear deformations may be, in a special case, equal to zero, but it does not tell about shearing stresses throughout the depth of section being equal to zero. Consequently, in order to determine shearing stresses it is necessary to bring in equilibrium equations of elasticity theory.

This very approach is used hereinafter to solve the problems of bars transverse shear taking into account transverse shear deformations.

In the technical theory of bending the main unknown are some integral characteristics (beam deflection determined by bending, shear deflection, etc.). Then function φ can always be evaluated through the main unknown problems, for example,

$$\varphi = \frac{dv_c}{dz}.$$

If $\varphi = \gamma_{cp}$, where γ_{cp} – is an average heightwise displacement angle, then V_c can be interpreted as average heightwise points transverse shear displacements.

Then the law of changes of points heightwise tangential displacements will be determined by the functional connection

$$w(y) = -yv_{,z} + \left(\int_0^y f(y) dy \right) \cdot \frac{dv_c}{dz} \quad (5)$$

The undertaken studies showed that the results correctness in disturbed zone is mainly determined by the function $f(y)$ choice.

3. Study results

Function $f(y)$ must take into consideration the peculiarities of bar transverse section distortion allowing for transverse shear deformations. Usually the basis is the solution from the classical bar bending theory according to which shearing stresses in the points at the level of band and wall connection have the break of continuity.

As far as double-T section, to choose the function $f(y)$ in the works considering the problems of bars calculation with account of transverse shear deformations the following variants of functions $f(y)$ are proposed:

1. Parabolic law (fig. 1)

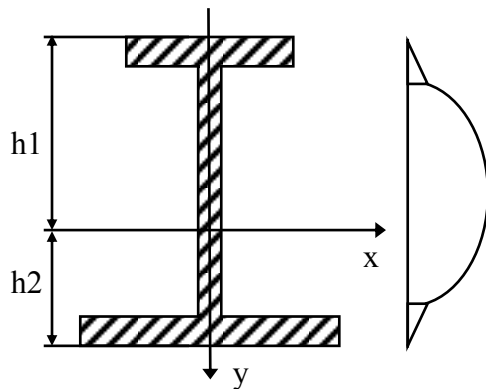


Figure 1. Shearing strain epure.

$$f(y) = k \left(1 - \frac{y}{h_1}\right) \left(1 + \frac{y}{h_2}\right); \quad (6)$$

where

$$k = \begin{cases} \delta_{c\tau}/b_1 & y \in [h_1, h_1 - \delta_1] \\ 1, & \\ \delta_{c\tau}/b_2 & y \in [h_1, -(h_2 - \delta_2)]. \end{cases}$$

In this very case a quadratic law of changes of transverse shear deformations with disconnections Fig. 1 Shearing strain of first genus epure. Disconnection coefficient reflecting section elements degree of participation in shear is determined from the obvious ratio

$$\frac{\tau_{xy}^{\Pi}(y_0)}{\tau_{zy}^{CT}(y_0)} = Q \cdot \frac{S_x(y_0)}{J \cdot b} / \frac{S_x(y_0)}{J \cdot \delta_{c\tau}} = \frac{\delta_{c\tau}}{b},$$

where b – belt width, $\delta_{c\tau}$ – wall depth.

2. Polygonal line law

$$f(y) = k;$$

where

$$k = \begin{cases} \delta_{c\tau}/b_1, & y < -(h_1 - \delta_{n1}); \\ 1, & -(h_1 - \delta_{n1}) < y < (h_2 - \delta_{n2}); \\ \delta_{c\tau}/b_2, & y > (h_2 - \delta_{n2}). \end{cases} \quad (7)$$

or

$$k = \begin{cases} 0, & y < -(h_1 - \delta_{n1}); \\ 1, & -(h_1 - \delta_{n1}) < y < (h_2 - \delta_{n2}); \\ 0, & y > (h_2 - \delta_{n2}). \end{cases} \quad (8)$$

Coefficient k is determined by the formula (7) in case of even distribution of transverse shear deformations in the section elements limits. It is determined by the formula (8) if we suppose that only beam wall works in shear and belts work only in bending due to their insignificant depths.

However, as professor K.N. Gorbachev's studies have shown [1] under certain relations of geometrical parameters and physical constants of the material application of the functions (7), (8) leads to incorrect laws of normal stresses distribution in the marginal zone of fixed or elastically restrained beams. In particular, normal stresses in supporting section change the sign within the limits of shelf depth (Fig.2), which is not natural and can be defined only by the formula of admitted functions of transverse shear deformations distribution.

As a matter of course, assuming linear law of stresses distribution within the limits of the wall, we in practice "harden" constrained shear zone because we locate the points of a possible signs change of the normal stresses epure to the zone of a beam belt in advance.

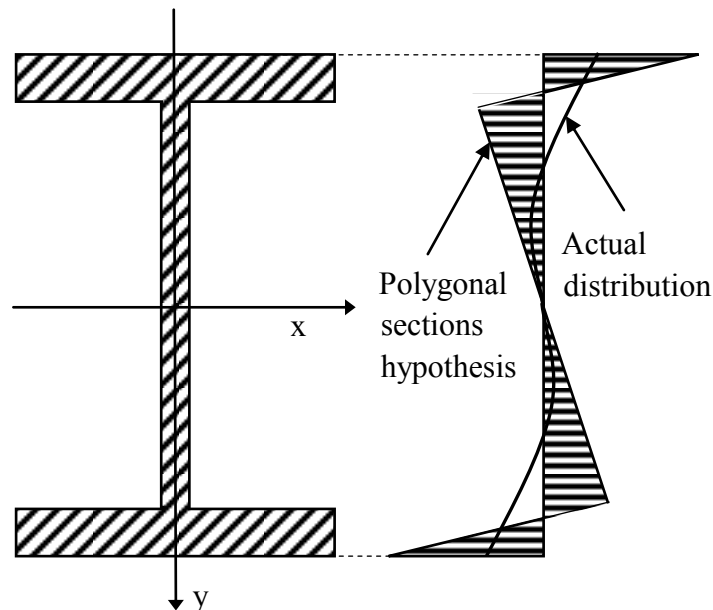


Figure 2. Distribution of normal stresses in restraint section.

4. Summary

Function $f(y, x = 0) = k \left(1 - \frac{y}{h_1}\right) \left(1 + \frac{y}{h_2}\right)$ has several peculiarities, which become obvious when doing simple formal transformations. If we introduce it in expanded form

$$f(y, x = 0) = k \left(1 - \frac{y}{h_1} + \frac{y}{h_2} - \frac{y^2}{h_1 h_2}\right).$$

Then, integrating, we will get

$$f(y, x = 0) = k \left(1 - \frac{y^2}{2h_1} + \frac{y^2}{2h_2} - \frac{y^3}{3h_1 h_2}\right). \quad (9)$$

Consequently, for non-symmetrical tangentially x beams the function (9) leads to a symmetrical constituent in the relation for points tangential displacements section heightwise (Fig. 3).

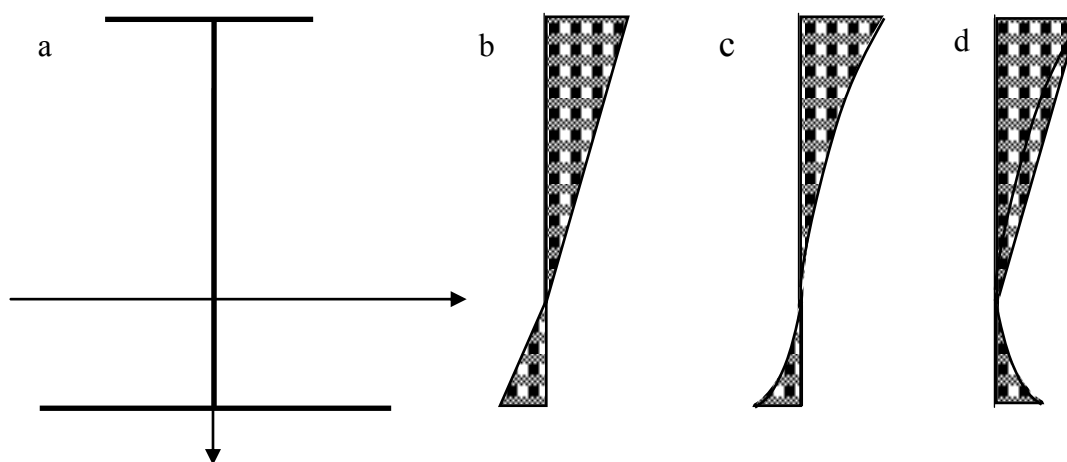


Figure 3. Full tangential displacement components epure: a – component from deflection; b, c – components from shear.

Fig. 3b demonstrates epure of points displacements cross-section heightwise in accordance with flat sections law

$$w_1(y) = -yv' \quad (10)$$

Fig. 3c and 3d show tangential displacements components taking into account transverse shear deformations influence. Epure ordinates exhibited in Fig. 3b are determined by the function changing according to cube law ($\sim y^3 \cdot \varphi_{cp}$).

The last epure (Fig. 3d) corresponds to quadratic law ($\sim y^2 \cdot \varphi_{cp}$).

Such distribution is characteristic for sections of beams non-symmetrical about X axis.

However, the calculations done with the finite elements method showed that in the case of applying function

$$f(y, x=0) = k \left(1 - \frac{y}{h_1}\right) \left(1 + \frac{y}{h_2}\right)$$

when $k < 1$ and $h_1 = h_2 = h/2$ shear forces

$$\text{determined as } \int_{-h/2}^{h/2} \tau_{zy} \delta(y) dy$$

, do not satisfy equilibrium conditions. This is explained by the fact that the beam thin belts degree of involvement in section shear can be adequately determined by parabola ordinates (Fig. 4). The square of epure in zone a is practically equal to the square of epure in b zone.

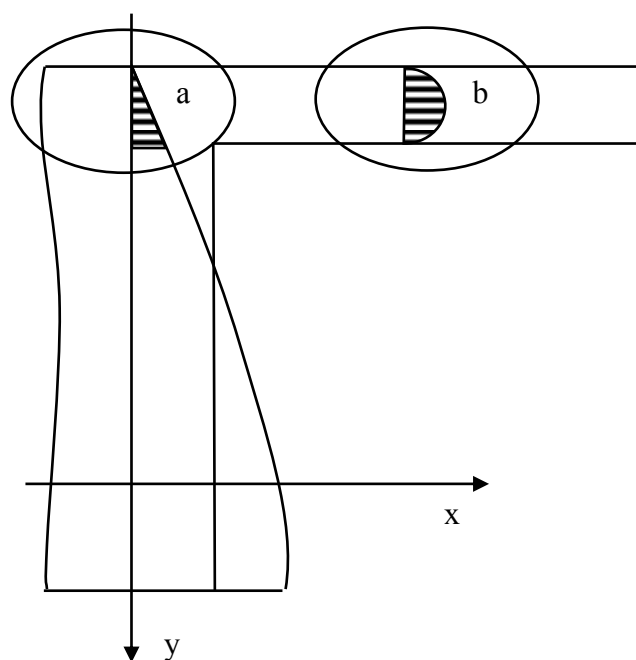


Figure 4. Shear stresses epures in the belt area.

5. Conclusion

Taking into consideration the abovementioned, in the case of thin belts beams calculation function $f(y)$ with reasonable accuracy can be chosen as follows:

For symmetrical sections about x axis

$$f(y) = \frac{1}{2} \left(1 - 4 \frac{y^2}{h^2} \right);$$

for non-symmetrical sections

$$f(y) = \left(1 + \frac{y}{h_1} \right) \left(1 - \frac{y}{h_2} \right)$$

Thus, the proposed shear function obtains a very simple form and can be applied while constructing computational algorithms, which take into account transverse shear deformation to solve the problems of bars bending.

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