Using the Binary Transformation for the Characterization of Structure-Texture Boundaries of Mineral Multifractal Surfaces

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Introduction. The possibilities of using fractal analysis for detecting atypical areas on digital images of feldspar (cracks, kinks, adhesions and structural changes) are investigated. The selection of texture heterogeneities and morphological features is carried out using the field of fractal dimension D, the self-similarity coefficient field K, and the measure field SRGB (triangle areas in the coordinate system of properties). The influences of the displacement step as well as the size of the scanning "window" are estimated. The proposed segmentation of digital images of minerals allows one to eliminate the regions with different ranges of fractal measure values and to describe complex multifractal surfaces of minerals taking into account the spectrum of fractal dimensions, because such fractals, along with geometric features, possess certain statistical properties. The application of the binary transformation of the obtained fractal measure fields to the selected threshold value revealed a sensitivity to a quantitative and qualitative description of the complexity of structural and texture boundaries of the surface. This fact which can be used to predict the results of the ores crushing.

1. Introduction

When processing digital images to date, great difficulty is caused by the allocation of chaotically placed small objects and their boundaries for the purpose of further analysis. Quantitative estimations of typomorphic features of the mineral aggregate structures (all elements of texture and structure) allow to study the conditions for their formation and further transformation. The variation in the structuring of the surface of mineral aggregates is affected both by the result of the intergrowth of the crystal boundaries and by the duration of the formation of the system of each level (ions, grains, crystals, aggregates). The degree of ordering in the emerging system determines the measure of the energy of interconnections between grain boundaries, the strength of minerals depends on that energy. Ordering in the granular structure of minerals is determined by several parameters, such as granulometric composition (close grain sizes), the morphology of the prevailing clusters, and the balance of the associated internal energy of the system. It is generally accepted that the crystal lattice of minerals is almost perfect and balanced energetically, despite the fact that in the real lattice dislocations, defects, etc. are always present. The most important contributions to the energy come from rough edges and meandering grain boundaries. The degree of roughness of the surface of the cluster boundary is determined by the kinetic parameters of grain growth and depends on the degree of cohesion of the structure of the boundary region, its thickness, the grain lattice relaxation rate in the

direction of the boundary, i.e. from the energy of the boundary. Surface energy depends mainly on the perimeter and the roughness of the cluster [1,2]. The fractal dimension of the inhomogeneous surface of the mineral cluster boundary demonstrates the sensitivity to a quantitative description of the complexity of structural and texture boundaries, as well as fractures, cleavage, hardness and brittleness of the surface of solids. A large number of such boundaries leads to an increase in the unevenness of the grinding of minerals [3-5].

To date, there are many algorithms for digital image processing. Studies in this area are still relevant. New more accurate and effective methods of image processing are being developed, including ones on the basis of fractal analysis, which allows to study images transformed into an inhomogeneous surface, for example, with a brightness index for height [6,7], any color code [8,9]. To exclude a volatility, a clear mathematical criterion for describing irregular textures is necessary and fractal dimension can serve as such a criterion. It is possible to use the Hurst index to determine whether the surface structure of the object is ordered and whether there is a certain regularity in the distribution of the random variable, even if the non-Gaussian series (not normally distributed) [10,11]. The proposed approach in this work can be useful for forecasting the grinding and crushing of ores, the opening in the process of crushing mineral clusters of vein and ore minerals.

2. Methods and materials

As an subject of research, digital images of feldspar, one of the most diverse, rock-forming minerals of magmatic, sedimentary and metamorphic rocks, characterized by perfect cleavage and hardness (Figure 1) has been chosen [12-14]. Various parameters describing the shape of mineral grains do not always give a complete characterization. Fractality does not always manifest itself clearly, it hides in the chaos of random statistics of irregular systems. A common characteristic of such objects is the spatial inhomogeneity with the invariance of the basic geometric features with respect to scaling. The quantitative measure of the distribution of an inhomogeneous structure in space is the fractal dimension D. The study of such systems is very interesting, because it allows one to study not only their structure, but also the relationship between it, the formation processes and their properties.

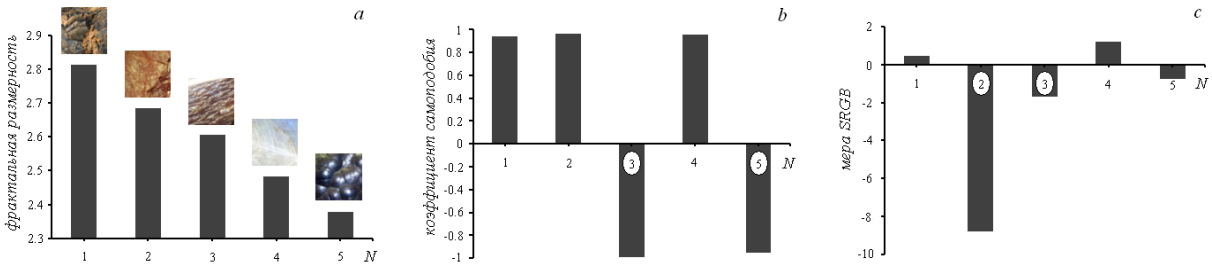

Figure 1. Histograms of fractal dimension *(a),* self-similarity coefficient *(b)* and measures *SRGB (c)* for the entire surface of the image of the investigated samples of minerals (feldspar): *1)* columbite (Ilmen Mountains, Chelyabinsk Region); *2)* common feldspar species; *3)* colored stones of the Transbaikal region; *4)* moonstone; *5)* hematite (Germany, Saxony).

The processing of parameters is based on the algorithm of fractal-multiple description of experimental data based on the approach of B. Mandelbrot [7]. Consider the set $A\left(N^2\right)$, where N^2 - is the number of elements $a_{i,j}$ in the set $a_{i,j} \in A\left(N^2\right)$, where $i, j = 1..N$. We assume that the elements of the set have certain properties $H_\xi(a)$ (dimension, color, volume, shape, etc), inherent only elements in the given set $\forall a_{i,j}\left(a_{i,j} \in \left\{a \middle| H_\xi(a)\right\}\right)$. If one considers few properties $(\xi > 1)$, then the set is described using a few fractal dimensions. We define the fractal dimension D_ξ of the set $A\left(N^2\right)$ on the property

$H_\xi(a)$ by the angular coefficient of the dependence of $\log \tilde{A}_\xi(n^2)$ on $\log s_\xi n^2$, where $\tilde{A}_\xi(n^2)$- the number of non-contiguous surfaces of cubes, covering subsets, $s_\xi n^2 = S_\xi(n^2)$ - is area of set elements

$$D_\xi = \sum_\gamma \frac{\log \tilde{A}_\xi\left(n^2_{\gamma+1}\right) - \log \tilde{A}_\xi\left(n^2_\lambda\right)}{abs\left(\log S_\xi\left(n^2_{\gamma+1}\right)\right) - abs\left(\log S_\xi\left(n^2_\gamma\right)\right)}\left(\frac{\alpha_{\gamma+1} - \alpha_\gamma}{N-1}\right). \quad (1)$$

Self-similarity coefficient detects minimal difference of an ideal fractal from self-similar structure. This coefficient can be defined as $K_\xi = \frac{D^0_\xi}{D_\xi}$, where D^0_ξ is fractal dimension of self-similar set

$$D^0_\xi = \frac{\log \tilde{A}_\xi\left(N^2\right) - \log \tilde{A}_\xi(1)}{abs\left(\log S_\xi\left(N^2\right)\right) - abs\left(\log S_\xi(1)\right)}.$$

For example, in the study of the fractal properties of the images the three colors can be selected as properties $H_\xi(a)$: red ($\xi = R$), green ($\xi = G$) and blue ($\xi = B$).Thus, the description of the image structure in this case is performed by three fractal dimensions D_R, D_G and D_B, with which one can calculate a some value *SRGB* (area of triangle in the property frame of references), which is highly sensitive to the changes in the structure of a color image [15,16]:

$$SRGB = \frac{1}{2} M[-2(D_R + D_B) + (D_B + D_G) + (D_G + D_B)] \quad (2)$$

Complex fractals, most often found in nature, are characterized by more than one scaling factor. Multifractal analysis of an inhomogeneous object allows one to describe the shape of a mineral cluster in such a way that each of its parts has self-similarity properties. To describe multifractals, it is necessary to take into account the spectrum of fractal dimensions, since such fractals, along with geometric features, possess certain statistical properties [8-10].

3. Results

Fragments of digital images of the investigated feldspar samples have $a = b = 273$ pixel sizes, preliminary image enhancement operations have not been performed. In accordance with the color code of the working pixel of the surface of the object, the program converted the pixel color code to the height of the point, with the formation of a three-dimensional object with an inhomogeneous rough surface. The obtained objects were investigated with respect to their fractality, according to the ideas of B. Mandelbrot's fractal geometry. The results of processing the entire image surface for fractal

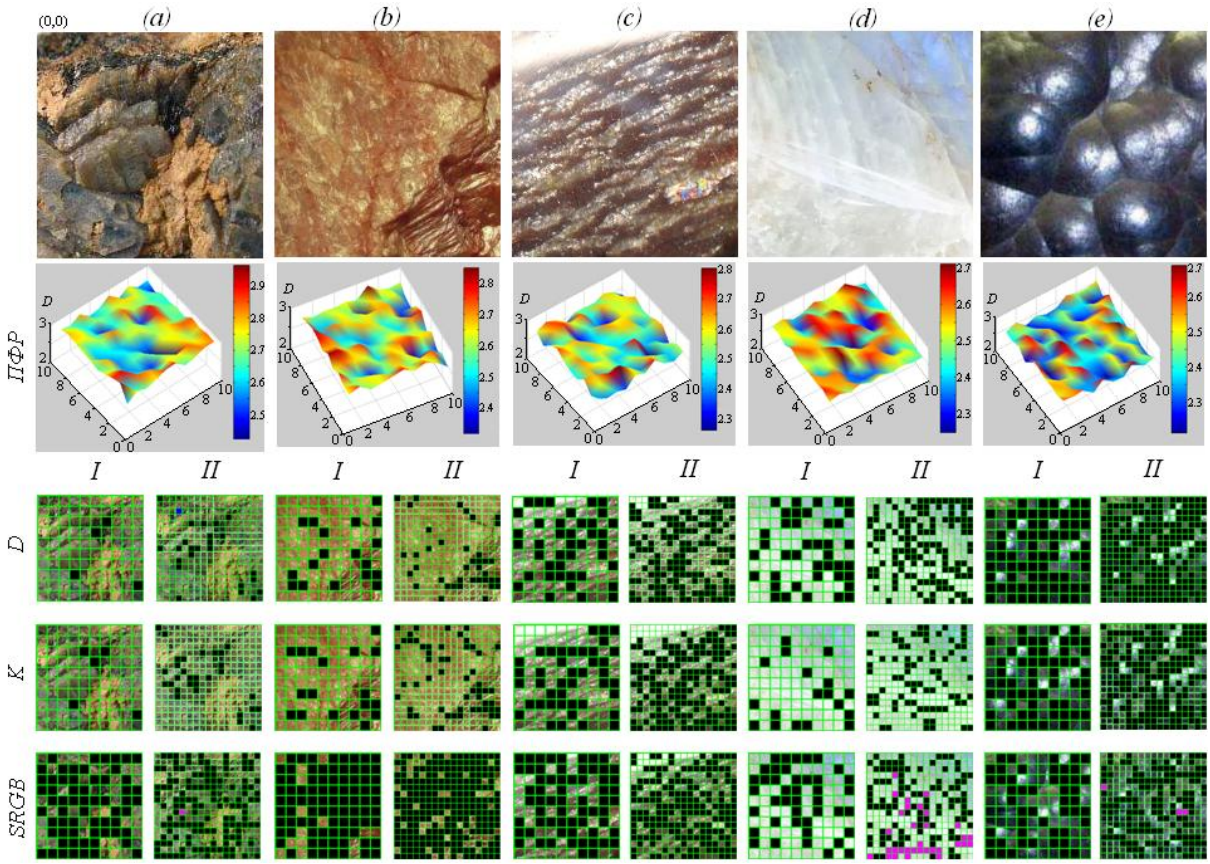

Figure 2. Fragments of the investigated digital images of feldspar (*a-e*) and the corresponding fields of fractal dimensions *(FFD)* (scanning with a "window" 20×20 pixels, $s=7$) and binary transformation of fractal measure fields (scanning with a "window" 20×20 pixels *(I)* and 16×16 pixels *(II)*, with offset step $s=7$ and $s=1$ respectively), where the "threshold" values of the binary transformation are: for a field of fractal dimension D $0\le H<0.5$ - "0", $0.5<H\le1$ - "1" ($D>3$ - "blue" colour); for the field of the self-similarity coefficient $K>0$ – "0", $K<0$ – "1"; for the measure field *SRGB* $SRGB>0$ - "0", $SRGB<0$ – "1", $SRGB=0$ - "pink" colour.

dimension *(a)*, self-similarity coefficient *(b)*, and *SRGB (c)* measure are presented in Figure 1. It is shown that the rugged, rough surface of columbite possesses the largest value of fractal dimension is (Figure 1*a.1*), while the smooth surface of hematite demonstrates the smallest value (Figure 1*a.5*). Atypical (anomalous) changes in the surface structure of minerals, characterized by the self-similarity coefficient ($K<1$), are present in samples of colored stone and hematite (Figure 1*b.3,5*). When the *SRGB* measure is analyzed for the last group, a sample 2 (Figure 1*c.2,3,5*).

Further processing of images of minerals was carried out by scanning the "window" with the $a\times b$ pixel size with the step of displacement s (with $s=1$ "window" being "sliding", with $s>1$ "jumping"). At each step, the numerical values of the fractal dimension in the "window" (segment) were determined, with the formation of the matrix and the field of fractal dimensions (FFD) (Figure 2, *FFD*) [17,18]. Similarly, fields of the self-similarity coefficient *K* and measures *SRGB*. It is revealed that in the center of the region surrounded by segments $D<2.5$ there is a segment with a larger fractal dimension, which is possibly due to a change in the degree of ordering of the system due to the varying degree of cohesion of the structure of the boundary regions, due to the energy balance in the direction of this or that boundary of the mineral aggregate (Figure 2.*D.I,II*).

Figure 3. Structural and textural boundaries of multifractal segmented FFD aggregates for a common type of feldspar and colored stone ($2.5 < D < 2.6$ -"grey" colour).

We apply a binary transformation to the obtained fractal measure fields. This transformation leads to a significant reduction in the large amount of information that we have to work with, by translating the color (or grayscale) image $S(x, y)$ to black and white with respect to the magnitude of a certain "threshold" S_{Π}, assigning the current image segment "1" ("black" color) or "0" ("white" color or transparent "window" when applying a binary mask to the image), if the original value is greater than (less than) the threshold [19,20]. We take as the "threshold" value of the fractal dimension the value of the Hurst index H, for the self-similarity coefficient and the measure of *SRGB*, the numerical value "0". The self-similarity parameters of the Hearst index H as well as the fractal dimension are measures of stability of a statistical phenomenon or a measure of the duration of a long-term dependence of the stochastic process. The binary transformation of multifractal surfaces of minerals allows us to distinguish previously visually hidden features of the structural and texture boundaries of mineral clusters inherent in each type of feldspar (Figure 2, 3). It is shown that next to the segment characterized by the value $D \approx 3$, with a high probability there are segments $2.5 < D < 2.6$ with close to the value $D = 2.5$ ($H = 0.5$), which is a sign of proximity to a random, non-fractal dependence. The closer the value of the index H to 1, the higher the degree of stability of the long-term dependence of the stochastic process. For $0 \le H < 0.5$, the dependence is stable, more variable than the random series [10,11]. The binary transformation of the field of the self-similarity coefficient increases the informativeness of the fractal analysis, visually identifying the slightest atypical (anomalous) changes in the surface structure of the minerals, which is possibly related to the degree of boundness of the structure of the boundary regions of mineral clusters and the degree of roughness (Figure 2.*K*).

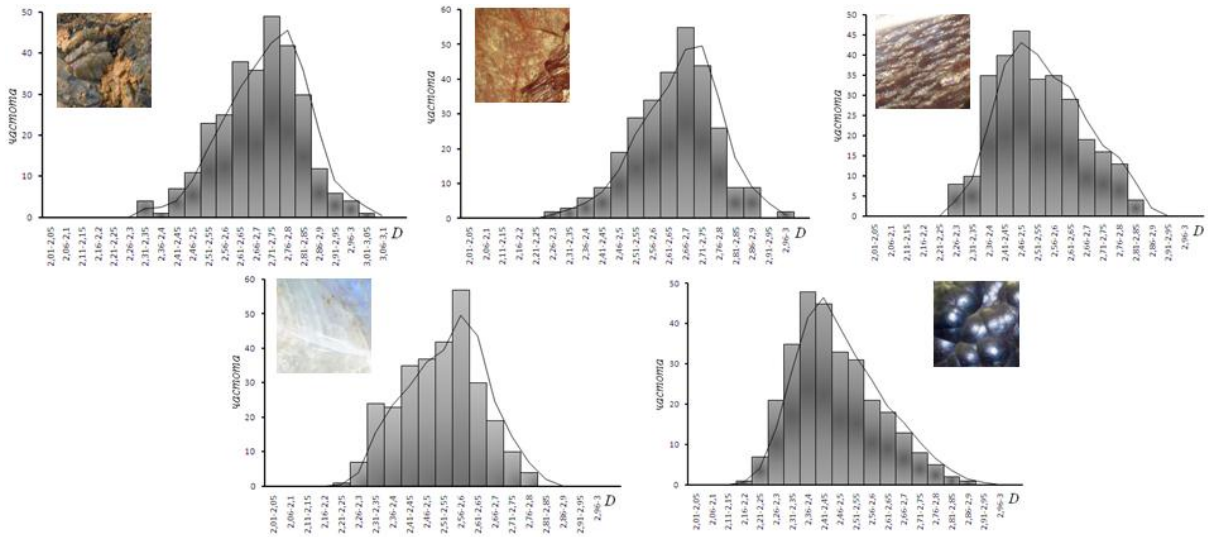

Figure 4. Histograms of frequency distribution of fractal dimension *D* (scanning of feldspar images with a "window" of 16×16 pixels, with offset step $s = 1$).

The binary conversion of the *SRGB* measure field reveals high sensitivity, even to small changes in the color gamut of the image (Figure 2.*SRGB*). Reducing the linear size of the scanning "window"

leads to greater detail in the binary transformation process and to a smoother frequency distribution of histograms of fractal dimension. Further reduction of the linear size of the "window" increases the number of segments with $D > 3$ and regions with $SRGB = 0$ $(D(R) = D(G) = D(B))$, which indicates the disappearance of self-similarity properties in small geometric regions. For different types of feldspar, there is a pronounced displacement of the maximum frequency distribution of fractal dimension, with the presence of a certain set of distribution of the accepted values of fractal dimension, which can be a pattern in the comparative characteristic of morphological and textural changes (tortuosity, cracks, pores) (Figure 4). The increase in the size of the scanning “window” and the choice of the “jumping” mode ($s > 1$) leads to a loss of detail of the visually identifiable features of the inhomogeneous surface of the minerals, while preserving the characteristic distribution spectrum of the fractal dimension, with the loss of smoothness in the frequency distribution of the fractal dimension.

4. Appendices

It is shown that the surface structure of feldspar with inhomogeneous inclusions (microcracks, fractures, etc.) has the well pronounced fractal properties. The sensitivity of the binary transformation of the fractal measure fields to the quantitative description of the typomorphic features of the structural and texture boundaries of multifractal mineral aggregates has been revealed, allowing identification, quantification and forecasting of unevenness in the grinding and crushing of minerals.

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