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## Creep Fibre Method of Simply-supported Composite Beams

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# Creep Fibre Method of Simply-supported Composite Beams

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**Abstract.** Creep effects of composite beams can not be neglected in service-time. Creep Fiber method is used to obtain the time-depended results, which proved to be quiet effect in both elastic and visco-elastic analysis. Only two individual central loading points or bars need considering, and then the system of two sumultaneous differential equations becomes simple in a clear fashion. It can be solved analytically by Rüsck constitutive equation of creep. For high and large beams of bridge structures the fibres of the two points are constant over time because the radius of inertial of steel beam is far larger than that of concrete slab, which simplifies the procedure further. The results show that under sustained loading creep of concrete produces significant negative effects. It may increase the modular ratio to 2.3 times of the initial, reduce the whole flexural stiffness by 76%, and cause the local buckling risk of the top flange and web of steel beam. These should be considered in the design of structure.

## 1. Introduction

Composite beam is widely used in structures for its large economic benefit caused by the considerable composite action between two individual sections (steel beam and concrete slab) . However with the development of time concrete creeps and steel doesn't. This deformation incompatibility causes eigen stresses and hence significant negative effects. Many methods were analysed in the literatures [1-6]. Here one important analytical method (yet almost forgotten) is deduced.

## 2. Creep fibre method

According to linear elastic theory, conjugate points mean that the stress response will be zero in one point if an axial force ( $N$ ) is applied to the other. For a homogenous section, there is one centre of gravity (Figure 1), pair of conjugate points (point 1 and 2) should satisfy  $ab=i^2$ , therefore graphical method can be used to draw the two points and there are obviously countless solutions for the two points in the case of one centre of gravity.

The two conjugate points can be used to solve visco-elastic problem known as Creep Fibre method named by Busemann [7]. In service-time composite beam is in the state of the elasticity. As shown in Figure 1, the composite section has two centres of gravity (point C and S) which do not change over time, while point V ( the centre of gravity of composite section) is time-dependent that moves downwards slowly because creep reduces the modular of concrete. In the case of two centres of gravity the coupled conjugate points are unique, and for bridges the two points will not change over time as well, because the radius of inertial of concrete slab ( $i_c$ ) in bridges is small enough and can be



neglected in practical calculations ( $i_c \approx 0$ ), compared with that of the steel beam. Considering the individual steel section, the distance  $d_2$  can be deduced,

$$\frac{N}{A_{st}} - \frac{Nd}{I_{st}} d_2 = 0 \Rightarrow d_2 = \frac{I_{st}}{A_{st}d} \quad (1)$$

where  $A_{st}$  and  $I_{st}$  are the steel area and second moment of area respectively.

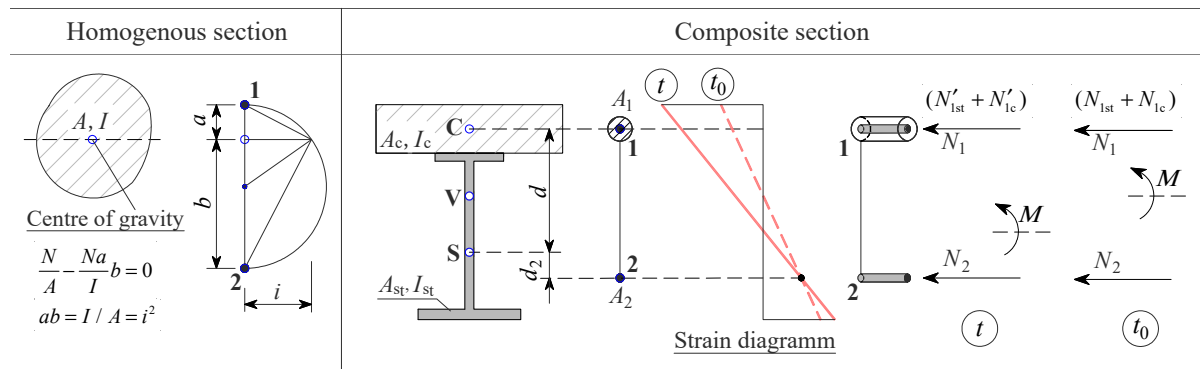


Figure 1. Calculation diagram by Creep Fibre method

Therefore just as shown in Figure 1, Point 1 representing a RC bar under axial load  $N_1$  has the same height level with point C and point 2 representing a steel bar under axial load  $N_2$  has a constant distance ( $d_2$ ) from point S. Then the two individual sections of composite beam under sustained loading can be transformed into the two individual points or bars under centrally loading.

$A_1$  and  $A_2$  are denoted as the transformed area of point 1 and 2 respectively. Considering the sections of steel beam individually, the areas can be deduced by equilibrium relationship,

$$\begin{cases} A_1 = A_{cr} + A_{1st}, & A_{cr} = A_c \frac{E_c}{1.4E_s}, & A_{1st} = A_{st} \frac{d_2}{d + d_2} \\ A_2 = A_{st} \frac{d}{d + d_2} \end{cases} \quad (2)$$

where  $A_{cr}$  is the transformed area of concrete slab,  $A_{1st}$  is the equivalent area of steel beam in point 1. In the calculation of  $A_{cr}$ , assuming that all the delayed elastic deformations of creep are produced at time  $t_0$  and the creep coefficient of delayed elasticity is 0.4.

The stress, strain and internal force of point 2 are time-independent because its material (steel) is time-independent, shown in Figure 1. In other word creep has no influence on point 2, also known as Creep Zero Point [8]. Then according to the stiffness distribution principle,

$$N_2 = \frac{M}{d + d_2} \quad (3)$$

where  $M$  is the bending moment of the composite section.

For point 1 the centrally loading RC bar, only the axial force of steel ( $N_{1st}$ ) is considered here for the convenience of its elasticity. The axial force before creep (time  $t_0$ ) is

$$N_{1st} = -\alpha \frac{M}{d + d_2}, \quad \alpha = \frac{A_{1st}}{A_1} \quad (4)$$

where  $\alpha$  is the restraint coefficient of the steel beam on the concrete slab.

And then the axial forces after creep (time  $t$ ) can be obtained without difficulty if Rüsche constitutive equation of creep [9] is used,

$$N'_{1st} = - \left[ 1 - (1 - \alpha) e^{-\frac{1}{1.4} \alpha (\varphi_t - 0.4)} \right] \cdot \frac{M}{d + d_2} \quad (5)$$

With these axial forces the rest variables (strain, stress and time-transformed parameters of cross-section) can be calculated according to the knowledge of Material Mechanics.

### 3. Example

Consider one high and large composite beam (Figure 2).

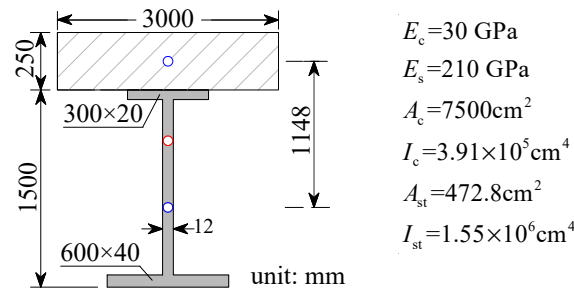


Figure 2. Dimension and parameter of cross-section

Consider time  $t_0$  (7 day) and  $t$  (six month, 1 year, 2 year and 3 year). The creep coefficient  $\varphi$  is calculated by MC2010 (*fib* Concrete Model Code) [10]. Let the mean compressive strength at an age of 28 days  $f_{cm}=40\text{Mpa}$ , the relative humidity of ambient environment  $RH=75\%$ , the notional size of the member  $h=11.5\text{cm}$ . In the case of sustained bending moment, the results of cross-section (Table 1) can be calculated reversely from the results of Equation 3 to Equation 5. Here the modular ratio ( $n$ ) only considers the results of axial deformation because the composite stiffness of the two individual parts ( $S_i \cdot d$ ) is the main part.

Table 1. Time-transformed parameters of cross-section.

Time	$\varphi$	$n$	$A_{cr}$ (cm <sup>2</sup> )	$I_{cr}$ (cm <sup>4</sup> )		$I_{st}$ (cm <sup>4</sup> )		$S_i \cdot d$ (cm <sup>4</sup> )		$I_t$ (cm <sup>4</sup> )
				Value	Ratio (%)	Value	Ratio (%)	Value	Ratio (%)	
$t_0$	0	9.8	765	39860	0.73	1554271	29	3851543	71	5445674
six month	1.46	17.5	428	22278	0.49	1554271	34	2959589	65	4536138
1 year	1.67	19.2	392	20391	0.46	1554271	35	2822460	64	4397122
2 year	1.84	20.5	366	19076	0.44	1554271	36	2719891	63	4293238
3 year	1.92	21.1	356	18536	0.44	1554271	37	2675997	63	4248804
100 year	2.11	22.6	332	17270	0.42	1554271	38	2568509	62	4140050

Table 1 shows that the flexural stiffness of concrete slab ( $I_{cr}$ ) is very small (0.73%), the composite stiffness ( $S_i \cdot d$ ) is very large (71%), therefore the neglecting  $I_{cr}$  is reasonable. With the development of time creep reduces  $A_{cr}$  and  $I_{cr}$  gradually, finally reaches about 43% of the elasticity, for the composite stiffness ( $S_i \cdot d$ ) and whole flexural stiffness ( $I_t$ ) the ratios are 67% and 76% respectively, and the final modular of ratio ( $n$ ) is about 2.3 times of the initial. Therefore the influences of the creep of sustained load on the bending stiffness of composite beams must be considered carefully. It is specially worth mentioning that the results in Table 1 are independent on  $M$ .

Creep-induced stress of statically determinate structure is self-equivalent, which means that only internal stresses will be influenced. Table 2 shows how the redistributed strain and stress change over time under  $M=3000\text{kN}\cdot\text{m}$ . As seen from Table 2, it is important to note that the compression stress at the top of the steel beam reaches about 2.3 times of the initial, which arises local buckling risk remarkably and then influences the section classification. For concrete slab the top and bottom stress increase about 1.8 and 2.3 times of the initial respectively, which means the concrete slab tends to be centrally loading.

Table 2. Time-dependent strains and stresses

Time	Curvature ( $10^6\text{mm}^{-1}$ )	Strain (‰)		Stress ( $\text{N/mm}^2$ )				
		Point 1	Point 2	Top of slab	Point C	Bottom of slab	Top flange	Bottom flange
$t_0$	0.26	-0.12	0.26	-3.19	-2.48	-1.77	-17.4	65.9
six month	0.32	-0.15	0.26	-2.48	-2.28	-2.09	-31.8	67.9
1 year	0.33	-0.16	0.26	-2.44	-2.25	-2.05	-34.5	68.3
2 year	0.33	-0.17	0.26	-2.41	-2.22	-2.202	-36.7	68.7
3 year	0.34	-0.18	0.26	-2.4	-2.2	-2.01	-37.6	68.8
100 year	0.35	-0.19	0.26	-2.36	-2.17	-1.98	-40.0	69.1

It should be noted that stress of concrete dose not equal strain times  $E_c$  because of creep. It is equal to force divided by area.

#### 4. Conclusions

For composite bridges the fibres of the two conjugate points are constant as time develops by Creep Fibre method, which simplifies the analytical procedure in a stunning manner. Here Rüsç constitutive equation of creep is used to obtain the time-dependent results. For high and large composite beams the results by using the creep coefficient of MC2010 show that:

(a) The bending stiffness of concrete slab is small enough to be neglected; Under sustained bending moment the final modular ratio and whole flexural stiffness are about 2.3 times and 76% of the initial respectively.

(b) In the top flange compression stress increases about 2.3 times of the initial and it causes the problem of the local buckling risk.

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