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To cite this article: Jietang Zhu *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **267** 042073

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Application of adaptive square root cubature Kalman filter in turbofan engine gas path performance monitoring

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Abstract. Kalman filters are very popular in turbofan engine community for health monitoring purposes. In this study, in order to get better performance in terms of filtering accuracy and noise adaptability, an Adaptive Square Root Cubature Kalman Filter (ASRCKF) was proposed to estimate health parameters of the turbofan engine gas path components. In the ASRCKF algorithm, the mean value and covariance of the engine nonlinear function were calculated by cubature rule-based numerical integration method and used as a substitute for nonlinear model in nonlinear Kalman filter. The latest information of measurement parameters in the recursion and filtering process was used to estimate and self-adjust the noises cross-covariance by removable window method to get higher filtering accuracy. Compared with the Extended Kalman Filter (EKF) and Square Root Cubature Kalman Filter (SRCKF), the simulation results in the gradual and rapid deterioration process of turbofan engine gas path components indicate that the higher accuracy and faster convergence can be obtained through ASRCKF, which can be used in health parameters estimation and condition monitoring of turbofan engine gas path.¹

1. Introduction

In recent years, life cycle costs of engines have been reduced despite their gradually increased complexity, which can be attributed to improved capacity of health management and condition-based maintenance of engines [1, 2]. Tracking down the performance parameters and health condition of a turbofan engine and its components is critical for its self-diagnosis and fault analysis. As one of key links of an engine, the gas path components are quite prone to failure due to harsh operating environments of high temperature and pressure. Therefore, how to effectively evaluate the healthy status of gas path components plays an important role in engine performance monitoring.

Kalman filtering, as a real-time recursive optimal estimation method, has been widely used in engine condition monitoring and gas path performance analysis. Luppold [3] firstly introduced the using Kalman filter to engine monitoring and performance analysis. However, it has a strong dependency on linear models because the health and parameter assessment is conducted on the basis of linear models. In order to overcome this drawback, some researchers proposed the Kalman filtering methods based on engine nonlinear models, such as Extended Kalman Filter (EKF) [4] and Unscented Kalman Filter (UKF) [5]. Since EKF is actually the linearization of nonlinear system, it inevitably brings some approximate errors [6]. UKF can be superior to EKF in precision and adaptability, and can also avoid linearization of engine nonlinear system by approximating nonlinear distribution via Weighted Statistical Linear Regression (WSLR). However, it relies much on the experiences to choose parameters, and different parameters differ greatly in impact on UKF performance [7].



Square Root Cubature Kalman Filter (SRCKF), as a new state estimation method proposed recently [8, 9], is to obtain a posteriori distribution via WSLR, and can directly calculate the mean values and covariance of random variables. Compared with EKF, SRCKF can avoid linearization of nonlinear system and Jacobian matrixes computation. Compared with UKF, SRCKF avoids parameters selection by experience and its impact on filters, although it shares similar basic principles of approximated nonlinear system distribution [10]. Considering its good performance in approximating nonlinear function, numerical precision and stability, and simple implementation, SRCKF is adopted in this study to conduct effective estimation of gas path components health parameters. Also, a new algorithm is proposed to improve its adaptability to noises and filtering accuracy. In this algorithm, a removable window method is used to estimate the noise covariance matrixes. Finally, effectiveness of this algorithm is tested through typical health parameter degradation of gas path components. The performance of SRCKF is illustrated and compared with EKF and SRUKF, and ASRCKF may be encountered on similar turbofan engines.

2. Turbofan engine gas path components health monitoring

Engine performance degradation, due to erosion, fouling or foreign/domestic object damage for instance, can be split into two groups: gradual deterioration and rapid deterioration. Performance degradation is generally represented by deterioration of component health parameters, i. e., compressor and turbine efficiency indices and flow capacity indices, which has great impact on engine performance [11]. Therefore, in order to realize gas path components health monitoring, health parameters are seen as state variables in engine nonlinear model.

$$\begin{cases} x_{k+1} = f(x_k, u_k) + w_k \\ y_k = g(x_k, u_k) + v_k \end{cases} \quad (1)$$

Where, $x_k \in R^n$ is extended state variable, in which the gas path components health parameter h_k is included. $u_k \in R^l$ is input control variable, and $y_k \in R^m$ is output measurement variable. w_k and v_k are process and measurement noises, which are uncorrelated white noises. $Q \in R^{n \times n}$ and $R \in R^{m \times m}$ are variance matrixes of process and measurement noises. Notably, these parameters are all normalized underground test condition according to the similarity theory.

3. Method description

The aim of this section is to provide the mathematical background of engine gas path components health monitoring system based on ASRCKF. The schematic diagram of health monitoring system is shown in Fig.1. The basic idea of the method is that health parameters deterioration will lead to changes of corresponding output variables during the process of gas path components performance deterioration. Therefore, SRCKF is used to estimate the health parameters according to the changes of output variables in health monitoring system. In the meantime, there are noises and uncertain disturbance during the engine working process, which would affect the accuracy of the method. Therefore, the removable windows method is exploited to estimate the noises according to the changes of output variables, and the estimated values of noises can be integrated into SRCKF to improve adaptability of the monitoring system to noises.

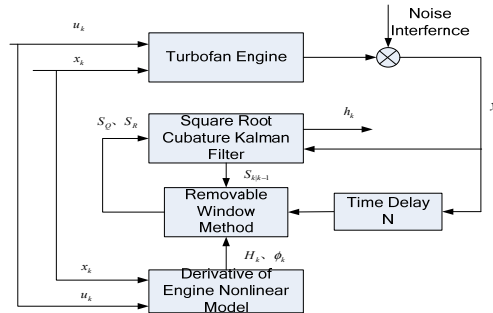


Figure 1. Engine health monitoring system based on adaptive square root cubature Kalman filter.

3.1. Square Root Cubature Kalman Filter (SRCKF)

According to Bayesian reasoning principle, the core of nonlinear Gaussian filtering is to solve the multi-dimensional integral of nonlinear function multiplied by Gaussian density function. The Gaussian weighted integral of function $f(x)$ can be computed by weighted summation of $2n$ cubature points, on the basis of the three order cubature integral rule [12].

$$\begin{aligned}
 I(f) &= \int_{R_n} f(x) N(x; \mu, P) dx \\
 &\approx \frac{1}{2n} \sum_{i=1}^{2n} f(\sqrt{P} \sqrt{n} [1]_i + \mu) \\
 &= \sum_{i=1}^{2n} w_i f(\sqrt{P} \xi_i + \mu)
 \end{aligned} \tag{2}$$

Where $N(x; \mu, P)$ reflects that x obeys the normal distribution with mean μ and covariance P . \sqrt{P} is square root of P , that is, $\sqrt{P} \sqrt{P}^T = P$. (ξ_i, w_i) , $i = 1, 2, \dots, 2n$ is cubature points set, in which ξ_i is cubature point and w_i is corresponding weighting. $\xi_i = \sqrt{n} [1]_i$ stands for basic volume point, where $[1]_i$ is the i th point in the whole holosymmetric points set $[1]$, and n is n -dimensional column vector, and the form of $[1]$ is

$$\begin{bmatrix} \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} & \cdots & \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ -1 \end{pmatrix} \end{bmatrix}$$

In the meantime, round-off errors will lead to the loss of positive definitiveness or holosymmetry. Therefore, the square root of error covariance matrix is used to improve filtering stability. The procedure of SRCKF algorithm is described as follows.

- 1) Initial values are set of state variables and covariance matrix square root.

$$x_{0|0}, S_{0|0} (P_{0|0} = S_{0|0} S_{0|0}^T) \tag{3}$$

- 2) Current state cubature points are computed.

$$X_{i,k-1|k-1} = S_{k-1|k-1} \xi_i + \hat{x}_{k-1|k-1}, \quad i = 1, \dots, 2n \tag{4}$$

- 3) Cubature points are transmitted in state equation.

$$X_{i,k|k-1}^* = f(x_{i,k-1|k-1}, u_{k-1}), \quad i = 1, \dots, 2n \tag{5}$$

- 4) Mean values and covariance square root of predicted state points are estimated.

$$\begin{aligned}\hat{x}_{k|k-1} &= \sum_{i=1}^{2n} \omega_i X_{i,k|k-1}^* \\ S_{k|k-1} &= \text{Tria}([\chi_{k|k-1}^*, S_Q])\end{aligned}\quad (6)$$

Where, $\text{Tria}()$ is exploited to obtain the square matrix by triangularization, S_Q is the square root of process noise covariance Q_k , $\chi_{k|k-1}^*$ is

$$\begin{aligned}\chi_{k|k-1}^* &= \frac{1}{2n} [X_{1,k|k-1}^* - \hat{x}_{k|k-1}, X_{2,k|k-1}^* - \hat{x}_{k|k-1}, \\ &\quad \dots, X_{2n,k|k-1}^* - \hat{x}_{k|k-1}]\end{aligned}\quad (7)$$

Measurement update

5) Updated state cubature points are computed.

$$X_{i,k|k-1} = S_{k|k-1} \xi_i + \hat{x}_{k|k-1}, \quad i = 1, \dots, 2n \quad (8)$$

6) Cubature points are transmitted in measurement equation.

$$Y_{i,k|k-1}^* = g(X_{i,k|k-1}, u_{k-1}), \quad i = 1, \dots, 2n \quad (9)$$

7) Mean values and covariance square root of predicted measurement points are estimated.

$$\begin{aligned}\hat{y}_{k|k-1} &= \sum_{i=1}^{2n} \omega_i Y_{i,k|k-1}^* \\ S_{yy,k|k-1} &= \text{Tria}([\zeta_{k|k-1}, S_R])\end{aligned}\quad (10)$$

Where, S_R is the square root of process noise covariance R_k , $\zeta_{k|k-1}$ is

$$\begin{aligned}\zeta_{k|k-1} &= \frac{1}{2n} [Y_{1,k|k-1}^* - \hat{y}_{k|k-1}, Y_{2,k|k-1}^* - \hat{y}_{k|k-1}, \\ &\quad \dots, Y_{2n,k|k-1}^* - \hat{y}_{k|k-1}]\end{aligned}\quad (11)$$

8) Correlated covariance matrix is estimated.

$$P_{xy,k|k-1} = \chi_{k|k-1} \zeta_{k|k-1}^T \quad (12)$$

Where,

$$\begin{aligned}\chi_{k|k-1} &= \frac{1}{2n} [X_{1,k|k-1} - \hat{x}_{k|k-1}, X_{2,k|k-1} - \hat{x}_{k|k-1}, \\ &\quad \dots, X_{2n,k|k-1} - \hat{x}_{k|k-1}]\end{aligned}\quad (13)$$

9) Gain matrix is computed.

$$W_k = P_{xy,k|k-1} (S_{yy,k|k-1} S_{yy,k|k-1}^T)^{-1} \quad (14)$$

10) Updated state estimated values are estimated.

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_k (y_k - \hat{y}_{k|k-1}) \quad (15)$$

11) square root of updated error covariance matrix is estimated.

$$S_{k|k} = \text{Tria}([\chi_{k|k-1} - W_k \zeta_{k|k-1}, W_k S_R]) \quad (16)$$

3.2. Adaptive estimation of noise matrix

Accurate information about state variables and noises is needed, for the sake of good filtering performance. However, disturbed by numerous uncertain factors, the dynamic model and noise model of engine cannot be accurate enough. As the dynamic data processing of filters is excessively dependence on inaccurate models, it will lead to error accumulate on, even filtering divergence [13]. Therefore, in this paper, a removable window method based on data of measurement variables is exploited to estimate the process and measurement noise matrixes adaptively, and the noise matrixes are used for models

self-adjustment during the filtering process.

Assume that the relationship between health parameters estimation and output measurement variable is determined, and noise variance matrixes are unknown completely or partly. Then, the removable window method is used for estimation of noise matrixes S_Q, S_R .

The state argument $\hat{x}_{k|k-1}$ and new measurement values y_k are exploited to define the innovation vector.

$$z_k = y_k - g(\hat{x}_{k|k-1}) \approx y_k - H_k \hat{x}_{k|k-1} \quad (17)$$

Where, H_k is the first-order derivative of engine measurement equation $g(x)$ at $x = \hat{x}_{k|k-1}$, and covariance matrix of y_k is

$$R_{y_k} = R_k + H_k P_{k|k-1} H_k^T \quad (18)$$

Where, $R_k = S_R S_R^T$, $P_{k|k-1} = S_{k|k-1} S_{k|k-1}^T$. The value of R_{y_k} is computed through sample mean of designated window length in removable window method.

$$R_{y_k} = \frac{1}{N} \sum_{j=0}^N y_{k-j} y_{k-j}^T, \quad k > N \quad (19)$$

Then, estimation square root values of measurement noise variance matrix are obtained through Eqs.(18) and (19).

$$S_R = \text{chol}(R_{y_k} - H_k S_{k|k-1} S_{k|k-1}^T H_k^T) \quad (20)$$

R_{x_k} is defined as the correction quantity of state argument vector $\hat{x}_{k|k-1}$.

$$Z_{x_k} = \hat{x}_{k|k} - \hat{x}_{k|k-1} \quad (21)$$

Then,

$$Q_k = R_{x_k} + P_{k|k} - \phi_k P_{k-1|k-1} \phi_k^T \quad (22)$$

Where ϕ_k is the first-order derivative of engine state equation $f(x)$ at $x = \hat{x}_{k-1|k-1}$. Then, R_{x_k} can be gained by Eq. (15).

$$R_{x_k} = W_k R_{y_k} W_k^T \quad (23)$$

Since a turbofan engine works steadily most of the time, Q_k can be replaced by R_{x_k} .

$$Q_k = W_k R_{y_k} W_k^T \quad (24)$$

The square root of state error variance matrix is

$$S_Q = \text{chol}(Q_k) \quad (25)$$

The estimation square roots S_Q 、 S_R of process and measurement noise matrixes are introduced into Eq. (5) and Eq. (10) for iterative operation; thereby, the adaptive estimation of noise matrixes through the removable window method can be integrated into SRCKF to form ASRCKF.

4. Application and analysis

A low bypass ratio, separated-flow turbofan engine is used as a test case. The engine configuration is given in Fig. 2. The test engine is a two-shaft turbofan engine with one fan, one High-pressure Compressor (HPC), one High-pressure Turbine (HPT) and one Low-pressure Turbine (LPT).

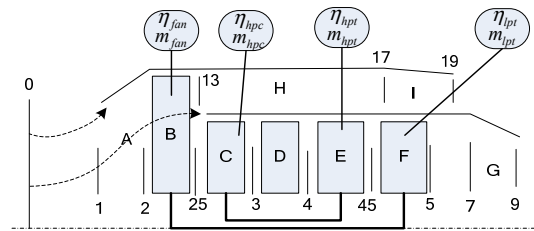


Figure 2. Engine layout with station numbering.

A: Intake; B: Fan; C:HPC; D: Burner; E: HPT;
 F: LPT; G:Primary nozzle; H: External bypass;
 I: Secondary nozzle.

An engine nonlinear model is manipulated in Matlab, which has been validated through the engine test. Assume that cruise conditions are ($H = 0\text{km}$, $Ma = 0$, $Nl = 70\%Nl_{design}$) are assumed. As shown in Table I, $h_k = [\eta_{fan}, m_{fan}, \eta_{hpc}, m_{hpc}, \eta_{lpt}, m_{lpt}, \eta_{hpt}, m_{hpt}]$ Where, the right side of the equation are respectively indices of fans, HPC, HPT and LPT efficiency, as well as of flow capacity, which reflect health conditions of main gas path components. Eight measurement parameters, which are representatives of instrumentation available onboard contemporary turbofan engines, are considered to perform health diagnosis in Table II. Engine wear is simulated by drifting values of health parameters and the data have been obtained. By using such simulated data, ASRCKF is exploited to track the gradual and rapid deterioration of health parameters. In this work, the process and measurement noise matrixes are set respectively as $Q = 0.0002^2 I_{n \times n}$ $R = 0.0002^2 I_{m \times m}$, and the window length of removable window method is $N = 100$. The tracking results of ASRCKF method are compared with those of EKF and SRCKF methods.

Table 1. Health parameters of turbofan engine gas path components.

Health parameter	Symbol	Nominal value
Fan efficiency indices	η_{fan}	1
HPC efficiency indices	η_{hpc}	1
HPT efficiency indices	η_{lpt}	1
LPT efficiency indices	η_{hpt}	1
Fan flow capacity indices	m_{fan}	1
HPC flow capacity indices	m_{hpc}	1
HPT flow capacity indices	m_{hpt}	1
LPT flow capacity indices	m_{lpt}	1

Table 2.Measurement parameters of a turbofan engine.

Measurement parameter	Symbol	Unit
LPT speed	N_l	rpm
HPT speed	N_h	rpm
HPC inlet total pressure	P_{25}	atm
HPC total temperature	T_{25}	K
Combustor inlet total pressure	P_3	atm

Combustor inlet total temperature	T_3	K
LPT inlet total pressure	P_{45}	atm
LPT inlet total temperature	T_{45}	K

4.1. Gradual deterioration

Gradual deterioration of engine gas path components assumes a relatively slow change of the health parameters. Here, the flight sequence is 5000s, and the following gradual deterioration of fans is simulated during the flight: -2% efficiency indices, -3% flow capacity indices [14]. Gradual deterioration simulations with the three algorithms namely the EKF method, the SRCKF method, and the ASRCKF method are presented, as shown in Figs. 3~5. Fig. 3 is the tracking result of fan gradual deterioration with the EKF method, Fig. 4 the SRCKF method, and Fig.5 the ASRCKF method. It can be seen clearly that the three algorithms can track the fan gradual deterioration effectively. In comparison with the others, the ASRCKF method produces the best accuracy.

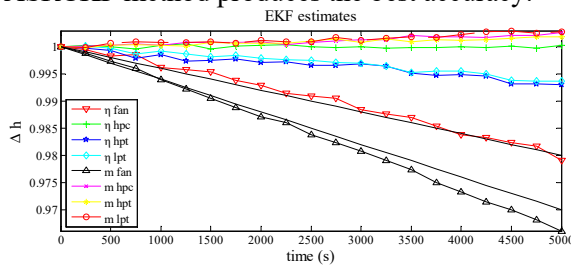


Figure 3. Filtering results of fan gradual deterioration with EKF.

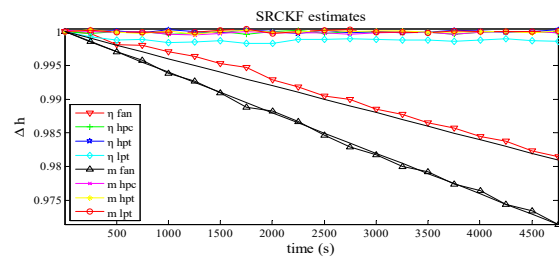


Figure 4. Filtering results of fan gradual deterioration with SRCKF.

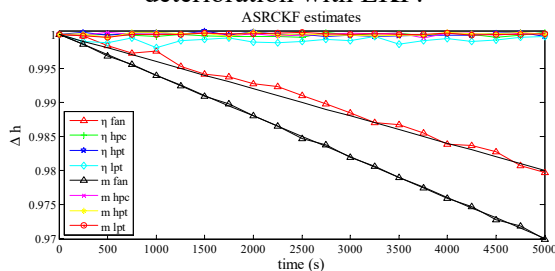


Figure 5. Filtering results of fan gradual deterioration with ASRCKF.

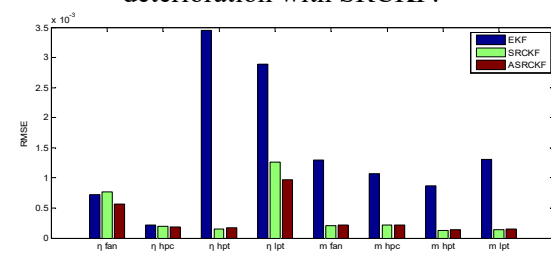


Figure 6. Comparison of RMSEs for EKF, SRCKF and ASRCKF.

In order to compare EKF, SRCKF and ASRCKF more rationally, the average square root error (RMSE) is used during the whole filtering process.

$$RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^n (\hat{x}_k - x_k)^2} \quad (26)$$

Where, n is the number of sample points, and x_k is the real value of health parameters. Fig. 6 depicts the RMSE comparison of the three algorithms. It can be seen clearly from Fig. 6 that both SRCKF and ASRCKF perform more accurate tracking of the fan gradual deterioration than EKF.

4.2. Rapid deterioration

Rapid deterioration of engine gas path components gives a sudden variation of the health parameters. Here, the following rapid deterioration of a fan is simulated at 2s: -2.5% efficiency indices, -2% flow capacity indices. The tracking results of rapid deterioration of fans are shown in Figs. 7 to 9. In Fig. 7, EKF has bad filtering effect of fan rapid deterioration, and the maximum error of HPT efficiency indices is about 1.5% at the rapid deteriorative point. This phenomenon is due to the fact that EKF has poor adaptability of nonlinearity, which exists at the health parameters sudden variation points. Fig. 8 and Fig.

9 are the results of SRCKF and ASRCKF, and the results clearly show that the accuracy of SRCKF and ASRCKF is superior to that of EKF. SRCKF and ASRCKF can obtain nearly three order accuracy of engine nonlinear object through the numerical integral method; therefore, the accurate estimation of rapid deterioration can be achieved. In the meantime, the adaptive estimation of noise matrixes is integrated into the SRCKF to modify the engine model, which makes ASRCKF get faster convergence speed and higher accuracy. It can be seen clearly from the comparison of errors in Fig. 10, and accuracy of the three algorithms is listed as: ASRCKF>SRCKF >EKF.

As for gradual and rapid deterioration of other components, ASRCKF is also able to achieve a good effect in filtering tracking. It cannot be discussed here for lack of space.

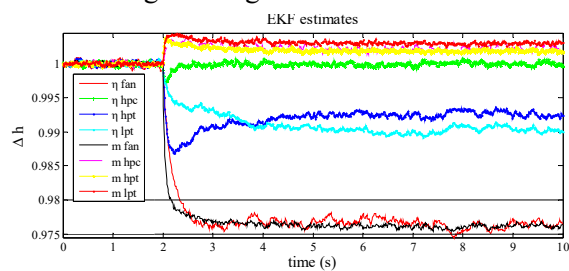


Figure 7. Filtering results of fan rapid deterioration with EKF.

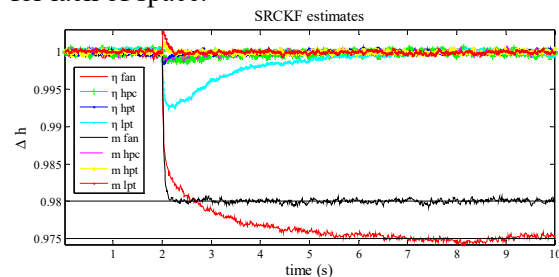


Figure 8. Filtering results of fan rapid deterioration with SRCKF.

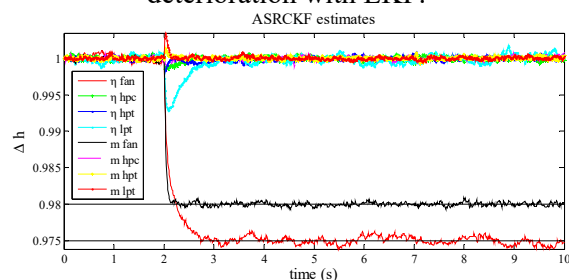


Figure 9. Filtering results of fan rapid deterioration with ASRCKF.

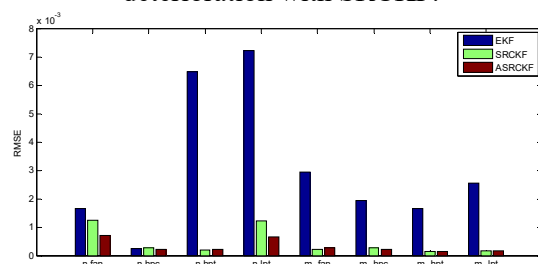


Figure 10. Comparison of RMSEs for EKF, SRCKF and ASRCKF.

5. Conclusions

In this contribution, an adaptive estimation algorithm based on SRCKF and the removable window method is proposed to improve the accuracy and noise adaptability of engine gas path components health parameters. In this algorithm, SRCKF performs a good tracking ability of components deterioration based on the data from the measurement variables, and the removable window method estimates the noise matrixes to improve the accuracy and noise adaptability. The simulated results, which are presented and compared with EKF and SRCKF, show that ASCKF has the advantages of higher accuracy, better noise adaptability and simpler implementation, and can be used for engine gas components health monitoring.

Acknowledgment

This work was supported by the Natural Science Foundation of Shaanxi Province (Grant No. 2017JQ5067) & the Science and Technology Planning Project of Xi'an (Grant No. 201805048YD26CG32(2))

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