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## Elastoplastic analysis of anchored tunnel under seepage

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# Elastoplastic analysis of anchored tunnel under seepage

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**Abstract.** In order to study the stress and displacement distribution of surrounding rock of full-length bonded anchor support tunnel under seepage. The surrounding rock is regarded as a uniform and continuous porous medium. The supporting reaction force and seepage force of the surrounding rock are converted into the axisymmetric radial volume force of the circular tunnel and act on the stress field. Considering the reinforcement of the surrounding rock parameters by the bolt, the mechanical model is established. The elastoplastic analysis method is used to derive the analytical solution of the stress and displacement elasticity of the surrounding rock and plastic analytical solution based on Mohr-Coulomb yield criterion, which can be used to obtain the expression of the radius of the plastic zone of the tunnel and the displacement of the wall. Finally, the calculation results of this paper are compared with the numerical simulation analysis. The results show that the influence of the seepage on the plastic zone range and the wall displacement is more serious. The length and pretension of the bolt are inversely related to the plastic zone range and the displacement of the wall. The bolt row spacing is positively correlated with the plastic zone range and the displacement of the wall. Under the same conditions, considering the seepage requires the anchor to provide greater support. In order to avoid the instability of tunnel caused by seepage, the length of the bolt can be reasonably increased, the bolt spacing can be reduced, and the bolt pretension can be increased.

## 1. Introduction

Underground tunnel is widely used in underground transportation, water conservancy and hydropower, mining and other important projects with the development of science and technology. Groundwater is one of the environmental factors in the occurrence of rock mass, and seepage has a serious impact on the deformation and stability of rock mass. Not considering seepage, domestic and foreign scholars have done a lot of research on the distribution of stress and displacement of tunnel surrounding rock. Fang Yong [1], Wen Jingzhou [2] analyzed the mechanical nature between the full-length bonded anchor and the surrounding rock, and obtained the expression of the plastic zone range of the tunnel surrounding rock; Gu Shuancheng, Zhou Pan et al [3] considered the anchor and surrounding rock as homogeneous solidification, and carried out elastoplastic analysis of the model. SeokWon Lee[4] deduced the analytical solution of the characteristic curve of surrounding rock under the action of permeability. Sun Zhenping [5], Li Zongli [6] et al studied the elastic-plastic analytical solution of circular tunnel under the action of seepage. Fan Hao, Liu Wanrong, Fu Tengfei, et al [7] analyzed the coupling relationship between initial lining reaction and surrounding rock deformation based on seepage, dilatancy and intermediate principal stress. The above scholars have proposed different research ideas, but there are also some shortcomings: when considering the seepage, the support and the seepage of the surrounding rock are less considered and the stress field and displacement field of the surrounding rock are less analyzed.



In summary, by considering the influence of seepage, the reinforcement effect of the anchor and the attenuation of the groundwater seepage effect equivalent for the additional mass force in the surrounding rock at the same time, through the anchor parameters of surrounding rock reinforcement, using Mohr-Coulomb criterion for elastic-plastic analysis of tunnel surrounding rock, derived the elastic-plastic analytical expressions of displacement and stress of surrounding rock, and obtained the analytical solution of the radius of plastic zone of surrounding rock of tunnel. On this basis, the influence of different bolt supporting parameters on the plastic zone of tunnel surrounding rock under the action of seepage is analyzed.

## 2. Seepage field calculation and mechanical model

### 2.1. Seepage field calculation

The seepage of the rock mass satisfies Darcy's law: assuming that the material permeability coefficient is the same, the seepage direction is mainly radial, ignoring the influence of buoyancy and water self-weight [8], and  $R$  is the influence radius of the seepage. According to Darcy's law, the osmotic differential equation is

$$\frac{dp_w(r)}{dr^2} + \frac{1}{r} \frac{dp_w(r)}{dr} = 0, \quad (1)$$

where  $p_w(r)$  is pore water pressure.

Boundary conditions are

$$p_w|_{r=R_0} = 0, \quad p_w|_{r=R} = p_m, \quad (2)$$

where  $p_m$  is the pore water pressure at the radius  $R$  of the seepage flow,  $R_0$  is the tunnel radius.

The pore water pressure distribution form obtained by the formulas (1) and (2) is

$$p_w(r) = p_m \frac{\ln R_0 - \ln r}{\ln R_0 - \ln R}. \quad (3)$$

Then the seepage volume force in the surrounding rock can be expressed as

$$f_w(r) = \eta \frac{dp_w(r)}{dr} = -\frac{\eta p_m}{r(\ln R_0 - \ln R)}. \quad (4)$$

### 2.2. Establish mechanical model

Considering the deformation of surrounding rock supported by the seepage action of bolt, the following assumptions should be made: the tunnel is circular; the surrounding rock is regarded as a uniform and continuous porous medium, and the surrounding rock property is an ideal elastoplastic body, which is in a plane strain state; the bolt is in the elastic state and exists in the plastic zone of surrounding rock; the original rock stress is in the isotropic state.

Under the action of external pressure, the bolt interacts with the surrounding rock, and in the anchorage body, the supporting reaction force of the bolt to the surrounding rock is transformed into the additional volume force  $f(r)$  of the surrounding rock in the anchorage area. The surrounding rock strength parameters are  $c$  and  $\varphi$ , and after anchoring are  $c_s$  and  $\varphi_s$ . The length of the bolt is  $L$ , the radius of the plastic zone of the surrounding rock is  $R_p$ , the pressure of the original rock is  $p_0$ , and the mechanical model is shown in Figure 1.

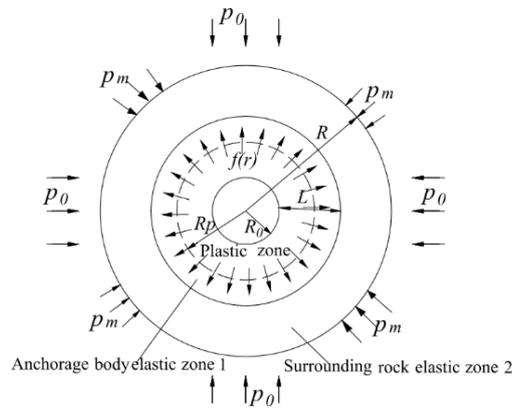


Figure 1. Mechanical model

### 3. Mechanical parameters

#### 3.1. Anchor additional volume force $f(r)$

Take a single anchor micro-body with a length of  $dr$  for analysis, as shown in Figure 2.

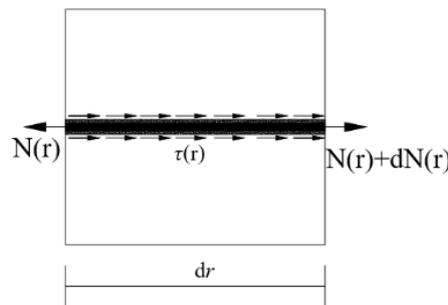


Figure 2. Anchor micro-body

It can be known from the neutral point theory that the shear stress at the anchor point at the neutral point is 0, so the displacement of the surrounding rock at the neutral point can be used to represent the overall displacement of the anchor [9]. The shear stress at the anchor interface is

$$\tau(r) = \frac{G_m}{t_m} \Delta u = \frac{G_m}{t_m} (u(a) - u(r)), \tag{5}$$

where  $G_m$  is the shear modulus of the anchoring agent part,  $t_m$  is anchoring agent thickness,  $a$  is the distance from the neutral point of the bolt to the center of the cave.

Apply a pretension to the anchor as shown in Figure 2, and analyze the overall bolt.  $P = \int_L \tau(r) \pi d dr,$

(6)

where  $d$  is the diameter of the bolt,  $P$  is the bolt pretension,  $L$  is the length of the bolt.

For the analysis of the anchor micro-element (Figure. 2), the static balance equation can obtain the anchor axial force  $N(r)$  as

$$N(r) = P - \pi d \int_{R_0}^r \tau(r) dr, \tag{7}$$

Assume that the effect of a single anchor is half the distance between two adjacent anchors. Then the volumetric force distribution of the surrounding rock is

$$f(r) = -\frac{\pi d \tau(r)}{r S_r S_L}, \tag{8}$$

where  $S_r$  is the circumferential spacing and  $S_L$  is the longitudinal spacing.

### 3.2. Intensity parameter

The bolt support improves the stress state of the rock surrounding the anchor, and the strength index is improved, that is, the cohesion force and the internal friction angle, thereby improving the bearing capacity of the surrounding rock and finally achieving the reinforcement and support.

Studies have shown that the internal friction angle of the surrounding rock before and after bolt support changes very little [10], so the internal friction angle after bolt reinforcement can be expressed as

$$\varphi_s = \varphi. \quad (9)$$

The maximum principal stress direction of the anchorage body is perpendicular to the anchor, and the angle between the main fracture surface and the direction of the maximum principal stress is  $\alpha = \pi/4 - \varphi/2$  [11].

The cohesive force provided by the transverse action of the bolt is

$$c_t = \frac{F_{b\max}}{rS_rS_L \cos \alpha}, \quad (10)$$

where  $F_{b\max}$  is the maximum shear force that the bolt can withstand when shearing purely,

$F_{b\max} = \sigma_s \pi d^2 / 4\sqrt{3}$  by Von-Mises criterion,  $\sigma_s$  is anchor yield strength.

The cohesive force provided by the axial action of the anchor is

$$c_m = \frac{N(r)}{rS_rS_L} \cos \alpha \tan \varphi. \quad (11)$$

From the above analysis, the cohesive force of the anchorage body is

$$c_s = c + \frac{\sigma_s \pi d^2}{4\sqrt{3}LS_rS_L \cos \alpha} \ln\left(1 + \frac{L}{R_0}\right) + \frac{\int_{R_0}^{R_0+L} \frac{N(r)}{r} dr \cos \alpha \tan \varphi}{LS_rS_L}. \quad (12)$$

## 4. Elasto-plastic analysis

When considering seepage, the stress component of the plastic zone of the anchorage body should meet the Mohr-Coulomb criterion.

$$\frac{\eta p_w(r) - \sigma_r + c_s \cot \varphi_s}{\eta p_w(r) - \sigma_\theta + c_s \cot \varphi_s} = \frac{1 - \sin \varphi_s}{1 + \sin \varphi_s}. \quad (13)$$

According to the elastic-plastic mechanics theory of rock and soil, when surrounding rock is supported by bolt and pore water pressure, the equilibrium differential equation is expressed as

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + f(r) - f_w(r) = 0. \quad (14)$$

The simultaneous equations (4), (8), (13) and (14) are brought into the boundary condition  $\sigma_r^p(R_0) = 0$ , and the stress distribution of the plastic zone of the anchor is solved.

$$\begin{aligned} \sigma_r^p &= r^{n-1} \left[ E - \left( \int f(r) r^{1-n} dr \right) \right] + c_s \cot \varphi_s + K \ln \frac{R_0}{r}, \\ \sigma_\theta^p &= nr^{n-1} \left[ E - \left( \int f(r) r^{1-n} dr \right) \right] + c_s \cot \varphi_s + K \ln \frac{R_0}{r}, \end{aligned} \quad (15)$$

where  $E = \int f(R_0) R_0^{1-n} dR_0 - R_0^{1-n} c_s \cot \varphi_s$ ,  $n = \frac{1 + \sin \varphi_s}{1 - \sin \varphi_s}$ ,  $K = \frac{\eta p_m}{\ln(R_0 - R)}$

Through the equilibrium equation (Equation 14), geometric equations, physical equations, the stress and displacement expressions at various points in the elastic region can be solved according to the boundary conditions.

$$\begin{aligned} \sigma_{ri}^e &= -\frac{A}{(1-\mu)r^2} - \frac{B}{r^2} + \frac{C}{2} + \frac{D}{r^2} \\ \sigma_{\theta i}^e &= \frac{A}{(1-\mu)r^2} + \frac{B}{r^2} - \frac{Z}{1-\mu} + \frac{C}{2} - \frac{D}{r^2} \\ u_i^e &= \frac{r(1+\mu)}{E_s} \left[ \frac{A}{(1-\mu)r^2} + \frac{B}{r^2} - Z + \frac{(1-2\mu)C}{2} - \frac{D}{r^2} \right], \end{aligned} \tag{16}$$

Where  $i=1$  stands for anchorage body elastic zone,  $i=2$  stands for surrounding rock elastic zone, and  $A = \int r \left( \int f(r) - f_w(r) dr \right) dr$ ,  $B = \int r^2 (f(r) - f_w(r)) dr$ ,  $Z = \int f(r) - f_w(r) dr$

Boundary conditions are

$$\begin{aligned} \sigma_{r2}^e \Big|_{r=R} &= -(p_0 + \eta p_m), \quad u_{r2}^e \Big|_{r=R_0+L} = u_{r1}^e \Big|_{r=R_0+L} \\ \sigma_{r2}^e \Big|_{r=R_0+L} &= \sigma_{r1}^e \Big|_{r=R_0+L}, \quad \sigma_{r1}^e \Big|_{r=R_p} = \sigma_r^p \Big|_{r=R_p} \end{aligned} \tag{17}$$

Substituting the boundary condition into equation (18) can solve  $C$  and  $D$ .

$$C = \frac{2R_p^2 \left[ K \ln\left(\frac{R_0}{R_p}\right) + c_s \cot \varphi_s \right]}{R_p^2 - R^2} + \frac{2R^2(\eta p_m + p_0)}{R_p^2 - R^2} + \frac{2R_p^{n+1}(E - F(r))}{R_p^2 - R^2} \tag{18}$$

$$D = \frac{KR^2 R_p^2 \ln\left(\frac{R_0}{R_p}\right)}{R^2 - R_p^2} + \frac{R^2 R_p^{n+1}(E - F(r))}{R^2 - R_p^2} + \frac{R^2 R_p^2(\eta p_m + p_0 + c_s \cot \varphi_s)}{R^2 - R_p^2} + B(r) + \frac{A(r)}{1-\mu},$$

where  $F(r) = \int_{R_0}^{R_0+L} f(r)r^{1-n} dr$ ,  $A(r) = \int_{R_0}^{R_0+L} r \left( \int f(r) - f_w(r) dr \right) dr$ ,  $B(r) = \int_{R_0}^{R_0+L} r^2 (f(r) - f_w(r)) dr$

From the continuous condition of radial stress, the radial stress  $\sigma_{r1}^e(R_p)$  and hoop stress  $\sigma_{\theta 1}^e(R_p)$  on the elastoplastic interface should satisfy the Mohr-Coulomb criterion, and the simultaneous equations (15) and (16) can be solved. The transcendental equation of the radius  $R_p$  of the plastic zone requires the Matlab software to solve the value of  $R_p$ .

$$\frac{2KR^2 \ln\left(\frac{R_0}{R_p}\right)}{R_p^{n-1}(R^2 - R_p^2)} + \frac{R^2 + R_p^2}{R^2 - R_p^2} - n = \frac{2R^2(\eta p_m + c_s \cot \varphi_s + p_0)}{R_p^{n-1}(R^2 - R_p^2)(E - F(r))} + \frac{Z(r)R^2}{R_p^{n-1}(R^2 - R_p^2)(E - F(r))(\mu - 1)}, \tag{19}$$

where  $Z(r) = \int_{R_0}^{R_0+L} f(r) - f_w(r) dr$

### 5. Case analysis

The radius of a circular tunnel is  $R_0=3.5\text{m}$ , and the influence range of seepage flow is  $R=40\text{m}$ . The physical and mechanical parameters of the rock mass are: Poisson's ratio  $\mu=0.3$ , cohesion force  $c=0.8\text{MPa}$ , internal friction The angle  $\varphi = 30^\circ$ , the original rock pressure  $p_0 = 10 \text{ MPa}$ , and the pore water pressure  $p_m=2\text{MPa}$ . The physical and mechanical parameters of the anchor are: the circumferential spacing of the anchors is  $S_r=2/7\text{rad}$ , the longitudinal spacing is  $S_L=1.0\text{m}$ , the bolt pretension is  $P=60\text{kN}$ , the diameter is  $d=22\text{mm}$ , and the yield strength is  $235\text{MPa}$ .

The following is an example to analyze the influence of tunnel radius and bolt support strength on the radius of plastic zone of surrounding rock.

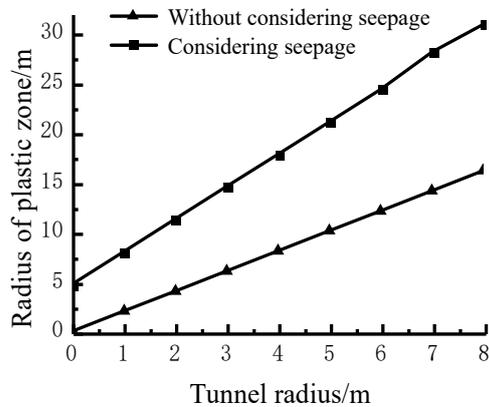


Figure 3. Influence of tunnel radius on radius of plastic zone.

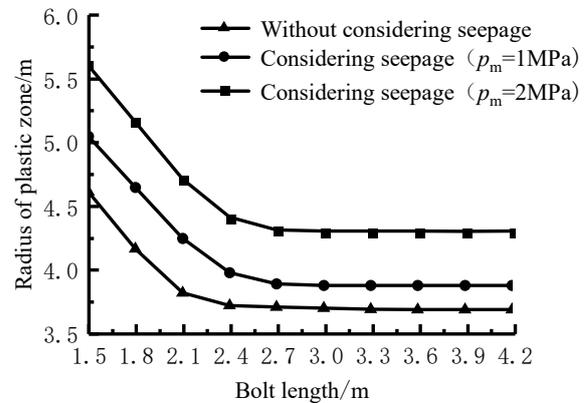


Figure 4. Influence of bolt length on radius of plastic zone.

As shown in Figure 3, it can be seen that when the tunnel radius increases from 0m to 8m without bolt support, the plastic zone radius increases from 0m to 16.235m without considering the seepage effect, and the plastic zone radius increases from 5.15m to 32.03m under the seepage effect ( $p_m=2\text{MPa}$ ). Therefore, the influence on the plastic zone range is more serious when considering the seepage effect.

The effect of the length of the bolt is shown in Figure 4. It can be seen that the radius of the plastic zone decreases with the increase of the length of the bolt without considering the seepage and considering the seepage ( $p_m=1\text{MPa}$  and  $p_m=2\text{MPa}$ ). When the length of the bolt is increased from 1.5m to 4.2m, the radius of the plastic zone is reduced by 13%, 12% and 8%, and the displacement of the wall is reduced by about 12%, 12% and 10% with and without considering the seepage.

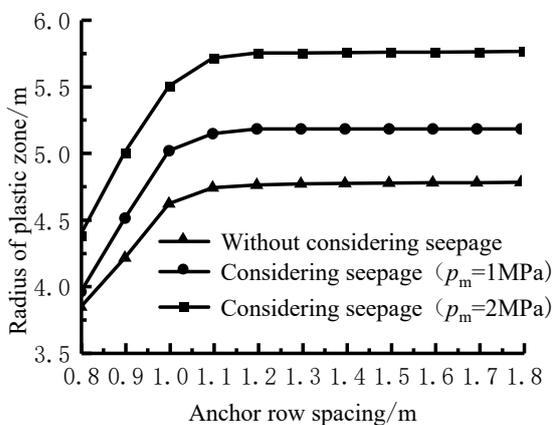


Figure 5. Influence of bolt spacing on the radius of plastic zone.

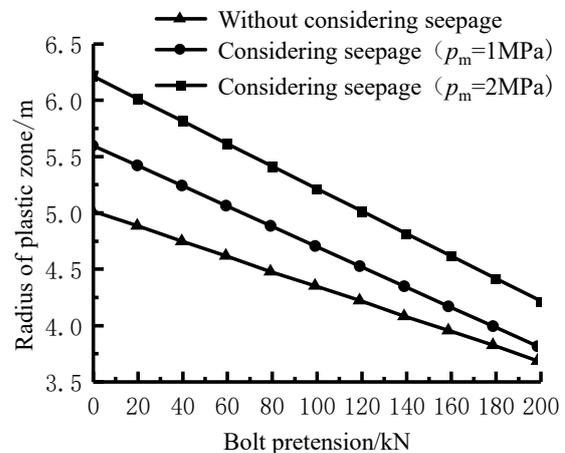


Figure 6. Influence of bolt pretension on radius of plastic zone.

The influence of bolt spacing is shown in Figure 5. It can be seen that the radius of the plastic zone increases with the increase of the bolt spacing without considering the seepage and considering the seepage ( $p_m=1\text{MPa}$  and  $p_m=2\text{MPa}$ ). When the bolt spacing increases from 0.8m to 1.8m, the radius of the plastic zone increases by about 14%, 13% and 10%, and the displacement of the wall increases by about 14%, 12% and 8% with and without considering the seepage.

The influence of bolt pretension is shown in Figure 6. It can be seen that the radius of the plastic zone decreases with the increase of bolt pretension without considering the seepage and considering the seepage ( $p_m=1\text{MPa}$  and  $p_m=2\text{MPa}$ ). When the bolt pretension is increased from 0kN to 200kN, the radius of the plastic zone is reduced by about 32%, 30% and 26%, and the displacement of the wall is reduced by about 36%, 34% and 27% with and without considering the seepage.

## 6. Conclusions

1. The effect of coordinated support of bolt support and surrounding rock is equivalent to the support volume force on the anchor, the anchorage strength parameter  $c$ ,  $\varphi$  value is increased, and the seepage flow is considered, and the seepage force is converted into the volume force, and the circumference is derived. The stress and displacement distribution of the rock is used to find the radius of the plastic zone of the tunnel under the effect of the seepage.

2. According to the example, the influence factors of tunnel plastic zone range under the action of seepage and no seepage are analyzed, and the influence of tunnel radius, bolt length, anchor row spacing and pretension on the plastic zone radius of the tunnel is obtained. Therefore, the seepage caused by groundwater should be fully considered in the design of tunnel support.

3. The anchor length and pretension are inversely related to the plastic zone range, and the bolt row spacing is positively correlated with the plastic zone range. Under the same condition, considering that the seepage requires the bolt to provide greater supporting force. In order to avoid the instability of the tunnel caused by the seepage, the length of the bolt can be reasonably increased, the bolt spacing can be reduced, and the pretension can be increased.

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