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Symplectic Solutions in Two-dimensional Viscoelastic Polymers

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Symplectic Solutions in Two-dimensional Viscoelastic Polymers

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Abstract. As applications of viscoealstic polymers increase, a comprehensive understanding of physical properties of these kind of materials has been proved to be critical in the modern science and technology. In this paper, a new symplectic system method is proposed to solve the classical Saint-venant problem for viscoelastic polymers. Based on the Laplace integral transformation and the symplctic character, the dual governing equations are established, and all the general eigensolutions are obtained in analytical form. In numerical examples, some boundary condition problems are studied, which well exhibit the viscoelastic behavior of polymers.

1. Introduction

In recent years, the time dependent viscoelastic model has been widely applied to simulating the mechanical property of polymers. A large amount of research appears to describe the stress and deformation responds when polymers are subjected to external force conditions or displacement constraints [1]. Bottoni et al present an efficient finite element model for orthotropic thin-walled beams subject to long-term loadings by adopting a generalized linear Maxwell viscoelastic model [2]. Drozdov derived that stress–strain relations are for the time-dependent behavior of a polymer melt at arbitrary three-dimensional deformations with small strains based on the theory of temporary heterogeneous networks [3]. Khan developed a simple and flexible phenomenological constitutive model to characterize the observed time and temperature dependent mechanical response of soft polymers under finite deformation [4]. Reese has also found a material model for the thermo-viscoelastic behavior of rubber-like polymers based on transient network theory [5]. More extensive uniaxial experimental data with the experimental details for polymers is presented in the research of Lopez-Pamies and Khan [6].

On the other hand, Zhong established the symplectic solution method for boundary condition problems in elasticity, and gave a direct solution method [7]. Xu et al studied the traditional Saint-Venant problem and replenished semi inverse method [8]. Because the stress-strain equations and the geometrical equations of viscoelastic solids in the Laplace domain are similar to the corresponding elastic counterparts in the time domain, the symplectic solution method can be generalized, and accordingly the general eigensolutions can be derived.

The main objective of this paper is to construct a new symplectic solution method for viscoelastic polymers. Based on the Laplace transformation and separation of variables, the dual governing equations are established, in which both the displacement and stress components are used as basic

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variables, and thus all the general eigensolutions are found. In addition, an efficient method is proposed for non-homogeneous equations and boundary conditions.

2. The symplectic system

A homogeneous isotropic elastic media of the strip plane-domain is considered. The Cartesian coordinate (x, y) is selected such that the x-axis is along the longitudinal direction with origin located at the central point of the cross section where $2b$ is the width of the strip and l the length (shown in Figure 1).

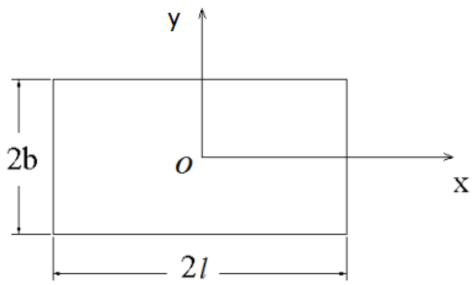

Figure 1. The sketch map of the coordinates.

Introduce the energy density function

$$L = \{(\lambda^* + 2G^*)[\dot{\bar{u}}^2 + (\partial_y \bar{v})^2] + 2\lambda^* \dot{\bar{u}} \partial_y \bar{v} + G^* (\partial_y \bar{u} + \dot{\bar{v}})^2\} / 2 \quad (1)$$

and the displacement and stress vectors

$$\bar{\mathbf{q}} = \{\bar{u}, \bar{v}\}^T \quad (2)$$

$$\bar{\mathbf{p}} = \frac{\partial L}{\partial \dot{\bar{\mathbf{q}}}} = \begin{Bmatrix} (\lambda^* + 2G^*)\dot{\bar{u}} + \lambda^* \partial_y \bar{v} \\ G^*(\partial_y \bar{u} + \dot{\bar{v}}) \end{Bmatrix} = \begin{Bmatrix} \bar{\sigma} \\ \bar{\tau} \end{Bmatrix} \quad (3)$$

Based on the separation of variable method, the dual equations under the symplactic system is obtained as

$$\begin{Bmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ \dot{\bar{\sigma}} \\ \dot{\bar{\tau}} \end{Bmatrix} = \begin{bmatrix} 0 & -a_3\partial_y & a_2 & 0 \\ -\partial_y & 0 & 0 & a_1 \\ 0 & 0 & 0 & -\partial_y \\ 0 & -a_4\partial_y^2 & -a_3\partial_y & 0 \end{bmatrix} \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{\sigma} \\ \bar{\tau} \end{Bmatrix} \quad (4)$$

where $a_1 = 1/G^*$, $a_2 = 1/(\lambda^* + 2G^*)$, $a_3 = \lambda a_2, a_4 = 4G^*(\lambda^* + G^*)a_2$, For briefty Eq. (4) are rewritten as

$$\dot{\psi} = \mathbf{H}\psi \quad (5)$$

3. Numerical results

In this section, the coefficient take is taken as

$$\gamma = 3K/(2G) = 2.5 \tag{6}$$

and the physical dimension l/b=5.

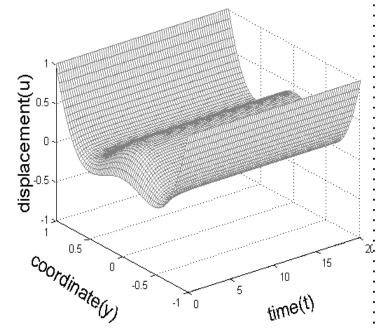

Figure 2. Displacement u

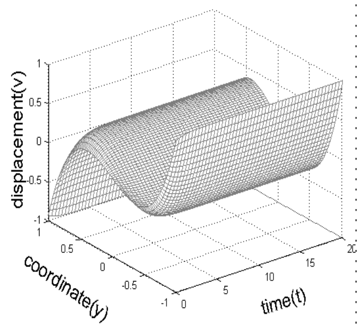

Figure 3. Displacement v

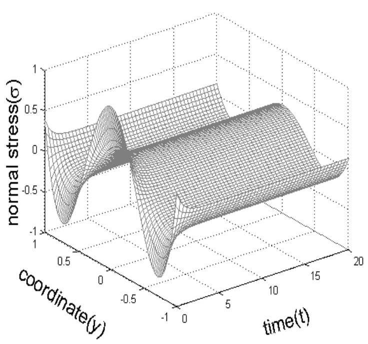

Figure 4. Normal stress

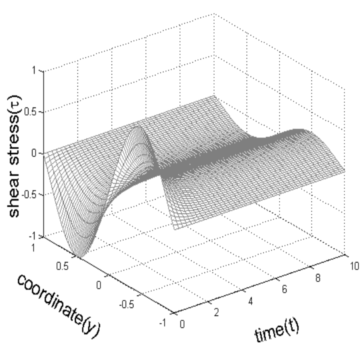

Figure 5. shear stress

On the basis of dual governing equation, the general eigensolutions can be found easily. These general solutions can well describe the local effect for boundary condition problems. Gererally, these eigensolutions can be divided into two parts, symmetry solution and anti-symmetry one. Figures 2-5 present the stress and strain components, which in turns are displacement u, v, normal stress σ and shear stress τ. These figures exhibit local effect of the boundary, and can be well explained the triditional Saint-Venant theory.

4. Conclusion

In the Laplace domain, viscoelastic problems can be transformed into ones of conservative system, in which the Hamiltonian formulation is applicable. With the proposed method, all the general solutions are derived in analytical form. Using the equivalent force systems, Saint-Venant solutions can satisfy effective boundary conditions. However, these solutions are incomplete to describe local effets. In fact, the accurate solution should be composed of zero eigenvalue solutions and non-zero eigenvalue solutions.

Acknowledgments

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