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An Interactive Two-Stage Fractile Optimization Model for Agricultural Irrigation Systems under Uncertainty—A Case Study of Sanjiang Plain

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Abstract. An interactive two-stage fractile optimization (ITFO) model is applied to a case study of agricultural irrigation systems in Sanjiang Plain. The advantages of ITFO are as follows: (1) ITFO can reflect the uncertainties and complexes of agricultural irrigation systems, and deal with dual uncertainties expressed as interval fuzzy numbers and stochastic distributions; (2) The optimal water-allocation strategies for agricultural irrigation systems under different feasibilities and fractile values can be generated through ITFO in order to reflect the preference to the system cost; (3) ITFO combines objectivity of mathematical programming method and subjectivity of human beings into a framework which can increase the applicability. ITFO could provide feasible schemes for decision makers to support the rational development and utilization for agricultural irrigation systems in Sanjiang Plain.

1. Introduction

Agriculture is basic industry in China. Rapid economy and population growth have resulted in increasing reliance on water resources for agricultural irrigation systems [1,2]. Many methods and models have been proposed in order to deal with unbalance supply and uncertainties presented as interval numbers, probability distributions and fuzzy random variables in agricultural irrigation systems[3,4]. Thus, how to allocate water to satisfy agricultural demands and generate optimal water-allocation strategies is concerned by many researchers. Lu et al. provided an exact programming method through combining two-stage stochastic programming and interval-parameter programming into a framework[5]. The application of proposed method to agricultural irrigation systems shows that reasonable solutions can be obtained. Adin concluded methods of irrigation[6]. The proper match between water quality of the source and within the distribution system and the type of treatment is the key to solve clogging problems.

However, the methods mentioned above can hardly deal with irrigation problems where dual uncertainties exist in system. The uncertainties of parameters in the agricultural irrigation systems are presented as not only random distributions but also interval fuzzy numbers. Therefore, a model that can deal with two-stage planning problems with dual uncertainties is desired to be developed.



2. Interactive two-stage fractile optimization (ITFO) model

2.1 Model formulation

An interactive two-stage fractile optimization (ITFO) model is proposed to determine water-allocation strategies. Decision makers need to promise water users an water supply target. Cost can be generated through allocating water to farms. And penalties may arise if the water target is not attained, which may seriously affect the productivity of farms. Therefore, it is necessary to introduce a modeling framework to determine optimal water allocation in agricultural irrigation systems. However, uncertain parameters expressed as interval fuzzy numbers and probability distributions may exist in agricultural irrigation systems. In this study, an ITFO model is proposed to deal with these problems.

$$\begin{aligned} \text{Min } f^{\pm} = & \sum_{j=1}^n \sum_{i=1}^m \sum_{d=1}^D B_{ijd}^{c\pm} X_{ijd}^{\pm} - \sum_{j=1}^n \sum_{i=1}^m \sum_{d=1}^D p_{ne} \delta_B |X_{ijd}^{\pm}| \\ & + \left(\sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^{K_t} \sum_{d=1}^D p_k D_{ijd}^{c\pm} Y_{ijk}^{\pm} - \sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^{K_t} \sum_{d=1}^D p_k p_{ne} \delta_D |Y_{ijk}^{\pm}| \right) \end{aligned} \quad (1)$$

s.t.

$$\sum_{i=1}^m \sum_{d=1}^D (X_{ijd}^{\pm} - Y_{ijk}^{\pm}) (1 + (1-\theta) E_1^{\tilde{\gamma}^{\pm}} + \theta E_2^{\tilde{\gamma}^{\pm}}) \leq \theta E_1^{\tilde{q}_{jk}^{\pm}} + (1-\theta) E_2^{\tilde{q}_{jk}^{\pm}}, \quad \forall j=1,2,\dots,n, k=1,2,\dots,K_t, \quad (2)$$

$$X_{ijd \max} \geq X_{ijd}^{\pm} \geq Y_{ijk}^{\pm}, \quad \forall i=1,2,\dots,m, j=1,2,\dots,n, d=1,2,\dots,D_t, k=1,2,\dots,K_t, \quad (3)$$

$$\sum_{j=1}^n X_{ij}^{\pm} \geq X_{id \min}, \quad \forall i=1,2,\dots,m, d=1,2,\dots,D_t \quad (4)$$

$$Y_{ijk}^{\pm} \geq 0, \quad \forall i=1,2,\dots,m, j=1,2,\dots,n, d=1,2,\dots,D_t, k=1,2,\dots,K_t. \quad (5)$$

Where $f^{\pm}$ is system cost (\$); $\tilde{B}_{ijd}^{\pm} = (B_{ijd}^{c\pm}, \delta_B)$ is cost rate offered by region i from water source j to farm d (\$/m³); $X_{ijd}^{\pm}$ is allowable target promised to region i from water source j to farm d (m³); $\tilde{D}_{ijd}^{\pm} = (D_{ijd}^{c\pm}, \delta_D)$ is penalty rate offered by region i from water source j to farm d (\$/m³); $Y_{ijk}^{\pm}$ is water shortage of region i from water source j to farm d under scenario k (m³); $\tilde{q}_{jk}^{\pm}$ is available water amount from water source j for scenario k (m³); $E_1^{\tilde{q}_{jk}^{\pm}}$ and $E_2^{\tilde{q}_{jk}^{\pm}}$ are lower and upper bounds of expected value $\tilde{q}_{jk}^{\pm}$, respectively; $X_{ijd \max}$ is the maximum of available water amount of region i from water source j to crop d ; $X_{id \min}$ is the minimum water amount satisfied the basic demands of region i for crop d ; $\tilde{\gamma}^{\pm}$ is rate of water loss during transportation.; $E_1^{\tilde{\gamma}^{\pm}}$ and $E_2^{\tilde{\gamma}^{\pm}}$ are lower and upper bounds of expected value $\tilde{\gamma}^{\pm}$, respectively; p_k is probability of scenario k ; p_{ne} is fractile value, representing the preference of decision makers; $\theta \in [0,1]$ is feasibility degree, representing the constraint preference of decision makers; $i=1,2,\dots,m$ is number of regions; $j=1,2,\dots,n$ is number of water sources; $d=1,2,\dots,D_t$ is number of crops; $k=1,2,\dots,K_t$ is number of scenarios; ‘+’ and ‘-’ are symbols of upper and lower bounds, respectively; ‘~’ is symbol of fuzzy numbers. ITFO can effectively deal with dual uncertainties in both objective function and constraints in agricultural irrigation systems.

2.2 Solution method

The two-step method proposed by Huang et al. is used to solve the ITFO model [7]. Let $X_{ij}^{\pm} = X_{ij}^{-} + \Delta X_{ij} y_{ij}$, where $\Delta X_{ij} = X_{ij}^{+} - X_{ij}^{-}$, $y_{ij} \in [0,1]$. The ITFO model can be decomposed into two sub-models.

Lower bound sub-model f^- :

$$\begin{aligned} \text{Min } f^- = & \sum_{j=1}^n \sum_{i=1}^m \sum_{d=1}^D B_{ijd}^{c-} (X_{ijd}^- + \Delta X_{ijd} y_{ijd}) - \sum_{j=1}^n \sum_{i=1}^m \sum_{d=1}^D p_{ne} \delta_B |X_{ijd}^- + \Delta X_{ijd} y_{ijd}| \\ & + \left(\sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^{K_t} \sum_{d=1}^D p_k D_{ijd}^{c-} Y_{ijk}^- - \sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^{K_t} \sum_{d=1}^D p_k p_{ne} \delta_D |Y_{ijk}^-| \right) \end{aligned} \quad (6)$$

s.t.

$$\sum_{i=1}^m \sum_{d=1}^D (X_{ijd}^- + \Delta X_{ijd} y_{ijd} - Y_{ijk}^-) (1 + (1-\theta) E_1^{\tilde{y}^-} + \theta E_2^{\tilde{y}^-}) \leq \theta E_1^{\tilde{q}_k^+} + (1-\theta) E_2^{\tilde{q}_k^+}, \quad \forall j=1, 2, \dots, n, k=1, 2, \dots, K_t, \quad (7)$$

$$X_{ijd \max} \geq X_{ijd}^- + \Delta X_{ijd} y_{ijd} \geq Y_{ijk}^-, \quad \forall i=1, 2, \dots, m, j=1, 2, \dots, n, d=1, 2, \dots, D_t, k=1, 2, \dots, K_t, \quad (8)$$

$$\sum_{j=1}^n (X_{ijd}^- + \Delta X_{ijd} y_{ijd}) \geq X_{id \min}, \quad \forall i=1, 2, \dots, m, d=1, 2, \dots, D_t \quad (9)$$

$$Y_{ijk}^- \geq 0, \quad \forall i=1, 2, \dots, m, j=1, 2, \dots, n, d=1, 2, \dots, D_t, k=1, 2, \dots, K_t. \quad (10)$$

Upper bound sub-model f^+ :

$$\begin{aligned} \text{Min } f^+ = & \sum_{j=1}^n \sum_{i=1}^m \sum_{d=1}^D B_{ijd}^{c+} X_{ijd, opt} - \sum_{j=1}^n \sum_{i=1}^m \sum_{d=1}^D p_{ne} \delta_B |X_{ijd, opt}| \\ & + \left(\sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^{K_t} \sum_{d=1}^D p_k D_{ijd}^{c+} Y_{ijk}^+ - \sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^{K_t} \sum_{d=1}^D p_k p_{ne} \delta_D |Y_{ijk}^+| \right) \end{aligned} \quad (11)$$

s.t.

$$\sum_{i=1}^m \sum_{d=1}^D (X_{ijd, opt} - Y_{ijk}^+) (1 + (1-\theta) E_1^{\tilde{y}^+} + \theta E_2^{\tilde{y}^+}) \leq \theta E_1^{\tilde{q}_k^-} + (1-\theta) E_2^{\tilde{q}_k^-}, \quad \forall j=1, 2, \dots, n, k=1, 2, \dots, K_t, \quad (12)$$

$$Y_{ijk}^+ \geq Y_{ijk}^-, \quad \forall i=1, 2, \dots, m, j=1, 2, \dots, n, d=1, 2, \dots, D_t, k=1, 2, \dots, K_t. \quad (13)$$

The optimal solution of ITFO model can be finally obtained: $f_{opt}^\pm = [f_{opt}^-, f_{opt}^+]$, $X_{ij, opt}$ and $Y_{ijk, opt}^\pm = [Y_{ijk}^-, Y_{ijk}^+]$. Therefore, the optimal water-allocation amount is $A_{ijk, opt}^\pm = X_{ij, opt} - Y_{ijk, opt}^\pm$.

3. Case study

The ITFO model is applied to Sanjing Plain, Eastern China (129°12'~135°05' E ~ 44°51'~48°28' N), with total area 10.89×10⁴ km². The Sanjiang Plain locates in humid and subhumid monsoon climate region. The annual average temperature of Sanjiang Plain is 1°C~4°C, and the annual average precipitation is 500~650 mm. As an important Grain reserves base and marketable grain productive base, the total cultivated area in Sanjing Plain is 5.46×10⁴ km², where Hegang, Shuangyashan, Jiamusi, Qitaihe and Jixi are main grain-producing areas. The decision maker is responsible for allocating water from surface and underground water to three farms cropping with corn, rice and bean. The water resources are rich in Sanjiang Plain. However, the agricultural irrigation in Plain is based mainly on underground water, leading to unbalance of utilization of water resources. Therefore, the decision maker desire to know how to rationally allocate the water resources to gain the minimum cost.

Data collected in this study mainly from statistical yearbook (from 2005 to 2014). ITFO is applied to locate water in agricultural irrigation systems of Shuangyashan and Jiamusi in Sanjiang Plain in order to verify its applicability and generate reasonable water-location strategies. The probability distributions of surface and underground water are shown in Table 1. The feasible degree θ is 0.5, 0.7 and 0.9. The fractile degree p_{ne} is 0.6 and 1.0.

Table 1. Streamflow data under various probabilities of occurrences

Regions	Streamflow levels	Probability (%)	Streamflow $\tilde{q}_j^{\pm}$ (10^8 m^3)	
			Underground water ($j = 1$)	Surface water ($j = 2$)
Shuangyashan	Low	0.2	$[(4.8, 5.0, 5.2), (5.8, 6.0, 6.2)]$	$[(11.1, 11.5, 11.9), (12.2, 12.6, 13.0)]$
	Medium	0.6	$[(6.3, 6.5, 6.7), (6.9, 7.1, 7.3)]$	$[(13.2, 13.6, 14.0), (15.3, 15.7, 16.1)]$
	High	0.2	$[(8.9, 9.1, 9.3), (10.7, 10.9, 11.1)]$	$[(17.8, 18.2, 18.6), (19.0, 19.4, 19.8)]$
Jiamusi	Low	0.2	$[(10.3, 10.6, 10.9), (13.5, 13.8, 14.1)]$	$[(22.0, 22.5, 23.0), (24.0, 24.5, 25.0)]$
	Medium	0.6	$[(14.5, 14.8, 15.1), (16.7, 17.0, 17.3)]$	$[(26.5, 27.0, 27.5), (29.5, 30.0, 30.5)]$
	High	0.2	$[(19.8, 20.1, 20.4), (23.6, 23.9, 24.2)]$	$[(31.5, 32.0, 32.5), (34.0, 34.5, 35.0)]$

4. Results analysis

Figure 1 provides the total cost of agricultural irrigation systems $f^{\pm}$ under different feasible degrees θ and fractile degrees p_{ne} . The results show that (1) the lower bound of total system cost $f^{\pm}$ will increase when the feasibility degree θ increases given fractile degree p_{ne} . The corresponding upper bound of total system cost will decrease when θ increases. For example, when $p_{ne}=0.6$, the lower and upper bounds of total system cost are 907.26 and 1023.17 under $\theta=0.5$, respectively. The lower and upper bounds of total system cost are 913.95 and 1022.87 under $\theta=0.9$, respectively. (2) the upper and lower bounds of total system cost $f^{\pm}$ will increase when p_{ne} decreases given θ . For example, when $\theta=0.7$, the lower and upper bounds of total system cost are 910.63 and 1023.03 under $p_{ne}=0.6$, respectively. The lower and upper bounds of total system cost are 901.85 and 1015.51 under $p_{ne}=1.0$, respectively. This is because (1) the decision maker has to choose a conservative water-allocation strategy when the feasibility degree increase, resulting that the uncertainty of total system cost decreases. (2) A higher fractile degree means is a higher preference of decision maker to objective function, and it will lead to a lower total system cost.

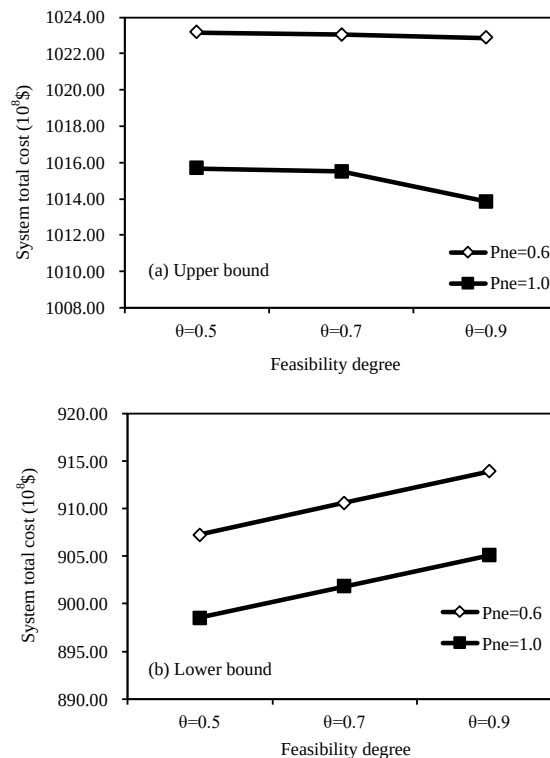


Figure 1. Total cost of agricultural irrigation system

Figure 2 provides the initial optimal water allocation target under feasibility degree $\theta=0.9$ and fractile degree $p_{ne}=0.6$. The water supplies of Shuangyashan and Jiamusi are mainly from surface water in order to obtain the minimum total cost of agricultural irrigation system given feasibility and fractile degrees. The highest initial optimal water allocation target in Shuangyashan is corn. The second highest initial optimal water allocation target is rice. And the lowest initial optimal water allocation target in Shuangyashan is bean. The initial optimal water allocation targets in Jiamusi is rice, corn and bean from high to low.

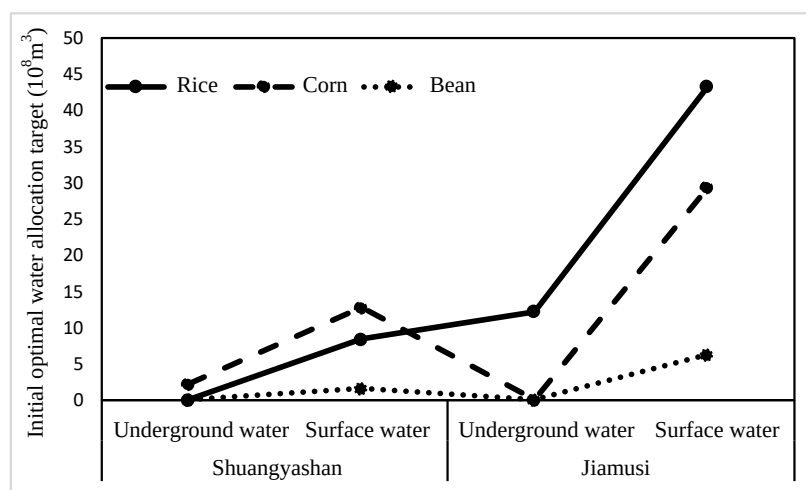


Figure 2. Allocation targets under feasibility degree $\theta=0.9$ and fractile value $p_{ne}=0.6$

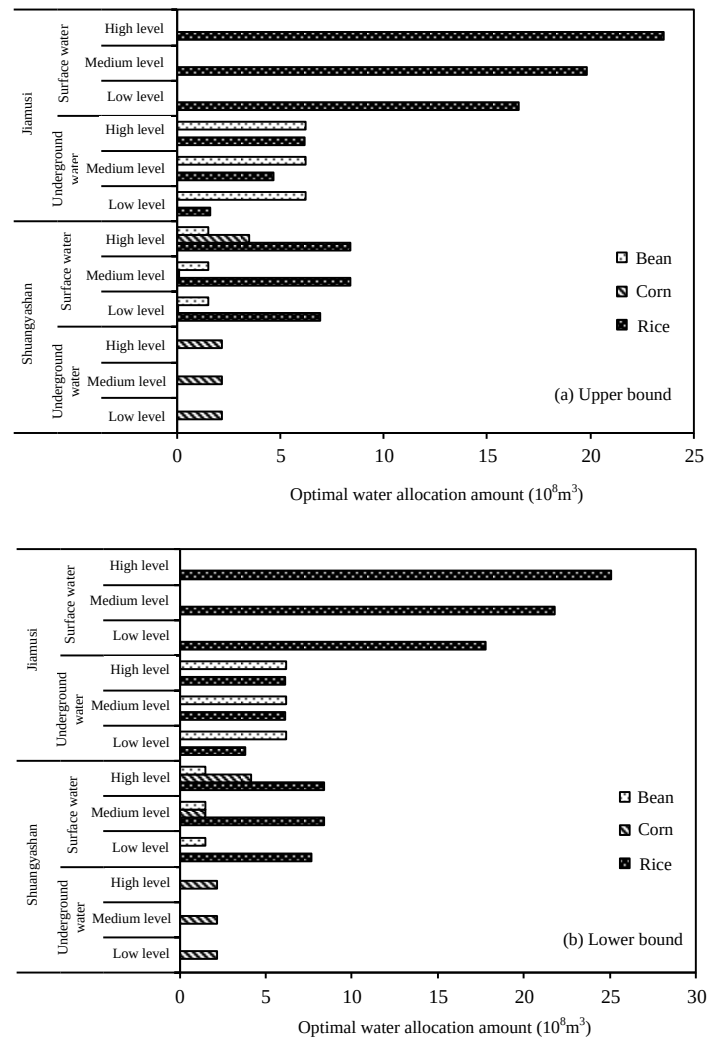


Figure 3. The optimal amount of water allocation under $\theta=0.7$ and $p_{ne}=1.0$

Based on the values of $Y_{ijk}^{\pm}$, an allocation scheme under $\theta=0.7$ and $p_{ne}=1.0$ is given as shown in Figure 3. The water source of rice is mainly from surface water in Jiamusi. The water allocation will decrease when the streamflow level of surface water decreases. The underground water is also supply water to rice, corn and bean. But the water source of corn is mainly from underground water in Shuangyashan. The optimal water allocation under different streamflow levels are almost the same. The surface water is also supply water to rice, corn and bean.

5. Conclusion

An interactive two-stage fractile optimization (ITFO) model is proposed to solve water-allocation problems in irrigation systems under uncertainty. It is capable for dealing with problems with inputs of random- and fuzzy interval-parameter. An irrigation management of Sanjiang Plain is provided to show the applicability of ITFO. Results show that (1) ITFO model can handle dual uncertainties in two sides of constraints and fuzzy information of objective function. (2) Optimal water-allocation strategies generated by ITFO model can reflect the preference of decision makers, balancing system satisfaction and feasibility degree. (3) The lower bound of total cost of agricultural irrigation system in Sanjiang Plain increases along with the increasing of feasibility degree. And the upper bound of total cost decreases along with the increasing of feasibility degree. (4) The upper and lower bounds of total cost decrease along with the increasing of fractile degree.

ITFO can deal with dual uncertainties exist in the agricultural irrigation systems. It is meaningful in theatrical and practical applications.

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