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# Natural stress estimation by solving inverse problem using acoustic sounding data

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**Abstract.** Under consideration is the approach allowing quantitative estimation of natural stress field components in the vicinity of underground objects based on solution of an inverse boundary-value problem using the data on acoustic sounding in an extended mine working. To ensure solvability of an inverse problem, the optimal sonic transmitter–receivers positioning and the limiting nominal error of measurements are determined. The objective function is set as a root-mean-square difference between the theoretical and actual head P-wave arrival times at the receivers; unimodality of the objective function is illustrated using synthetic input data.

## 1. Introduction

The quantitative information on stress-fields in the vicinity of study objects is of prime importance to solve theoretical and practical problems dealing with geomechanics, geophysics or mining engineering, viz., evaluation of rock mass stability in large-scale construction or mineral deposit development projects, protection of production wells in raw hydrocarbon production, selection of optimal mineral deposit development schemes.

The approach based on definition and solution of inverse problems is known as one of the most efficient methods to evaluate stresses. Its application in geomechanics to diagnose a state and mechanical properties of rocks is described in publications [1, 2]. Applicability of inverse problems to study geotechnical and natural objects is mainly restricted because of quasi-stationary character of most processes running in a rock mass, as well as high cost of in-situ measurements and, as a consequence, scanty volume of input data. Geophysical methods to receive input information are famous for relatively low costs, but interpretation of the obtained data requires special laboratory experiments to plot empirical relationships as “medium state equations” [3–5]. In particular, to realize the acoustic method implies availability of experimental relationships of velocities of elastic wave propagation in rocks versus stresses.

## 2. Estimation of vertical and horizontal components of the natural stress field

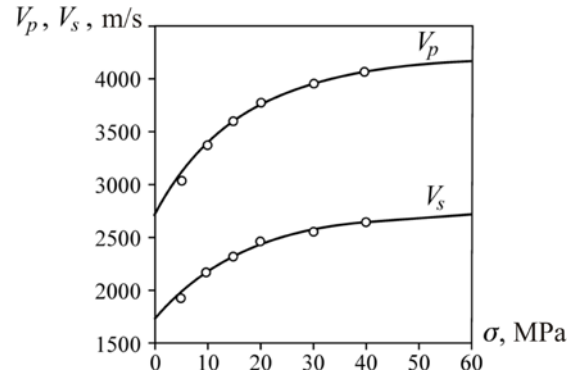
Let consider the approach based on inversion of acoustic sounding data in order to identify vertical and horizontal components of the natural stress field; the idea of the given approach is set forth in [6]. In the publication the input data were results on laboratory tests of argillite specimens under hydrostatical compression within pressure range  $0 \leq \sigma \leq 60$  MPa [7].

Figure 1 demonstrates experimental relationships of elastic waves  $V_p$  and  $V_s$  velocities vs. hydrostatic stress with approximation with function:



$$V_m(\sigma) = A_m - B_m \exp(-\alpha_m \sigma / \sigma_0), \quad m = p, s, \quad (1)$$

where  $\sigma_0 = 5$  MPa.



**Figure 1.** Empirical relationships of elastic wave propagation velocities, obtained from the laboratory experimental data [7].

In the Table 1, the coefficients (1) are calculated by the least squares method, and approximation error is  $\omega$ .

**Table 1.** Coefficients of approximation function and relative error.

$m$	$A_m$ , m/s	$B_m$ , m/s	$\alpha_m$	$\omega$
$p$	4212	1408	0.296	0.008
$s$	2740	1008	0.290	0.012

The study object is a horizontal arched mine working, which extension length is substantially greater than its cross-section dimensions. The study working is in a rock mass, modeled by a homogeneous isotropic elastic medium. The operative direction of the maximum horizontal component  $\sigma_H$  of the natural stress field coincides with orientation of the study working; the minimum component  $\sigma_h$  (cross line) is characterized by lateral factor  $q$ . This is in compliance with terms of the plane strain state [8].

Calculation domain  $R$  is a vertical section across extension of the working (Figure 2). The following boundary terms are formulated on  $\partial R$ :

$$\begin{aligned} \sigma_{zz}(x, 0) &= S, \quad \sigma_{xz}(x, 0) = 0; \\ u_z(x, Z) &= 0, \quad \sigma_{xz}(x, Z) = 0; \\ u_x(0, z) &= 0, \quad \sigma_{xz}(0, z) = 0; \\ \sigma_{xx}(X, z) &= q(S + \rho g z) \quad c, \end{aligned} \quad (2)$$

where  $\sigma_{ij}$  are components of stress tensor;  $u_i$  are displacements;  $S$  is lithostatic stress,  $\rho$  is medium density;  $(x, y, z)$  are Cartesian coordinates, axis  $y$  is directed along extension of the working;  $X$  and  $Z$  are horizontal and vertical dimensions of the domain.

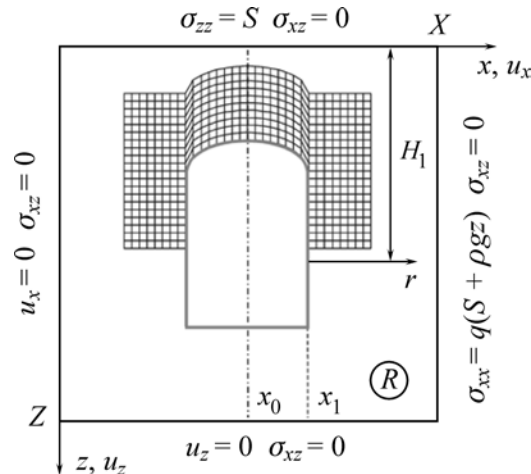
Deformation of the medium in its vicinity is described by equation set of the elasticity theory including equilibrium equation (3), Hook's law (4), and Cauchy relation (5):

$$\sigma_{ij,j} + \rho g \delta_{iz} = 0; \quad (3)$$

$$\sigma_{ij} = \lambda \varepsilon_{ij} + 2\mu \varepsilon_{ij}; \quad (4)$$

$$\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i}), \quad (5)$$

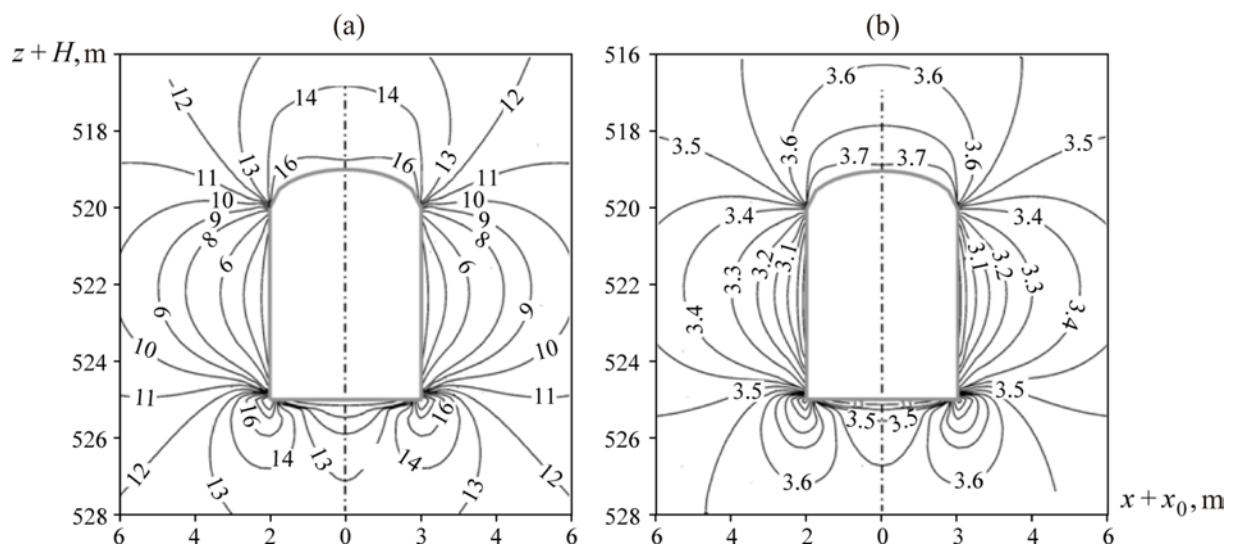
Direct problem (2)–(5) was solved by the finite element method using the original program code [9]. A fragment of calculation domain discretizing is presented in Figure 2.



**Figure 2.** Plan of calculation domain, a fragment of finite element grid, and boundary conditions.

In Figure 3 for  $X = 92$  m,  $Z = 46$  m,  $\rho = 2000$  kg/m<sup>3</sup>,  $\lambda = \mu = 8$  GPa,  $S=10$  MPa (stress corresponds to depth  $H=500$  m),  $q = 1.5$  lines of the average stress level  $\sigma = (\sigma_{xx} + \sigma_{zz})/2$  and velocity  $V_p(\sigma)$ , calculated from (1) are shown.

It is established based on the calculation results that the maximum variation in  $V_p$  is gained in the working wall at  $q > 1$ , and in its roof and floor at  $q < 1$ . It is imperative to consider direction of the sharpest variations in acoustic properties of the medium when it is necessary to select a measurement circuit in order to realize the proposed stress-evaluation process.



**Figure 3.** Isolines: (a) average stress  $\sigma$  (MPa); (b) P-wave velocity  $V_p$  (km/s).

Let us assume for certainty  $q > 1$ . We locate a sounding signal transmitter at depth  $H_1$  from the upper boundary of  $R$  area in one of working walls and receivers  $N$  with interval  $\Delta y$  between them longitudinally in the working. Let  $t_n$  be arrival time of the head wave at a receiver of coordinate  $y_n = n\Delta y$ . As  $t_n$  depends only on  $S$  and  $q$  (medium properties  $A_m$ ,  $B_m$ ,  $\alpha_m$ , and  $\rho$  being known), we introduce the objective function as a mean-square deviation between real  $t_n^0$  and “calculated”  $t_n$  time values

$$\Psi(S, q) = \sqrt{\frac{1}{N} \sum_{n=1}^N [t_n(S, q) - t_n^0]^2}. \quad (6)$$

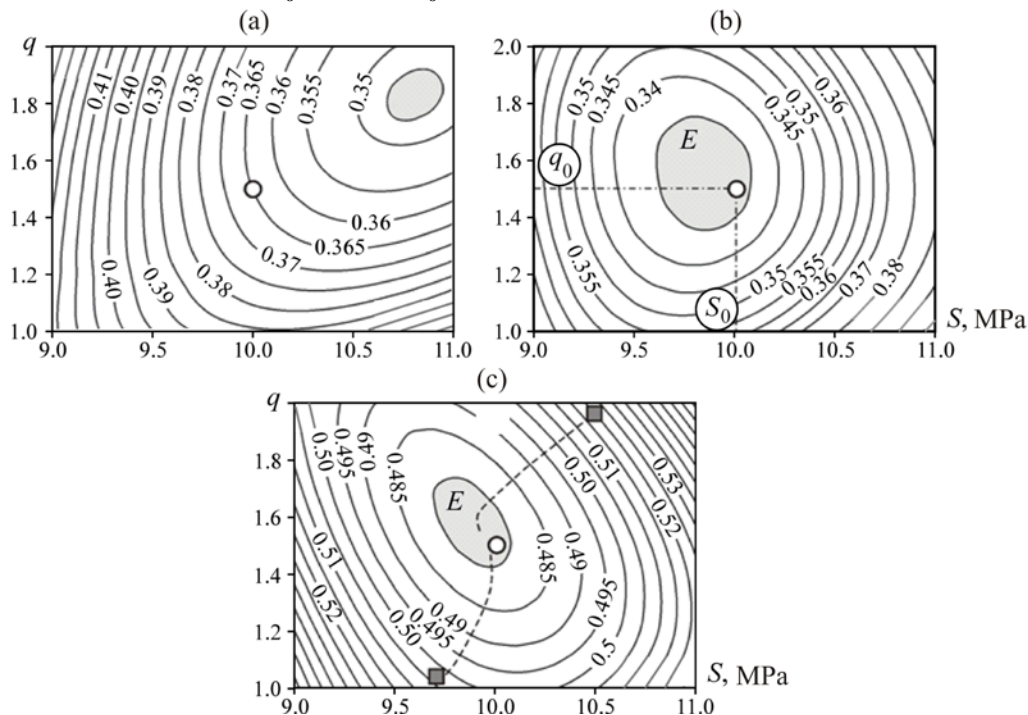
Values  $t_n$  can be determined by using the radial method for the gradient model of the medium in cross-section  $z = H_1$ , and the spatial distribution of wave velocities can be found from (1), considering (2)–(5). The head P-wave arrival-time curve is computed from relations [10]:

$$y_n(r) = 2 \int_0^r \frac{V_p(\xi) d\xi}{\sqrt{V^2(r) - V_p^2(\xi)}}, \quad t_n(r) = 2 \int_0^r \frac{V(r) d\xi}{V_p(\xi) \sqrt{V^2(r) - V_p^2(\xi)}}, \quad (7)$$

where  $V(r) = \max_{0 \leq \xi \leq r} V_p(\xi)$ ,  $r = x - x_1$ ,  $x_1$  — abscissa of working wall (Figure 2).

The minimum of the objective function  $\Psi$  is the solution of the boundary inverse problem to determine horizontal and vertical components of the natural stress field using the acoustic sounding data.

To investigate the solubility of the study problem, the researchers implement synthesizing of input data  $t_n^0 = (1 + \beta_n)t_n(S_0, q_0)$ , where  $\beta_n$  is a random magnitude, uniformly distributed on  $[-\gamma, \gamma]$ ,  $\gamma$  is a relative noise amplitude,  $S = S_0$  and  $q = q_0$  is a precise solution.



**Figure 4.** Isolines of target function  $\Psi$  (ms) at  $\gamma = 0.05$  for different number of receivers: (a)  $N = 25$ ; (b)  $N = 30$ ; (c)  $N = 35$ .

In Figure 4 isolines  $\Psi$  are for different number of receivers  $N$  at  $S_0 = 10$  MPa,  $q_0 = 1.5$ ,  $\gamma = 0.05$ ,  $H_1 = 22$  m,  $x_1 = 48$  m,  $r = 10$  m,  $\Delta y = 1.5$  m. Equivalence domain  $E$ , where a relative variation in  $\Psi$  does not exceed 2%, is shaded. Squares mark initial approximations, circles are for precise solution, dashed lines are computation paths. Modified gradient descent method is applied to search for minimum  $\Psi$  [11]. The objective function is unimodal at  $\gamma \leq 0.05$ , and its equivalence domain tends to reduce with increase in number of receivers. According to experimental numerical approximation results the extreme point of iteration process locates within domain  $E$  at any initial approximation, however, not at any number and/or spacing of receivers (Figure 4a):

$$E(S_0, q_0) \in E. \quad (8)$$

For  $S$  and  $q$  the variation range accepted in the present research and prescribed acoustic properties of rocks (Table) the optimal parameters of the recording system are  $N = 30\text{--}35$  and  $\Delta y = 1.5$  m (Figure 4b, c). Increase in relative noise amplitude  $\gamma$  in input data starting from 0.1, spacing variation  $\Delta y$ , number of receivers  $N$ , depth  $H_1$  are used to break unimodality conditions  $\Psi$  and/or (8).

### 3. Conclusions

To conclude, the approach set forth in the present research enables to make quantitative estimation of vertical and horizontal components of the natural stress field in the vicinity of a geomechanical object. The approach rests on solution of a boundary inverse problem using acoustic sounding data, recorded in an extended underground working, and empirical relationships of elastic wave velocities versus hydrostatic stress which were obtained based on laboratory experiments on rock specimens. It is established that the single-valued solubility of the inverse problem is feasible, provided that the noise level in input data should be no more than 10%.

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