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Strength of inhomogeneous rib pillars

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Abstract. The author analyzes stress state of a piecewise-homogeneous pillar with a curved interface. Based on the system of singular integral equations, relations are derived for stresses and displacements at the boundary of the pillar and at the interfaces of the pillar components. Examples of numerical implementation are given.

1. Introduction

In underground mines, various pillars serve as support, barriers or walls, i.e. elements intended to ensure safe and regular operation. Rib pillars are a kind of spacer assemblies which interact with enclosing rock mass in independent, conjoint or hybrid modes as practice shows [1].

Modern rock mechanics sets a stress on the analysis of nonuniform bodies. Knowledge on stress state is required to calculate strength, stiffness and stability of mine structures, all components of stresses and displacements at boundaries and interfaces are needed to anticipate possible failure.

A major difference of layered and uniform structures is more complex stress state of the former. It is impossible to predict which components of stresses and displacements can or cannot be neglected, which is finally governed by geometry and position of layers, and their mechanical constants.

The most of calculations in mechanics and engineering are carried out within the framework of elasticity. Mathematical modeling, permissive of a theoretical analysis of the solution, provides the fullest understanding of the behavior of a structure under loading.

2. The problem formulation and solution

This study addresses stress state at the boundary of a piecewise-homogeneous pillar with a curved interface for the primal boundary-value problems without actual finding of complex potentials $\varphi(z)$ and $\psi(z)$ in the domain under analysis [2, 3]. The speculation is based on the system of singular integral equations connecting boundary values of stresses and displacements [4].

Consider a piecewise uniform pillar, the scheme of which is shown in Figure 1. On the border of the presented region, various variants of boundary conditions are assumed within the framework of the three main problems of the theory of elasticity.
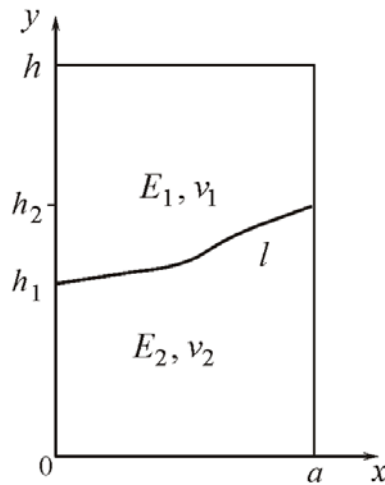
Figure 1. Schematic layout of piecewise-homogeneous pillar.

It is assumed that the interface of the pillar components experiences cohesion:

$$\sigma_n^+ = \sigma_n^-, \quad \tau_n^+ = \tau_n^-, \quad u^+ = u^-, \quad \upsilon^+ = \upsilon^-, \tag{1}$$

i.e. continuity of the normal and shearing stresses and displacements, where the upper sign implies belonging of the parameters to a respective pillar component.

The system of singular integral equations connecting boundary values of all stresses ad displacements in a general form for an arbitrary and simply connected domain [4] contains the function $f(t)$:

$$f(t) = i \int_0^t (X_n + iY_n) ds, \tag{2}$$

where X_n, Y_n—forces at the domain boundary in the lines of the axes x and y, respectively; t—point of the boundary; i—imaginary unit.

By way of illustration, we analyze a piecewise-homogeneous pillar (Figure 2) with the boundary $\Gamma = \Gamma_1 + \Gamma_2$ ($\Gamma_1 = \Gamma_{11} + \Gamma_{12} + \Gamma_{13} + \Gamma_{14}$, $\Gamma_2 = \Gamma_{21} + \Gamma_{22} + \Gamma_{23} + \Gamma_{24}$) and parabolic interface.

The boundary conditions are set as follows:

$$\sigma_n = \sigma_0 = -1, \quad u = 0 \quad \text{at the faces } 0 \le x \le a \text{ and } y = h,$$

$$\sigma_n = 0, \quad \tau_n = 0 \quad \text{at the faces } 0 \le y \le h, \; x = 0 \text{ and } x = a, \tag{3}$$

$$u = 0, \quad \upsilon = 0 \quad \text{at the faces } 0 \le x \le a \text{ and } y = 0,$$

where σ_n, τ_n are the normal and shearing stresses; u, υ are the horizontal and vertical displacements.

Based on [4–6] with regard to the condition (1), the relations are written to connect the boundary values of displacements and the function $f(t)$ for the pillar components and at their contact line l.
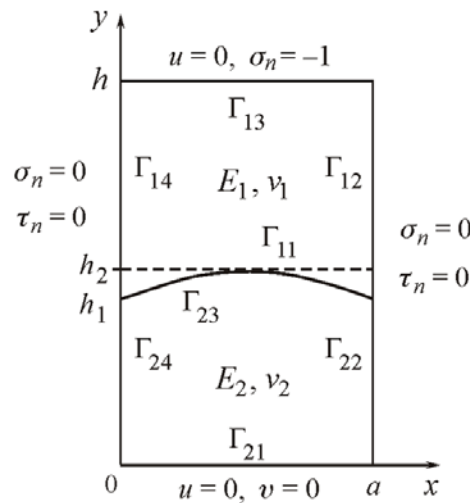
Figure 2. Analytical model of piecewise-homogenous pillar.

For the numerical implementation of the resultant system of equations, we go to dimensionless values such that the length dimension is related to the pillar width and the stress dimension is related to the value of stresses at the computational domain boundary.

The calculations presented in this paper were carried out at $h = 6$, $a = 1$, $v = 0.25$ for:

$$1)\ E_1 = 10^4, \quad E_2 = 3E_1; \quad 2)\ E_2 = 10^4, \quad E_1 = 3E_2, \tag{4}$$

as well as at different positions and shapes of the interface.

Figure 3 demonstrates deformation of the pillar boundary in variants (4) at $h_1 = 2$. The results are zoomed across the width for demonstrativeness.

The test case was a one-dimensional pillar with the interface represented by a parabola and a straight line. The calculated results agreed along the whole boundary of the pillar in both scenarios.
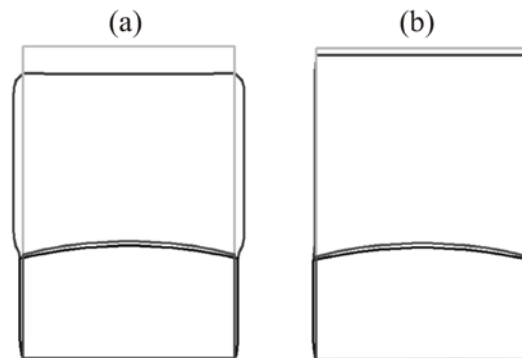

Figure 3. Deformation of pillar boundary: (a) $E_1 = 10^4$, $E_2 = 3E_1$; (b) $E_2 = 10^4$, $E_1 = 3E_2$.

Figure 4 depicts calculations for the horizontal and vertical displacements u, υ at the boundary of the upper pillar part. The boundary $\Gamma_1 = \Gamma_{11} + \Gamma_{12} + \Gamma_{13} + \Gamma_{14}$ is unfolded into a straight line. Curves *1–3* stand for the scenarios of $E_2 = 3E_1$, $E_1 = E_2$ and, $E_1 = 3E_2$, respectively.

The results are reflective of a complex structure of the stress state and the dependence of the solution on many parameters. The analysis of the functions (2) provides insight into the stress state both at the interface and the other branches of the boundary. To this effect, it is sufficient to differentiate the obtained results for the real and imaginary parts of *f*. The comparison of the results with the case

when $h_2 = h_1$ (i.e. l is a horizontal straight line) shows that even a minor increase in h_2 causes difference in the solution, which is the greatest in the neighborhood of the points $(0, h_1)$ and (a, h_1) (Figure 2).
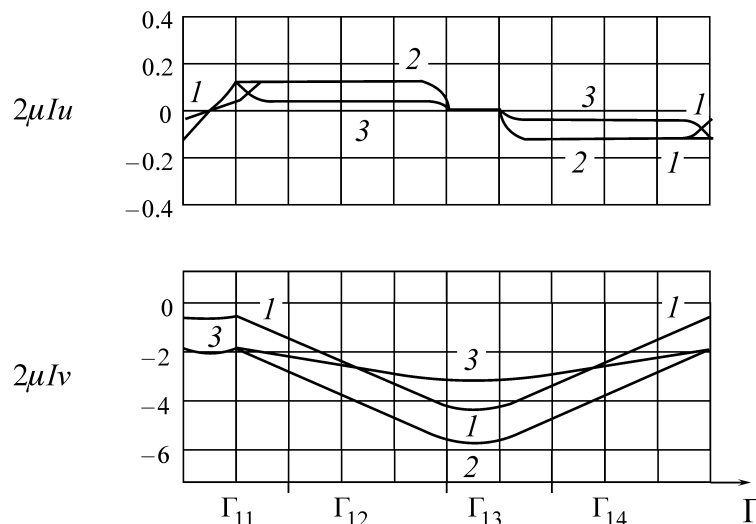
Figure 4. Displacements at the boundary of upper part of pillar.

3. Conclusions

The obtained relations and their implementation offer relevant information on all components of the normal and shearing stresses and displacements at the whole boundary of the pillar and at the interface of its components without factual determination of the stress state on the domain under study. The integral form of the solution allows both analyzing the solution itself and varying the involved parameters in order that solution possesses the wanted properties. The numerical implementation algorithm has been constructed. The obtained results have been analyzed.

References

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