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# A note on the strength and deformation properties of a some sandstone under three-point bending in the context of tension and compression behaviour

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**Abstract.** The results of the three-point bending test of the *Brenna* sandstone beam samples are presented. The values of strains at the peak strength of the tensile part was  $\varepsilon_{max}$ , the strength of three-point bending was  $\sigma_b$  and the modulus of *longitudinal* elasticity (of the tensile part) was  $E_b$  - all of these values were indicated. The values of constants obtained in the three-point bending tests were compared with those obtained in the uniaxial tensile and compressive tests and the Brazilian tests. For this *typical* sandstone, values of constants determined by the three-point bending tests are greater than those determined by the direct tensile tests. There were stated large differences in the tensile strength values ( $\sigma_b/\sigma_T \approx 3$ ) and relatively small differences in the elasticity modulus  $E$  ( $E_b/E_T \approx 1.2$ ).

## 1. Introduction

The uniaxial tensile strength of rocks  $\sigma_T$  is one of the fundamental material constants describing behaviour of rocks in the field of stresses. However, direct tensile tests are difficult to carry out. The accuracy requirements of rock samples, the surfaces of grips, the concentricity of joint system and the contact-live between the samples and the grips must be met. Strains measurement is also difficult.

These are the reasons why new methods are searched for- and the known indirect methods for determining the tensile strength of rocks and laboratory testing of rock behaviour in the field of tensile stresses are analysed [1].

Due to difficulties in preparing direct tensile tests using a sample-grip contact joint, comparative analyses are usually carried out based on the studies of other authors.

Bending tests of rock beams (e.g. [1]) are also very unique. Difficulties primarily come from challenges of providing appropriate rock samples with the recommended accuracy. Regardless of difficulties, knowledge about behaviour of rocks under bending is very important. *Deflection* of rock layers occurs when it comes to roofs of mine working chambers and longwall excavations, rectangular sectional tunnels or rising headings under high horizontal stress values.

Having the results of own tests obtained on the basis of direct tensile  $\sigma_T$  and Brazilian tests  $\sigma_B$  [2, 3], three-point bending tests of *Brenna* sandstone beam samples were conducted. The values of tensile strength were determined at three-point bending  $\sigma_b$ , strains at the peak strength  $\varepsilon_{max}$  and elasticity modulus of tensile part of a sample  $E_b$ .



## 2. Three-point bending test

In Polish sources, the phenomenon of rock bending was analysed first of all in the context of *deflection* of roof layers near a longwall mining.

Such introductory analyses were already published in 1955 by Sałustowicz [4], and later by Sałustowicz and Galanka [5]. Borecki and Chudek specify the strength of bending rocks as one of three fundamental material constants describing their strength properties [6]. Hobler [7] writes about the bending strength of rocks in the context of solving problems with roof stability of underground chambers and near the excavation faces. Bending strength is also one of the fundamental constants described by Kidybiński [8] in a comprehensive monograph about properties of rock subjecting the underground excavations and predicting behaviour of the rock mass during mining works. Nagaraj [9] writes about bending strength as the third of 4 constants describing the strength properties of rocks, in addition to compressive, tensile and shear strength.

Unfortunately, there are very few results of bending strength tests.

Hobler, identifying bending with tensile strength  $\sigma_T$ , describes that  $\sigma_{Tb}$  as calculated as:

$$\sigma_{Tb} = \frac{M_g}{W_g} \quad (1)$$

where:

$\sigma_{Tb}$  – tensile strength determined in the three-point bending test, Pa,

$M_g$  – the maximum bending moment corresponding to the failure (break) force, N,

$W_g$  – sectional modulus of bending strength, m<sup>3</sup>.

The maximal bending moment of the concentrated force  $F$  in the middle of the beam supported on two points is equal to:

$$M_g = \frac{F \cdot l}{4} \quad (2)$$

Moment of inertia of a rectangular section:

$$I = \frac{b \cdot h^3}{12}, \text{ m}^4 \quad (3)$$

Sectional modulus of bending strength:

$$W_g = \frac{b \cdot h^2}{6}, \text{ m}^3 \quad (4)$$

where:

$F$  – concentrated force, N,

$l$  – distance between point supports, m,

$h$  – height of the beam, m,

$b$  – width of the beam, m.

Thus for a rectangular section, the tensile strength under three-point bending is:

$$\sigma_{Tb} = \frac{2}{3} \cdot \frac{F \cdot l}{b \cdot h^2}, \text{ Pa} \quad (5)$$

However, few researches indicate that bending strength should not be identified with tensile strength, and  $\sigma_T = (0.5 \div 0.7) \sigma_{Tb}$ . According to Kidybiński, the ratio  $\sigma_T / \sigma_{Tb}$  is equal to: for coarse-grained sandstone 0.6, for fine and medium grained sandstone 0.7, for shales and coals 0.4.

Jastrzębski et al. [10] describe the bending effect as such during which "the initially straight axis of the beam curves, with the axial fibres of the beam from the convex side extending, and from the concave side they are shortening". If there are no lateral forces in the cross-section or they are negligible, and only bending moments along the main axis occur, then *pure bending* is considered.

The tasks on describing the strains of beam and accompanying stresses are easiest to solve if symmetrical pure bending occurs. "It occurs when all forces act in one plane, called the plane of forces, which is also the plane of beam symmetry [10]". Then, in cross-section of the beam only normal stresses  $\sigma_n$  act. There are no tangential stresses  $\tau$ .

Stresses and strains during bending can be only determined in a few simple load cases; the simplest is three-point bending. Nagaraj writes that in case of pure bending, determining the strength and modulus of deformations for tensioned external *fibres*, one can (probably) talk about determining the tensile strength and the elastic modulus of tensioning rocks.

### 3. Laboratory three-point bending tests of rock samples

Three-point bending tests of rock samples were carried out in the rock mechanics laboratory of the Department of Geomechanics and Underground Construction at Silesian University of Technology.

*Brenna* sandstone samples were tested. The properties of this sandstone were well known as a result of research conducted in previous years. At that time, uniaxial compression and tension, Brazilian and shear under compression were made [11].

The *Brenna* sandstone is a medium-solid grey sandstone with a green, fine-grained random texture.

In the sandstone, there is a grain matrix characteristic for the arenites. It consists of quartz grains, feldspars and pieces of rocks. The contacts between grains are numerous, which proves a large scale of deposit compaction; they run along irregular lines. There is no direction in distribution of matrix of grains; the matrix elements are bonded with a clay matrix and with a precipitated dolomite cement. The clay matrix is consisted of a fine aggregate illite. The clay adhesive has the characteristics of a pore-, occasionally, contact bonded.

The grain size distribution indicates that it is quite well sorted fine-grained sandstone. It consists of about 10% grains of medium-grains fraction.

The matrix grains are dominated by sharp-edged elements. There is a small amount of grains slightly coated. The degree of coating indicates a poorly matured sediment.

In the sandstone mineral composition there are quartz grains, feldspar, pieces of rocks and micas. The characteristic elements of *Brenna* are glauconitic aggregates uniformly distributed in the background of the rock matrix.

The quartz grains quench in a simple, sporadically wavy-grained mode. Few of them contain small inclusions of opaque minerals. The scales are represented by orthoclases, acidic plagioclases (oligoclase-andesines) and perestroits. These minerals are metamorphosed to varying degrees. In potassium feldspars, there is sericitization, and in plagioclases - caolithization.

A standard testing machine (compression/tension, 400kN) EDZ-40 by Werkstoffprüfmaschinen Leipzig (East Germany) was used for the tests (fig. 1). The piston moved out of the machine's working cylinder at a constant speed of 0.005 mm/s. Rectangular-beam samples measuring 15×5×5cm (h×a×b) were placed on a steel plate with hardness over 45 HRC on two bolts. The distance between the bolts was 11.5cm. The sample was loaded in the middle of its length by a ball-and-socket joint (figure 2).

To measure longitudinal strain  $\varepsilon_x$  were used electro-resistant strain gauges RL350/30/2.15 (manufactured by "Techno-Mechanik", Gdańsk) and one two-channel strain gauge extensometer CMT-831 (produced by "Techno-Mechanik", licensed by the Gdańsk University of Technology). A system of three strain gauges connected in series was placed in the middle of the sample height, vertically to its longitudinal axis (figure 3a, b).



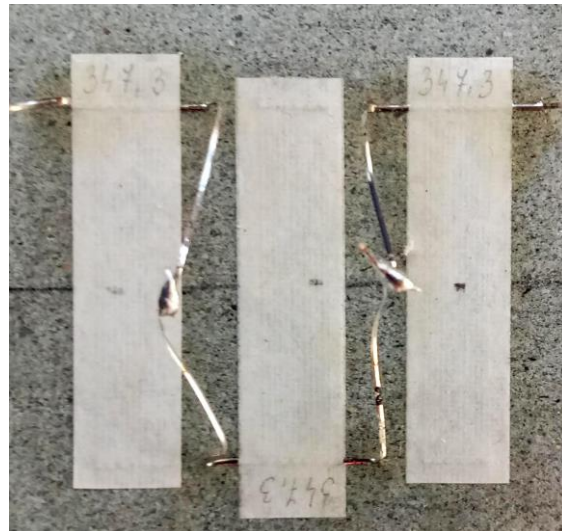
**Figure 1.** EDZ-40 testing machine with measuring and recording system.



**Figure 2.** Brenna sandstone beam placed on the pins of the device for performing three-point bending tests.

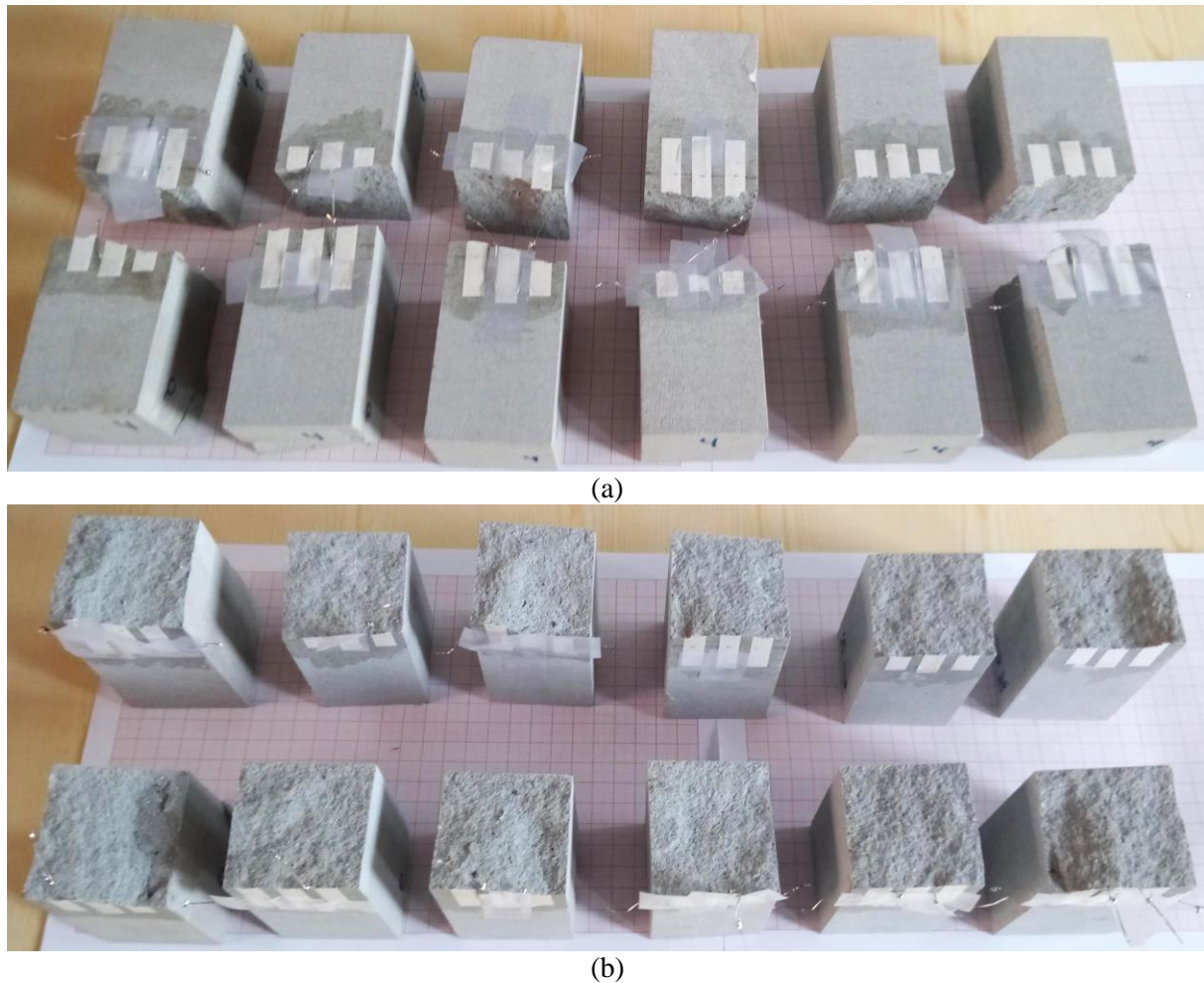


(a)



(b)

**Figure 3.** Rectangular *Brenna* sandstone samples with glued electro-resistant strain gauges (a) and view of RL350/30/2.15 strain gauges connected in series (b).



**Figure 4.** *Brenna* sandstone beams after bending tests: top view of the tensile part (a) and a view of the macro-cracking planes (b).

All *Brenna* sandstone beams got failure along their cross-sectional or close to it area (figure 4a, b). The failure was accompanied by a clear sound effect, displacement of both parts of the sample and a sharp drop of load force.

Due to the accuracy of the measurement recording system, it was not possible to obtain the full stress-strain characteristics. The maximum failure load  $F_{max}$  and (tensile) strain at the failure  $\varepsilon_{x_{max}}$  were recorded.

#### 4. Analysis of the results of three-point bending tests of *Brenna* sandstone samples. Comparison of bending strength with uniaxial compression and tension and Brazilian test strength

On the basis of the three-point bending tests, the values of maximum failure force  $F_{max}$  and strains at the failure  $\varepsilon_{x_{max}}$  were obtained.

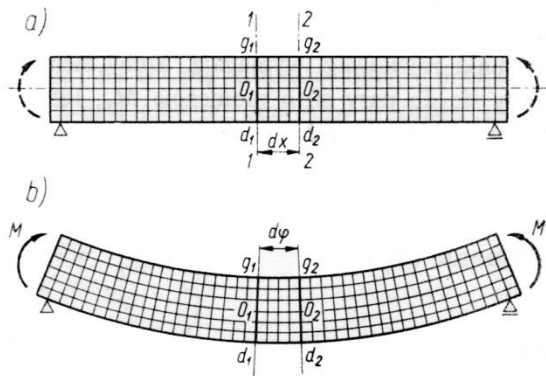
The value of bending strength was calculated:

$$\sigma_b = \frac{3}{2} \cdot \frac{F_{max} \cdot l}{b \cdot h^2}, \text{ Pa} \quad (6)$$

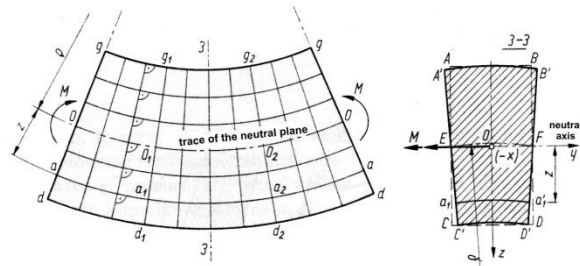
Assuming that the bending strength can be equal to the tensile strength when bending:

$$\sigma_b = \sigma_{Tb} \quad (7)$$

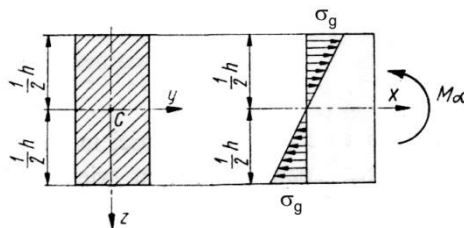




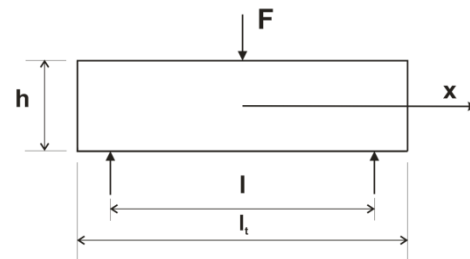
**Figure 5.** The beam with the applied grid has been bent; before (a) and during bending;  $M$  - concentrated moment (bending moment),  $d$ ,  $O$  and  $g$  - points on the lower, neutral and upper axis,  $\varphi$  - distance of deflection of the upper fibres [10].



**Figure 6.** Enlarged view of the middle part of the beam and cross section 3-3;  $A, a, B, C, D, E, F, G$  - characteristic points before starting the loading, " ' " - during loading. In practice, we can assume that the lateral strains are so small that the shape and dimensions of the neutral cross-section  $ABCD$  remain unchanged [10].



**Figure 7.** Stress characteristics  $\sigma_g$  of rectangular cross-section;  $M_\alpha$  - bending moment in cross section, corresponds to  $M_g$  [10].



**Figure 8.** Scheme of loading *Brenna* sandstone beams.

The values of the axial elasticity modulus:

If the relative *elongation* (strain) of the longitudinal *fibers* lying on the tensile part of the bar (figure 5 and 6) [10]:

$$\varepsilon_x = \frac{z}{\delta} \quad (8)$$

where:

$\delta$  – radius of curve, m,

$z$  – distance from the neutral axis; for a rectangular section  $z = 0.5h$ .

If the maximum normal stresses  $\sigma_x$  change linearly with the height of the cross-section of the beam and their extreme values occur in the outermost fibres furthest from the neutral axis, then we can write:

$$\sigma_x = \frac{E \cdot z}{\delta} \quad (9)$$

therefore:

$$E_b = \frac{\sigma_x \cdot \delta}{z} \quad (10)$$

where:

$E_b$  - modulus of axial elasticity for tensile fibers, Pa.

also (figure 7 and 8; following [10]):

$$\sigma_x = \sigma_b = \frac{M_g}{W_g} \quad (11)$$

Knowing the value of bending strength  $\sigma_b (= \sigma_x)$  and strain  $\varepsilon_x$ , it is possible to determine the value of elastic modulus  $E_b$  for the tensile fibres (parts) of the beam.

The results of previous *Brenna* sandstone tests subjected to uniaxial compression and tension and Brazilian tension tests are presented in table 1 [2, 3]. Average values of constants were equal to:

- uniaxial compression strength  $\sigma_c=94,86\text{MPa}$ ;
- axial strain at failure (under compression)  $\varepsilon_{max}=0.63\%$ ;
- elastic modulus (under compression)  $E_c=12.99\text{GPa}$ ;
- uniaxial tension strength  $\sigma_T=3,18\text{MPa}$ ;
- axial strain at failure (under tension)  $\varepsilon_{max}=0.07\%$ ;
- elastic modulus (under tension)  $E_T=4.81\text{GPa}$ ;
- tensile strength in Brazilian test  $\sigma_B=5.68\text{MPa}$ .

**Table 1.** *Brenna* sandstone parameters obtained on the basis of uniaxial compression, uniaxial tension and the Brazilian tests [2, 3].

Uniaxial compression tests $h:d=2.0, h=84\text{mm}$				Uniaxial tension tests $h:d=4.0, h=168\text{mm}$				Brazilian tests $h:d=0.5, h=21\text{mm}$	
No.	$\sigma_c$ (MPa)	$\varepsilon_{max}$ (%)	$E_c$ (GPa)	No.	$\sigma_T$ (Pa)	$\varepsilon_{max}$ (%)	$E_T$ (GPa)	No.	$\sigma_B$ (MPa)
B1	95.80	0.65	12.76	B20	3.15	0.07	4.89	B41	5.48
B2	97.40	0.62	13.85	B21	3.54	0.09	4.13	B42	6.42
B10	92.30	0.62	12.96	B23	3.63	0.08	4.46	B43	5.10
B11	93.70	0.62	12.44	B30	2.74	0.04	6.65	B44	5.74
B13	95.10	0.63	12.96	B31	2.84	0.09	3.93	B45	5.67
average	94.86	0.63	12.99	average	3.18	0.07	4.81	average	5.68

The values of constants determined and calculated on the basis of the bending tests that were carried out are presented in table 2 (see also figure 9 and 10). The values of strengths for three-point bending  $\sigma_{Tb}(=\sigma_b)$  from  $8.98\div 10.14\text{MPa}$ , axial strain at failure  $\varepsilon_{xmax}$  from  $0.16\div 0.17\%$  and modulus of elasticity  $E_b$  for tensioned fibres (part) from  $5,32\div 5,98\text{GPa}$ . Differences in the calculated values were small.

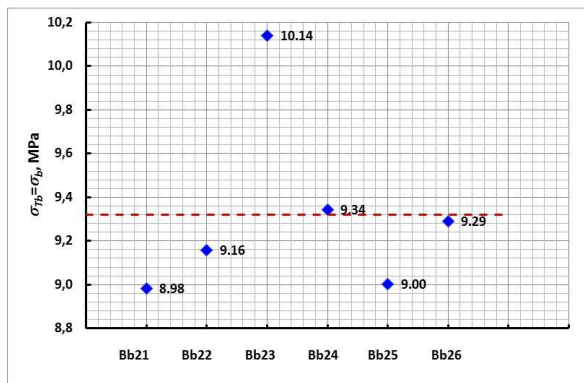
The average values were equal to:

- three-point bending strength  $\sigma_{Tb}(=\sigma_b)=9.32\text{MPa}$ ;
- axial strains at failure  $\varepsilon_{xmax}=0.17\%$ ;
- elastic modulus for tensioned fibres (part)  $E_b=5.62\text{GPa}$ .

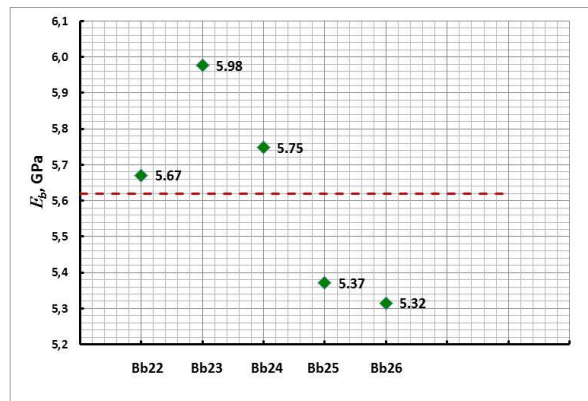


**Table 2.** Basic data on beam dimensions and calculated constant values;  $h$  - beam height,  $b$  - beam width,  $l_c$  - distance between supports (see figure 8),  $l$  - sample length,  $F_{max}$  - failure force,  $V_{bridge}$  - maximum voltage for the channel of strain at the moment of sample failure,  $\varepsilon_{xmax}$  - strains along the  $x$  axis at the strength,  $\sigma_{Tb}(=\sigma_b)$  - tensile strength in a three-point bending test,  $E_b$  - modulus of elasticity for fibres (parts) subjected to tensile stresses along the  $x$ -axis.

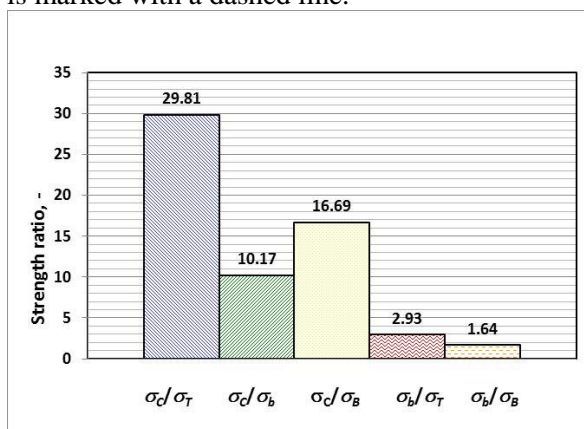
4. No.	$h$ (mm)	$b$ (mm)	$l_c$ (mm)	$l$ (mm)	$F_{max}$ (kN)	$V_{bridge}$ (mV)	$\varepsilon_{xmax}$ (%)	$\sigma_{Tb}(=\sigma_b)$ (MPa)	$E_b$ (GPa)
Bb21	53.2	51.9	153.0	115.0	7.65	-	-	8.98	-
Bb22	49.7	49.8	153.0	115.0	6.53	158	0.16	9.16	5.67
Bb23	49.5	49.9	153.0	115.0	7.19	166	0.17	10.14	5.98
Bb24	53.7	49.6	152.0	115.0	7.75	159	0.16	9.34	5.75
Bb25	49.8	50.0	152.0	115.0	6.47	164	0.17	9.00	5.37
Bb26	49.5	53.5	152.0	115.0	7.06	171	0.17	9.29	5.32
average							0.17	9.32	5.62



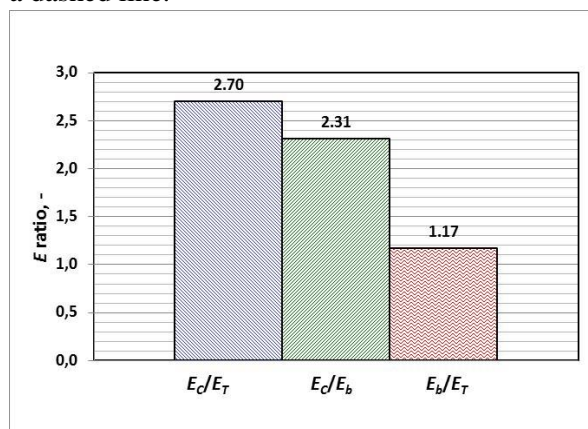
**Figure 9.** Values of three-point bending strength  $\sigma_{Tb}$  ( $=\sigma_b$ ) of Brenna sandstone; the average value is marked with a dashed line.



**Figure 10.** Elastic modulus  $E_b$  for tensioned fibres (part); the average value is marked with a dashed line.



**Figure 11.** Ratios of the values of strength  $\sigma$  determined for different types of tests.



**Figure 12.** Ratios of elastic modulus  $E$  for different types of tests.

Comparing the values of constants obtained on the basis of three-point bending tests with constants obtained on the basis of uniaxial compression and tension tests and with the Brazilian test, it was found that their ratios are as follows (figure 11 and 12):

- uniaxial compression strength  $\sigma_C$  to uniaxial tensile strength  $\sigma_T$   $\sigma_C/\sigma_T=29.81$ ;
- uniaxial compression strength  $\sigma_C$  to three-point bending strength  $\sigma_b$   $\sigma_C/\sigma_b=10.11$ ;
- uniaxial compression strength  $\sigma_C$  to tensile strength determined by the Brazilian test  $\sigma_B$   $\sigma_C/\sigma_B=16.69$ ;
- three-point bending strength  $\sigma_b$  to uniaxial tensile strength  $\sigma_T$   $\sigma_b/\sigma_T=2.93$ ;
- three-point bending strength  $\sigma_b$  to tensile strength determined by the Brazilian test  $\sigma_B$   $\sigma_b/\sigma_B=2.93$ ;
- elasticity modulus at the uniaxial compression  $E_C$  to the elasticity modulus under tension  $E_T$   $E_C/E_T=2.70$ ;
- elasticity modulus at the uniaxial compression  $E_C$  to the elasticity modulus at three-point bending (for the tensile part)  $E_b$   $E_C/E_b=2.31$ ;
- elasticity modulus at three-point bending (for the tensile part)  $E_b$  to the elasticity modulus at the uniaxial tension  $E_T$   $E_b/E_T=1.17$ .

The presented results of laboratory tests clearly indicate differences between the values of constants determined for three-point bending and tensile tests using the direct tension method. For *Brenna* sandstone, these differences are very clear. The bending strength  $\sigma_b$  is almost 3 times higher than the uniaxial tensile strength, and the modulus of elasticity  $E_b$  is around 1.2 times greater. Therefore, one cannot identify these constants determined by these two methods.

## 5. Conclusions

Three-point bending of *Brenna* sandstone beams was carried out. Laboratory tests were conducted to determine the basic constants. The values of strains at the failure for the tensile part  $\varepsilon_{xmax}$ , the three-point bending (ultimate) strength  $\sigma_b$  and the elasticity modulus (of the tensile part)  $E_b$  were determined. The following average values were obtained:  $\varepsilon_{xmax}=0.17\%$ ,  $\sigma_b=9.32\text{MPa}$  and  $E_b=5.62\text{GPa}$ . The values of constants obtained in three-point bending tests were compared, among others with values obtained in uniaxial tension tests.

There were large differences in tensile strength values ( $\sigma_b/\sigma_T\approx 3$ ) and relatively small differences in elastic modulus ( $E_b/E_T\approx 1.2$ );  $b$  - three-point bending,  $T$  - direct tension.

Summarizing:

- generally, there is very few data on constants of rocks determined in bending tests; this is probably due to difficulties in performing bending tests and direct tensile tests, among others, such as: high-grade accuracy of beam samples preparation, use of a precise contact resin-bonded joint, measuring axial small values strains and collinearity of grips;
- although generally it is assumed that the bending strength is greater than the direct tensile strength ( $\sigma_b > \sigma_T$ ) (e.g. Kidybiński [8] p. 51, table 1.3 and 1.4 also Vulukuri et al. [13] p. 134, table 2), there are also opinions that  $\sigma_b = \sigma_T$  (e.g. [7] and [9]);
- if *typical* sandstone tests are conducted the values of constants determined in the three-point bending tests are greater than those determined in the direct tensile tests; differences in values of parameters increase as the strength of rocks decreases;
- in mining we often talk about *deflection* (bending) of floor layers, including i.e. context of operation with a fall of roof layers, stability of chamber workings or tunnels with a rectangular cross-section shape. Also in the case of vertical workings the phenomenon of curving vertical surface of the contour is seen. Unfortunately, we have little information about the parameters of rock bending phenomenon [see i.e. [12)];

- if the strength and deformation properties of rocks are so different for uniaxial tension and three-point bending (for the tensile part of beam), it seems advisable to determine the properties of rocks during bending.

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