Elliptical shape and size approximation of a particle contour

IOP Conf. Series: Earth and Environmental Science **261** (2019) 012013 doi:10.1088/1755-1315/261/1/012013

Elliptical shape and size approximation of a particle contour

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Abstract. The paper presents an approximation of the particle contour by means of an ellipse with equivalent moments of inertia along both axes perpendicular to each other. This method ensures higher accuracy of size approximation than circular and rectangular approximation methods. The proposed method can be applied to machine vision systems used for particle size distribution analysis. In this method, there are simultaneously determined two (instead of one) characteristic dimensions. From these two dimensions as a decisive criterion there is chosen the smaller one, because it has the greatest importance for the particle behavior of the grain in the screening process.

1. Introduction

Mineral particles have very diverse shapes. These shapes are usually a result of the mechanical or explosive mining and of the crushing process. Therefore it is difficult to calculate their individual volume and mass and to predict the quantitative result of the sieving process. This is necessary in order to calculate the particle size distribution of the particulate material stream (e.g. on the conveyor belt). The problem is particularly important for machine vision based granulometric systems, as they use only an indirect method of size measurement, based on the analysis of recorded 2-dimensional or sometimes 3-dimensional images [1], [2], [3]. One of the most important tasks in the vision based granulometric analysis is an image segmentation. The image segmentation specifies a process of individual particle contour delineation on the background the selected particulate material stream area. Mineral particle as a 3-dimensional block can be generally characterized by the set of 3 main dimensions (*a,b,c*). These dimensions have been shown in figure 1. When the particle is placed in the mechanically stable position (with the lowest potential energy i.e. with lowest position of the center of gravity) e.g. lies on a flat surface like the conveyor belt, the dimensions can be ordered as

$$a \geq b \geq c \tag{1}$$

so only the largest (a) and the intermediate (b) particle dimensions are visible from the top and can be calculated using an 2-D image analysis technology. The third (usually lowest) dimension c can be only partially assessed using 3-d image acquisition technology or sometimes using some statistical dependencies based on empirical measurements of the stream sample.

But when these particle is subjected to the sifting process the result of the sieving depends on the two lowest dimensions (b and c). As usually there is valid a relation $b \geq c$ it can be deduced that the most decisive criterion for predicting the result of the sieving process is the intermediate b dimension. Fortunately, the intermediate b dimension can be extracted from the 2-d or 3-d image. The process of calculation of the characteristic (a & b) dimensions (perpendicular to each other parallel to the XY image plane) is the main topic of this paper. It should be noted, that the maximum dimension (a)can be also important for the performance of the sieving process in some special cases – particularly in the case of very elongated particles (when $a \gg b$). This is due to the fact that for the very elongated particles the probability of the vertical position during sieving like in figure 1b is quite low. Detailed analysis of this case is beyond the scope of this paper, but it can be noted that this maximum dimension can be also measured using a proposed method.

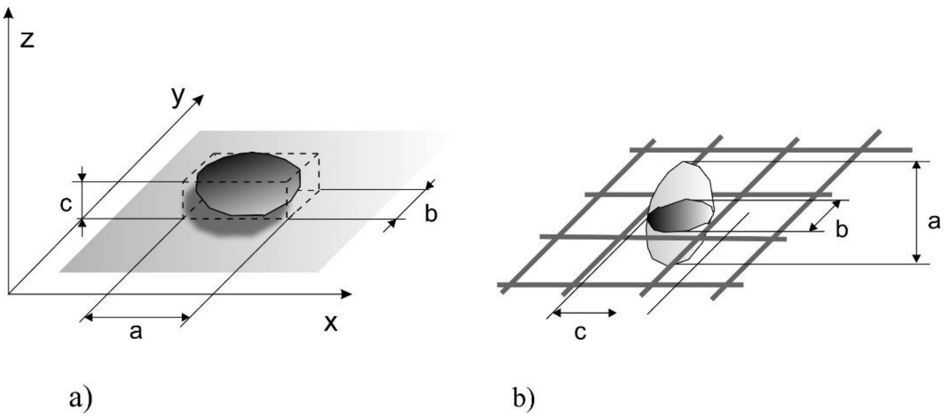

Figure 1. Mineral particle dimensions and their impact on the sieving process: a) hree orthogonal particle dimensions, b) Impact of these particular dimensions on the sieving process precision.

From the geometric point of view, the best description of the mineral particle dimension is given by a so-called Feret diameter [4]. Feret diameter (in a 2-dimensional case) is a distance between two parallel lines touching and tangent to the particle contour. In a 3-dimensional case it is a distance between two parallel surfaces touching and tangent to the particle surface. The process is like placing a particle between the calliper jaws (so the Feret diameter is sometimes called a calliper diameter). This measurement is highly dependent on the particle orientation. Therefore it is a quite cumbersome and time-consuming method, difficult for efficient implementation [5]. So there are widely used methods of equivalent diameter calculation, based on a kind of equivalence between real particle parameters and some simplified shape parameters. A comprehensive comparison of the shape coefficients definitions and their properties can be found in [6]. In some applications there can be even used machine learning methods as neural networks [7].

2. Circular approximation of the particle shape

The simplest way to approximate a characteristic dimension of the mineral particle is a circular approximation – i.e. approximation of an irregular shape by a circle with the same area. Area of the mineral particle projection onto the image plane can be calculated as

$$A = k_p \sum_{\substack{x \in \langle x_{min}, x_{max} \rangle \\ y \in \langle y_{min}, y_{max} \rangle}} I_B(x, y) \qquad (2)$$

where pixels of the binary image have values

$$I_B(x, y) = \begin{cases} 1 \; where \; point \; (x, y) \; is \; inside \; particle \; contour \\ 0 \; where \; point \; (x, y) \; is \; outside \; particle \; contour \end{cases} \qquad (3)$$

and k_p denotes a scaling coefficient.

Diameter of the equivalent circle can be therefore calculated as

$$D_0 = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{\sum_{\substack{x \in \langle x_{min}, x_{max} \rangle \\ y \in \langle y_{min}, y_{max} \rangle}} I_B(x, y)}{k_p \cdot \pi}} \qquad (4)$$

This approach is computationally simple, but can lead to significant errors, particularly for elongated particles. In case of these elongated particles this most important intermediate dimension is prolonged, and the maximum dimension is shortened. This effect has been shown in figure 2.

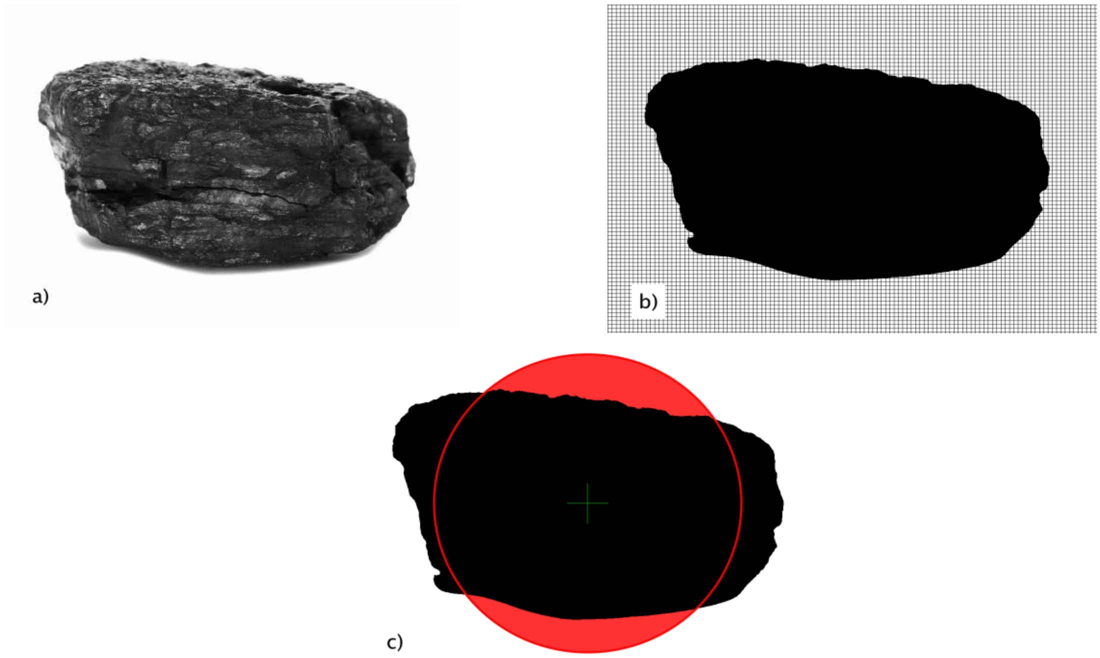

Figure 2. Circular approximation of particle with greater value of elongation coefficient: a) particle image, b) particle contour, c) comparison of characteristic dimensions of particle and its circular approximation.

3. Rectangular approximation of the particle shape

Rectangular approximation of the particle size is apparently even simpler as the dimension can be calculated just as the coordinate difference of the outermost particle contour points like

$$a = max(x_{max} - x_{min}, y_{max} - y_{min}) \tag{5}$$

$$b = min(x_{max} - x_{min}, y_{max} - y_{min}) \tag{6}$$

This simple approach works fine, only when the particle is placed alongside the main directions of the image coordinate system (image pixels rows and columns). When the particle is arranged at some angle in relation to the image rows and columns system the calculated dimensions differ from the real ones. Both situation have been shown in figure 3. For each of the cases, there can be easily calculated coordinates (x_c,y_c) of the particle projection gravity center from the following equations:

$$x_c = \frac{\sum_{x\in\langle x_{min},x_{max}\rangle}\sum_{y\in\langle y_{min},y_{max}\rangle} x\cdot I_B(x,y)}{\sum_{x\in\langle x_{min},x_{max}\rangle}\sum_{y\in\langle y_{min},y_{max}\rangle} I_B(x,y)} \tag{7}$$

$$y_c = \frac{\sum_{x\in\langle x_{min},x_{max}\rangle}\sum_{y\in\langle y_{min},y_{max}\rangle} y\cdot I_B(x,y)}{\sum_{x\in\langle x_{min},x_{max}\rangle}\sum_{y\in\langle y_{min},y_{max}\rangle} I_B(x,y)} \tag{8}$$

The gravity center coordinates (x_c,y_c) calculated for each of the delineated particles can be used as an origin of the local coordinate system. This approach greatly simplifies further calculations, as the ellipse centered at the coordinate system origin has much simpler mathematical description [8].

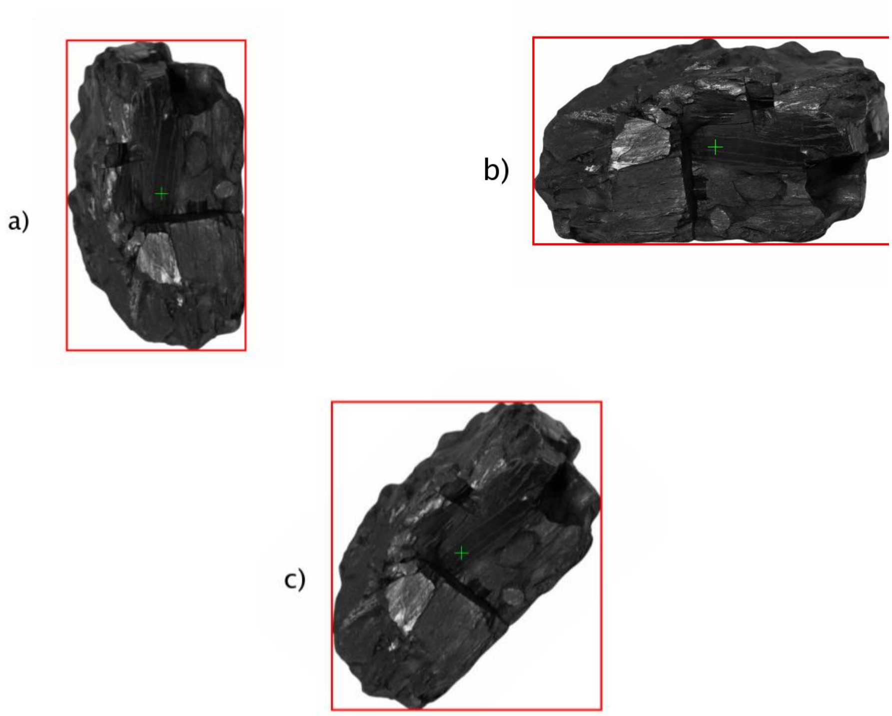

Figure 3. Effect of mineral particle position on the approximation result of a rectangle describing its contour: a) and b) coal particle aligned with the direction of the axis of the coordinate system c) coal particle not aligned with the direction of the axis of the coordinate system.

4. Elliptical approximation of the particle shape and size

Approximation of the particle shape by an ellipse instead of a circle can cope with with the problem of particle elongation as well as with the problem of slanted particle position. The second order (inertia) moments M_{xx}, M_{yy} M_{xy} of the particle along two main axes x and y can be calculated as

$$M_{xx} = \sum_{\substack{x\in\langle x_{min}, x_{max}\rangle \\ y\in\langle y_{min}, y_{max}\rangle}} (x - x_c)^2 I_B\,(x, y) \tag{9}$$

$$M_{yy} = \sum_{\substack{x\in\langle x_{min}, x_{max}\rangle \\ y\in\langle y_{min}, y_{max}\rangle}} (y - y_c)^2 I_B\,(x, y) \tag{10}$$

$$M_{xy} = \sum_{\substack{x\in\langle x_{min}, x_{max}\rangle \\ y\in\langle y_{min}, y_{max}\rangle}} (x - x_c)(y - y_c)^2 I_B\,(x, y) \tag{11}$$

When these inertia moments M_{xx}, M_{yy} M_{xy} are divided by the value of particle projection area A they form normalized inertia moments m_{xx}, m_{xy}, m_{yy} as

$$m_{xx} = \frac{M_{xx}}{A} \tag{12}$$

$$m_{yy} = \frac{M_{yy}}{A} \tag{13}$$

$$m_{xy} = \frac{M_{yy}}{A} \tag{14}$$

General ellipse centred at the local coordinate origin (0,0) can be described by an equation

$$Ax^2 + By^2 + 2Cxy = 1 \tag{15}$$

or in matrix form as:

$$[x \quad y] \cdot \begin{bmatrix} A & C \\ C & B \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 1 \tag{16}$$

When the particle approximating ellipse is aligned horizontally or vertically C=0, but in the general case C≠0. This particle approximating ellipse is limited by the set of extreme points [1] presented in figure 4 and described in table 1.

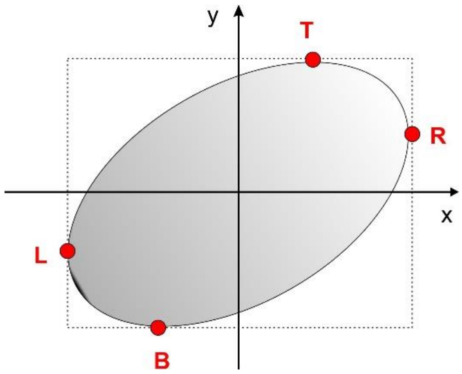

Figure 4. Extreme points defining integration limits for calculating geometric inertia moments of elliptical particle contour approximation.

Table 1. Extreme points defining ellipse integration limits.

Extreme points	Horizontal coordinate (image pixel column)	Vertical coordinate (image pixel row)
Topmost (T)	$x_T = \frac{-C}{\sqrt{A(AB-C^2)}}$	$y_T = \frac{\sqrt{A}}{\sqrt{AB-C^2}}$
Bottommost (B)	$x_B = \frac{C}{\sqrt{A(AB-C^2)}}$	$y_B = \frac{-\sqrt{A}}{\sqrt{AB-C^2}}$
Leftmost (L)	$x_L = \frac{-\sqrt{B}}{\sqrt{AB-C^2}}$	$y_L = \frac{C}{\sqrt{B(AB-C^2)}}$
Rightmost (R)	$x_R = \frac{\sqrt{B}}{\sqrt{AB-C^2}}$	$y_R = \frac{C}{\sqrt{B(AB-C^2)}}$

The area of the ellipse can be calculated as

$$S = \int_{x_L}^{x_R} \int_{y_{B(x)}}^{y_T(x)} dxdy = \frac{\pi}{\sqrt{AB-C^2}} \tag{17}$$

and the geometrical inertia moments as

$$M_{xx} = \int_{x_L}^{x_R} \int_{y_B(x)}^{y_T(x)} x^2 dxdy = \frac{\pi B}{4(AB-C^2)^{\frac{3}{2}}} \tag{18}$$

$$M_{yy} = \int_{x_L}^{x_R} \int_{y_B(x)}^{y_T(x)} y^2 dxdy = \frac{\pi A}{4(AB-C^2)^{\frac{3}{2}}} \tag{19}$$

$$M_{xy} = \int_{x_L}^{x_R} \int_{y_B(x)}^{y_T(x)} xydxdy = \frac{-\pi C}{4(AB-C^2)^{\frac{3}{2}}} \tag{20}$$

Normalized inertia moments can be therefore calculated as

$$m_{xx} = \frac{M_{xx}}{S} = \frac{B}{4(AB-C^2)} \tag{21}$$

$$m_{yy} = \frac{M_{yy}}{S} = \frac{A}{4(AB-C^2)} \tag{22}$$

$$m_{xy} = \frac{M_{xy}}{S} = \frac{-C}{4(AB-C^2)} \tag{23}$$

Image covariance matrix can be expressed as

$$M = \begin{bmatrix} m_{xx} & m_{xy} \\ m_{xy} & m_{yy} \end{bmatrix} = \begin{bmatrix} \frac{B}{4(AB-C^2)} & \frac{-C}{4(AB-C^2)} \\ \frac{-C}{4(AB-C^2)} & \frac{A}{4(AB-C^2)} \end{bmatrix} = \frac{1}{4(AB-C^2)} \begin{bmatrix} B & -C \\ -C & A \end{bmatrix} \tag{24}$$

Equation (24) can be also written as

$$M = \frac{1}{4} \begin{bmatrix} A & C \\ C & B \end{bmatrix}^{-1} \tag{25}$$

so the matrix of elliptical approximation coefficients can be calculated as

$$\begin{bmatrix} A & C \\ C & B \end{bmatrix} = \frac{1}{4} M^{-1} = \frac{1}{4(m_{xx}m_{yy} - m_{xy}^2)} \begin{bmatrix} m_{yy} & -m_{xy} \\ -m_{xy} & m_{xx} \end{bmatrix} \tag{26}$$

When the elliptical approximation matrix parameters are calculated the last step is to compute the maximum and minimum dimensions of the particle approximating ellipse. If the mineral particle is placed horizontally or vertically on the image plane (alongside the image pixel rows and column direction) it is an easy task, as in this case C=0. But when the particle and its approximating ellipse is placed at some angle, the task is more complex and it needs rotation of the local coordinate system. It can be done using an eigenvalue and eigenvector technique. Eigenvalues λ_1 and λ_2 of the $\begin{bmatrix} A & C \\ C & B \end{bmatrix}$ matrix can be computed as a solution of an equation system

$$\begin{bmatrix} A-\lambda & C \\ C & B-\lambda \end{bmatrix} = 0 \tag{27}$$

These solutions can be calculated as

$$\lambda_1 = \frac{A+B+\sqrt{(A-B)^2+4C^2}}{2} \tag{28}$$

and

$$\lambda_2 = \frac{A+B-\sqrt{(A-B)^2+4C^2}}{2} \tag{29}$$

In the temporary rotated local coordinate system (u,v) constructed individually for each particle on the basis of calculated eigenvalues the particle approximating ellipse equation can be described as

$$\lambda_1 u^2 + \lambda_2 v^2 = 1 \tag{30}$$

or

$$\left(\frac{u}{1/\sqrt{\lambda_1}}\right)^2 + \left(\frac{v}{1/\sqrt{\lambda_2}}\right)^2 = 1 \tag{31}$$

Therefore the ellipse semi-major and semi-minor axes a,b can be evaluated as

$$a = \frac{1}{\sqrt{\lambda_1}} \tag{32}$$

$$b = \frac{1}{\sqrt{\lambda_2}} \tag{33}$$

and after taking into account equations (28)(29):

$$a = \frac{\sqrt{2}}{\sqrt{A+B+\sqrt{(A-B)^2+4C^2}}} \tag{34}$$

$$b = \frac{\sqrt{2}}{\sqrt{A+B-\sqrt{(A-B)^2+4C^2}}} \tag{35}$$

Because the *A,B,C* coefficients according to (26) can be calculated as

$$A = \frac{m_{yy}}{4(m_{xx}m_{yy}-m_{xy}^2)} \tag{36}$$

$$B = \frac{m_{xx}}{4(m_{xx}m_{yy}-m_{xy}^2)} \tag{37}$$

$$C = \frac{m_{xy}}{4(m_{xx}m_{yy}-m_{xy}^2)} \tag{38}$$

the approximated mineral particle criterial dimensions (twice the semi-major and semi-minor axes) necessary for the assessment of the sieving process performance can be after some simplifications expressed finally as

$$d_{max} = 2\sqrt{2}\sqrt{m_{yy} + m_{xx} + \sqrt{\left(m_{xx} - m_{yy}\right)^2 + 4m_{xy}^2}} \tag{39}$$

$$d_{min} = 2\sqrt{2}\sqrt{m_{yy} + m_{xx} - \sqrt{\left(m_{xx} - m_{yy}\right)^2 + 4m_{xy}^2}} \tag{40}$$

Two examples of elliptical approximation of elongated and slanted mineral particle contour have been presented in figure 5.

Figure 5. Two examples of elliptical approximation of an elongated and slanted coal particle.

5. Conclusions

Elliptical approximation can be considered as an efficient method for calculating maximum and intermediate mineral particle dimensions. In order to compare the approximation methods described above, the calculations have been performed for the same example particle (presented in figs. 3 and 5a).

Results of these calculations (rescaled back from pixels to millimetres) have been presented in table 2. The proposed method performs quite well even for elongated and slanted particles and is much better than circular and rectangular (in the case of slanted particles) approximation. It can also provide initial data for the most exact (i.e. closest to the real sieving process performance, but also the most cumbersome one) Feret diameter calculation method, significantly reducing its computational effort.

Table 2. Comparison of particle sizes calculated by different approximation methods.

measurement method:	size [mm]
maximum rectangle dimension	54,3
minimum rectangle dimension	31,2
equivalent area circle diameter	42,6
minimum ellipse axis	33,2
maximum ellipse axis	52,7

From the computational point of view the proposed algorithm can be reduced to simple 2-dimensional integration (pixel counting) for inertia moment calculations, and then some non-iterative computations, Because of this simple non-iterative implementation it can be used in machine-vision based granulometric systems operating in real time.

6. References

[1] Heyduk A 2017 *Two-dimensional and three dimensional image acquisition and processing methods for machine vision granulometric analysis* (Gliwice: SUT editions)
[2] Krawczykowski D, Krawczykowska A and Trybalski K 2012 *Laser particle size analysis – the influence of density and particle shape on measurement results* (Min Res.Mgt) 28 pp.101-112
[3] Krawczykowski D 2018 *Application of a vision systems for assessment of particle size and shape for mineral crushing products*. (IOP Conf. Ser. Mater. Sci. Eng) 427
[4] Walton WH 1948 *Nature* **162** 329
[5] Drazic S, Sladoje N and Lindblad J 2016. *Patt. Rec. Lett*. **80** 3745
[6] Peszko B, Kordek J, Niedoba T, Krawczykowska A, 2007 *J Appl Sci* **7** 2084-87
[7] Krawczykowska A and Trybalski K 2008 *MiAG* **46** 24
[8] Stewart J, Redlin L, Watson S. 2014 *Precalculus – Mathematics for Calculus* (Boston: Cengage)