An Efficient FPGA Parallel Implementation for 2-D MUSIC Algorithm

To cite this article: Haoqiang Shi *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **252** 032168

IOP Conf. Series: Earth and Environmental Science **252** (2019) 032168 doi:10.1088/1755-1315/252/3/032168

An Efficient FPGA Parallel Implementation for 2-D MUSIC Algorithm

Haoqiang Shi[a], Zhanjun Jiang[*], Qianru Liu[b], Xiaoyu Cai[c]

School of Electronic and Information Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China

*Corresponding author e-mail: 59444069@qq.com, [a]452072337@qq.com, [b]496608361@qq.com, [c]513095613@qq.com

Abstract. The multi signal classification (MUSIC) algorithm is a classical one in the direction estimation of radar. An highly efficient implementation scheme is proposed in this paper. In order to solve the problem that computing two-dimensional(2-D) MUSIC algorithm is not fast enough in engineering implementation and the resource consumption is large, and we have completed the simulation implementation of the algorithm on field programmable gate array (FPGA). Through the analysis of the steps of Jacobi transformation, a high efficient serial control scheme is proposed to realize Jacobi transformation, and the characteristic value decomposition is realized by using cyclic Jacobi iterative algorithm. This design avoids finding the most value through threshold comparison, reduces the hardware complexity and improves the decomposition speed. In spectral peak search, a parallel computing method is presented, which realizes parallel 2-D spectral peak search through spatial division, this method can effectively improve the effectiveness of search, and also can achieve the estimation of the target at the junction of airspace. Compared with the traditional implementation scheme, the design scheme has the advantages of high precision, high real time and saving hardware resources. Theoretical analysis and MATLAB simulation test show that this method and its FPGA design have good performance and have high application value in the practical process.

1. Introduction

Array signal processing is widely used in communication, radar, acoustics, medicine and other fields. Direction of estimation(DOA) is a key technology in array signal processing, DOA estimation method, in which multiple signal classification [1] (MUSIC) algorithm is of good performance of high resolution DOA estimation algorithm of the most classic algorithm, MUSIC algorithm is universal applicability, as long as the known antenna disposal form, both linear array and circular array, and the interval distribution, such as whether or not the array element is to be able to get high resolution estimation results [2], because of its many advantages and has been a lot of research.

2-D MUSIC algorithm is a classic in the 2-D DOA estimation algorithm, this method has high resolution and can produce wave direction of asymptotically unbiased estimates, but because of its covariance matrix decomposition is required in the process of implementation and the 2-D spectrum peak search, both the need for a large number of operations, the hardware implementation is difficult, especially in the 2-D space spectrum peak search, calculation is very big.

MUSIC algorithm has changed from the previous parallel implementation of digital signal processing to the DSP+FPGA scheme in the actual engineering implementation, and then to the current implementation of FPGA. Literature [3] adopts DSP+FPGA implementation scheme, which mainly uses FPGA to achieve linear operations. However, it is difficult to decompose matrix eigenvalues due to the difficulty of FPGA implementation, so it is implemented by DSP. This combination architecture involving data communication between DSP and FPGA, fixed-point floating-point conversion, so makes the algorithm implementation difficulty increases, increases the hardware system to realize the complexity and cost, at the same time as a result of the existence of DSP, the final implementation time or stay in a millisecond. An method based on DSP + FPGA by coarse search and fine search and optimal spectral peak judge combination algorithm has realized the MUSIC algorithm [4], in which DSP is the core operation, complete the main computing algorithm, the FPGA is the core logic, realizing the function of chip scheduling, so that the whole project performance is not high enough.

The main difficulty of hardware implementation of 2-D MUSIC algorithm is that the singular value decomposition of its covariance matrix contains a large number of nonlinear operations and a large amount of searching computation of 2-D spectral peak. For matrix decomposition has a large number of researchers proposed various improvements to simplify the implementation process, many scheme mainly adopts CORDIC [5] (Coordinate Rotation Digital Computer) algorithm designed two kinds of calculation mode, namely Rotation model and Vector model to the realization of parallel computing for PFGA Jacobi algorithm of matrix eigenvalue decomposition [6]. As for the hardware implementation of spectral peak search, in literature [7], a block multi-path parallel scheme was adopted for one-dimensional MUSIC peak search. Four parallel schemes and eight parallel schemes were designed, both of which achieved good results, greatly reducing the search time and greatly improving the speed of realization. Literature [8] realized the FPGA implementation of 2-D MUSIC by conducting spectral peak rough search and fine search, which reduced the calculation amount and shortened the search time.

This paper focuses on the MUSIC algorithm in 2-D DOA estimation matrix decomposition and spectral peak search problem, put forward a kind of assembly line design of CORDIC algorithm was adopted to realize circular Jacobi iteration algorithm, the method in reducing the consumption of hardware resources at the same time, the consumption of time and parallel time, at the same time by using threshold comparison method. In order to solve the problem of 2-D spectral peak searching, the airspace is divided, and then the parallel computation is carried out. Finally, the overall implementation scheme can reduce the computation time and consume resources.

2. Implementation analysis of 2-D MUSIC Algorithm

Mathematical model bits of MUSIC algorithm:

$$X(t) = A(\theta)S(t) + N(t) \tag{1}$$

Where, $X(t)$ is the receiving data vector of the array at time t ; $A(\theta)$ is the direction vector; $S(t)$ is the incident source signal vector; $N(t)$ is the noise vector. By using the formula to calculate the covariance matrix R of $X(t)$, and then the feature decomposition of 8, the noise vector matrix U can be calculated. The calculation formula of R is:

$$R = E[XX^{\mathrm{H}}] \tag{2}$$

The expression of spatial general function of MUSIC algorithm is

$$P_{\text{MUSIC}}(\theta)=\frac{1}{A(\theta)UU^{\text{T}}A(\theta)^{\text{T}}} \tag{3}$$

Where, the maximum peak value of $P_{\text{MUSIC}}(\theta)$ and all corresponding θ values are the estimated direction of the signal source.

The spatial general function expression of the 2-D MUSIC algorithm is:

$$P_{\text{2D-MUSIC}}(\theta,\phi)=\frac{1}{a^{H}(\theta,\phi)GG^{H}a(\theta,\phi)} \tag{4}$$

Where, $\theta\in(0,\frac{\pi}{2})$ is pitch Angle, $\phi\in(-\pi,\pi)$ is azimuth Angle and $a(\theta,\phi)=e^{-j\varphi_m}, m=0,...,M\text{-}1$ is guide loss, $\varphi_m=-\frac{2\pi}{\lambda}(x_m\sin\theta\cos\phi+y_m\sin\theta\sin\phi+z_m\cos\theta)$, λ is the wavelength , G is the noise subspace.

It can be seen from the observation of 2-D MUSIC spatial general function expressions that we need to calculate pseudo-spectral and incoming wave direction estimation Calculate $P_{\text{2D-MUSIC}}(\theta,\phi)$, find N maximum peaks and their corresponding (θ,ϕ) , N are the signal source number

As it is not easy to calculate the inverse in FPGA hardware, the calculation is not accurate due to the high intercept caused by the bit width problem during the square calculation. The idea that the maximum problem can be equivalently converted to the minimum problem is applied, the calculation procedure of the spatial general function can be improved to not calculate the inverse, and the sum of the modulus squared can be changed to the sum of the modulus, in this way, the equivalent expression of equation (1) is obtained:

$$P_{\text{2D-MUSIC}}(\theta,\phi)=sum(abs(a^{H}(\theta,\phi)G)) \tag{5}$$

Find out N the trough of the minimum values of corresponding, N as the number of signal source, so although the result of calculation is not pseudo spectral values, but the original pseudo-spectral wave into the trough of the now, position has not changed. This effectively avoids the above problems.

3. Design of computational inversion scheme

Guidance vector $a(\theta,\phi)$ as a column vector of M dimensions, noise subspace G for a Mx (M, N) matrix, then $a^{H}(\theta,\phi)G$ for a (M, N) d row vector, its each element modulus and accumulative sum can get cepstrum. For steering vector $a(\theta,\phi)=\left[e^{-j\varphi_0} \quad e^{-j\varphi_1} \quad \cdots \quad e^{-j\varphi_{M-1}}\right]$, the key to calculation is based on (θ,ϕ) and m to calculate the corresponding, so you can use the ROM $\sin\theta$, $\cos\theta$, $\sin\varphi$, $\cos\varphi$ and $2x_m/\lambda$, $2y_m/\lambda$, $2z_m/\lambda$ values for storage, at the time of calculation can be read. This design has the following three advantages, one is complexity, reduces the project simplified the hardware design, the second is to improve the calculation accuracy data, the inside of the three is just by changing the ROM data can change the scope of scanning and the model of the array so that you can achieve any formation of 2-D MUSIC.

In order to improve the throughput of data, in the process of calculating equivalent expressions

$$P_{\text{2D-MUSIC}}(\theta,\phi)=sum(abs(a^{H}(\theta,\phi)G)) \tag{6}$$

The design adopts pipeline calculation method, in order to correspond and G (elements) input to the calculation module to calculate. By entering ϕ_m and g_{ij} into a CORDIC module to complete the calculation of $g_{ij}e^{j\phi_m}$, the sum of M modules is summed to get $a^H G(:,j)=\sum_{i=0}^{M-1} g_{ij}e^{j\varphi_m}$, and then the sum of M minus N pieces of (θ_1,ϕ_1) pieces of demodulated, So you get the inverse spectral value of time

$$P_{2D-MUSIC}(\theta_1,\phi_1)=\sum_{j=0}^{M-N-1}(abs(\sum_{i=0}^{M-1} g_{ij}e^{j\varphi_m})) . \tag{7}$$

In this way, the next inverted spectrum value can be calculated by changing the pitch Angle and azimuth Angle, and the corresponding inverted spectrum value can be obtained by calculating the required scanning space.

4. Design of eigenvalue decomposition scheme

The eigenvalue decomposition module mainly completes the covariance matrix decomposition so as to calculate the eigenvalue and eigenvector. Jacobi algorithm is one of the commonly used eigenvalue decomposition algorithm, it mainly through doing a series of rotating transform of matrix, the transformed into diagonal matrix, thus calculated matrix eigenvalue and eigenvector. Classic Jacobi iteration algorithm has faster convergence speed, but as a result of the algorithm needs of many elements in the matrix of the diagonal elements as the center of rotation, the absolute value of the largest element in the subsequent calculation operation again. In this way, every step should find the largest absolute value of the non-diagonal elements, which is not conducive to hardware implementation. Therefore, this paper designs a serial controlled loop Jacobi iteration algorithm for scanning the calendar line by line, so as to avoid the complicated process of finding the largest absolute value non-diagonal elements. The algorithm flow is shown in figure 1, where $A \in C^{n\times n}$ is the conjugate matrix, T is the maximum traversal number, and initializes the traversal counter and the eigenvector initial matrix, $t=0$, $V=E$, t is a natural number and $1 \le t \le \mathrm{T}$, i=1,2,3,...,n, $\mathrm{j}=1,2,3,\ldots\mathrm{n}$.

5. Parallel computing scheme for airspace division

Through the above scheme analysis shows that the 2-D MUSIC requires a 2-D cepstrum spectrum valley search, calculation and computational complexity is higher, even the valley and real-time spectrum is obtained by using the front of the line search scheme implementation, the consumption of the final time is quite big, therefore can make use of the advantages of parallel computing, reproduction peak when searching for multiple module calculation at the same time, the scheme design of so it can effectively reduce the waste of time.

So this article reproduction in the design of the peak search, analysis of literature [7] one dimensional MUSIC spectrum peak search design scheme, the whole airspace division of 2-D MUSIC algorithm [9], so that you can realize the parallel computing spectral peak searching, parallel computing division design is shown in figure 2. The number of scanned airspace is set as K, the scanning area of elevation Angle $[\theta_1 \quad \theta_2 \quad \cdots \quad \theta_n]$ and the scanning area of each airspace's respective azimuth $[\phi_{k1} \quad \phi_{k2} \quad \cdots \quad \phi_{km}]$. Elevation Angle and azimuth jointly constitute k 2-D airspace.

Through the following equation, the respective MUSIC spectrum of the K airspace is simultaneously $P(\theta_i,\phi_{kj})$, $k=1\sim K$.

$$P(\theta_i,\phi_{kj})=\frac{1}{a^H(\theta_i,\phi_{kj})GG^H a(\theta_i,\phi_{kj})} \tag{8}$$

$$j=1\sim m\ ,\ i=1\sim n\ ,\ a(\theta_i,\phi_{kj})=\left[e^{-j\varphi_0} \quad e^{-j\varphi_1} \quad \cdots \quad e^{-j\varphi_{L-1}}\right]^T \in C^{L\times 1} \ .$$

Where, L is the number of matrix elements; (x_l, y_l, z_l) is the coordinate information of the l matrix element; $G \in C^{L \times L-N}$ is the noise subspace and N is the number of signal sources.

At the same time, L maximum spectral peaks of $P(\theta_i, \phi_{kj})$ in each MUSIC spectrum were found in the airspace of K, so as to obtain the corresponding elevation Angle and azimuth Angle of $L \times K$ spectral peaks, namely $P(\theta_i, \phi_{kj})$ and $q = 1 \sim L \times K$.

Find out the N maximum values in $P_q(\theta_i, \phi_{kj})$ and $q = 1 \sim L \times K$, then the pitch Angle θ_i and ϕ_{kj} azimuth Angle corresponding to the maximum value of N are the pitch Angle and azimuth Angle corresponding to N signal sources.

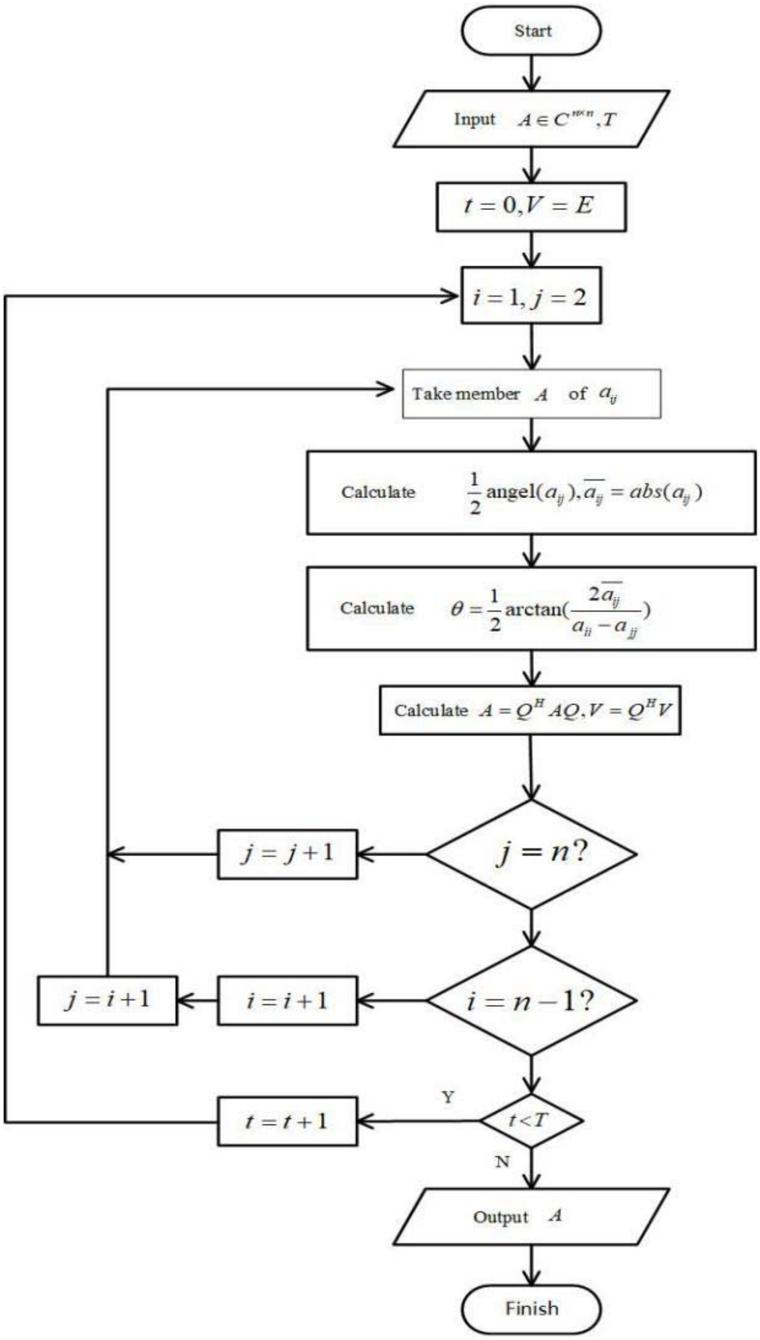

Figure 1. Circular iteration Jacobi algorithm flow chart.

Airspace border goal, in order to guarantee can estimates the airspace of the segmentation method such as shown in figure 3, horizontal azimuth, vertical is pitching Angle, different color areas represent different search airspace. The elevation scanning area of this K -space domain is $[\theta_1 \quad \theta_2 \quad \cdots \quad \theta_n]$. The scanning area of airspace 1 is $[\phi_{11} \quad \phi_{12} \quad \cdots \quad \phi_{1m}]$, the scanning area of airspace 2 is $[\phi_{21} \quad \phi_{22} \quad \cdots \quad \phi_{2m}]$, the scanning area of airspace 3 is $[\varphi_{31} \quad \varphi_{32} \quad \cdots \quad \varphi_{3m}]$ and the scanning area of airspace K is $[\varphi_{K1} \quad \varphi_{K2} \quad \cdots \quad \varphi_{Km}]$ .And needs to be satisfied, $\varphi_{1(m-1)} = \varphi_{21}$, $\varphi_{1m} = \varphi_{22}$, $\varphi_{2(m-1)} = \varphi_{31}$, $\varphi_{2m} = \varphi_{32}$ ..., $\varphi_{(K-1)(m-1)} = \varphi_{K1}$, $\phi_{(K-1)m} = \phi_{K2}$.

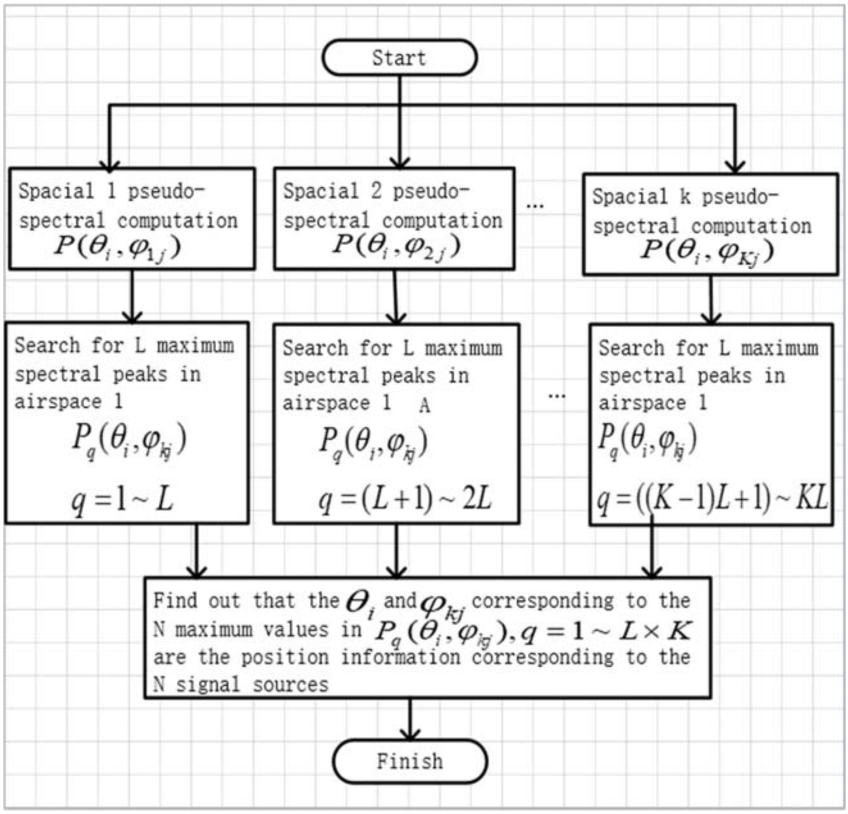

Figure 2. Parallel computing to segment the design drawing.

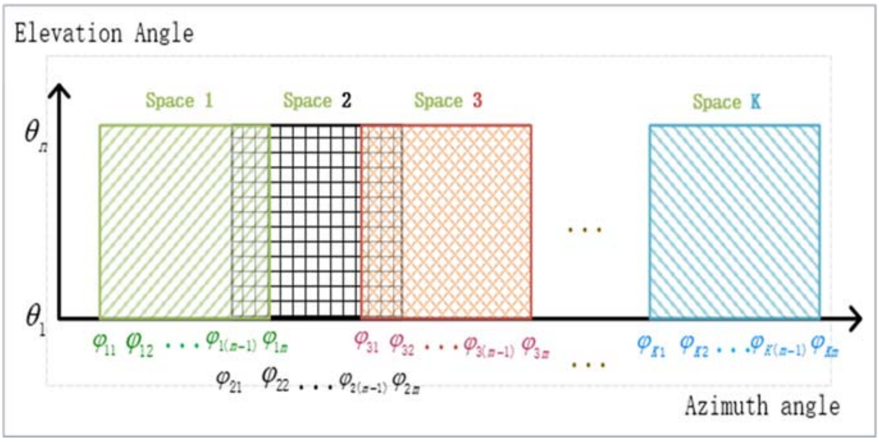

Figure 3. Parallel calculation of pitch Angle overlap.

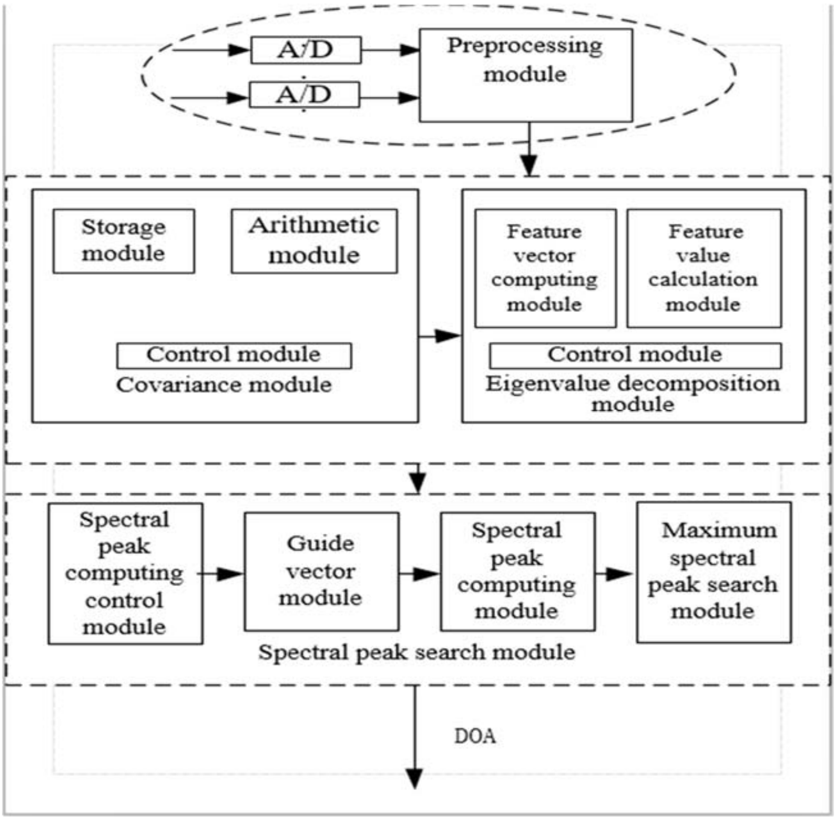

Figure 4. Overall hardware structure.

6. FPGA implementation and simulation

The overall hardware structure design is shown in figure 4 above, which consists of four parts: preprocessing module, covariance module, eigenvalue decomposition module and spectral peak search module. The spectral peak searching module consists of spectral peak computing control module, guide vector module, spectral peak computing module and maximum spectral peak searching module.

In the FPGA implementation of specific projects, the number of scanning the airspace N value of 4, the airspace is divided into four statures airspace, adopted four parallel spectrum peak search module, this can be achieved by four spatial spectrum peak search, four output spectrum peak search results compare to find four corresponding to the pitching Angle and phase Angle of the airspace of the smallest troughs. The pitch Angle and phase Angle of the minimum N wave valley are the wave direction of the signal according to the estimated signal source number N.

This MUSIC module with M=16 is designed and synthesized under Xilinx's XC7K325-TFFG900 FPGA. The FPGA runs at 312MHz the highest system clock. This design is compiled on ISE simulation software platform and simulated in Modelsim. Finally, the simulation results are imported into MATLAB simulation verification results.

Figure 5 shows a spatial spectrum output based on MATLAB when signal-to-noise ratio (SNR) is set to 15dB with D=2, $[\theta_1,\varphi_1]=[37^{\circ},-155^{\circ}],[\theta_2,\varphi_2]=[35^{\circ},-150^{\circ}]$. The FPGA hardware simulation results of the algorithm are drawn and verified in the MATLAB environment as shown in figure 6.

IOP Conf. Series: Earth and Environmental Science **252** (2019) 032168 doi:10.1088/1755-1315/252/3/032168

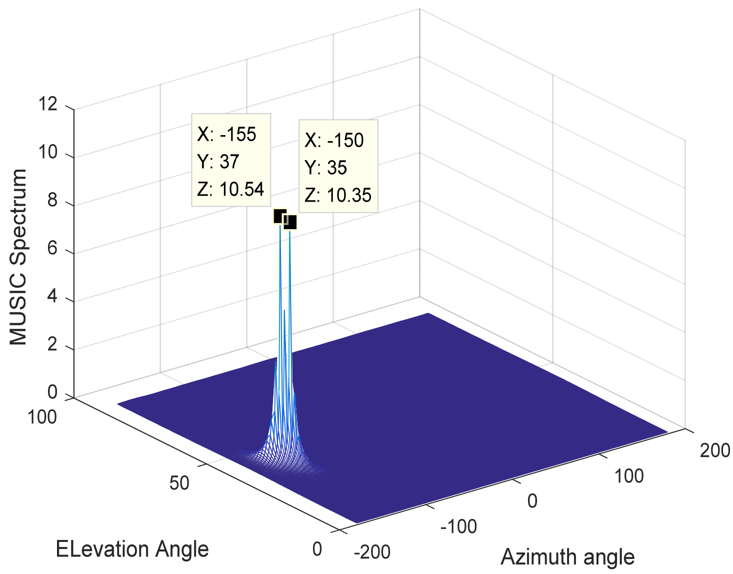

Figure 5. MATLAB 2-D DOA estimation.

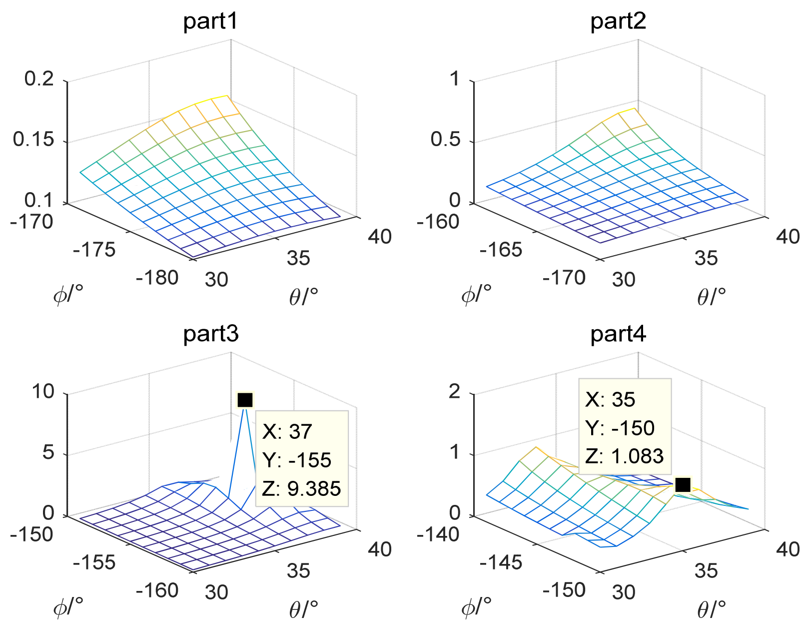

Figure 6. FPGA hardware results.

From the above pseudo-spectral graph, it can be seen that the simulation results and theoretical values are the same. Table1 gives the results of FPGA implementation and MATLAB simulation with the number of signal sources D=1,2,3,4. An acceptable degree error(±0.1~0.4) may occur for either MATLAB or our implemented FPGA hardware. Table 2 shows the device utilization summary.

Table 1. The results comparison.

Test	Number	Actual DOA	Results by MATLAB	Results by FPGA
1	1	(9.7°, 60°)	(9.8°, 60°)	(10°, 60°)
2	1	(30.4°, 100°)	(30.5°, 100°)	(30°, 100°)
	2	(34°, -132°)	(34°, -132°)	(34°, -132°)
3	1	(26.3°, 126°)	(26.5°, 126°)	(26°, 126°)
	2	(36.2°, 66°)	(36.4°, 66°)	(36°, 66°)
	3	(60°, 160°)	(60°, 159.9°)	(60°, 159.9°)

Table 2. Device utilization summary.

Logic Utilization	Used	Available	Utilization
Slice Registers	72056	407600	17%
Slice LUTs	75246	203800	37%
Block RAM/FIFO	28	445	6%
DSP48E1s	192	840	23%

7. Conclusion

In this paper, an efficient FPGA parallel implementation method is proposed by decomposing and analyzing the theoretical calculation formula steps of 2-D MUSIC algorithm. The method by comparing the threshold method for eigenvalue decomposition, airspace division design is adopted to improve the parallel searching spectral peak, at the same time in the design of computing peak equivalent to avoid the countdown, at the same time instead of sum of squares mode and the hardware design, the extensive use of module reuse and assembly line design, so as to greatly improve the computational efficiency of 2-D MUSIC algorithm, at the same time improve the computing speed reduces the resource consumption. Compared with the existing related research method, in this paper, the design of 2-D MUSIC algorithm implementation scheme has high precision, high speed, low resource consumption and high applicability, flexible parametric design makes this design have high practical engineering application value.

Acknowledgments

This work was financially supported by Foundation of A Hundred Youth Talent Training Program of Lanzhou Jiaotong University (Program No.1520220232).

References

[1] Schmidt R. Multiple emitter location and signal parameter estimation, IEEE Transactions on Antennas and Propagation, 34(3), 1986, pp. 276-280.

[2] X. F. Zhang, H. W. Chen, X. F. Chou. Array signal processing and MATLAB implementation [M]. Beijing: Publishing House of Electronics Industry, 2015.

[3] Qi Zhang. Study of DOA Estimation and Its Hardware Implement Based on Space-Time Two-Dimensions MUSIC Algorithm, Xidian University, 2014.

[4] W. F. Zhang. The Research on the Implementation of MUSIC Algorithm in the DSP System, Huazhong University of Science and Technology, 2011.

[5] Cavallaro J R, Luk F T. CORDIC Arithmetic For An SVD Processor [J]. Journal of Parallel and Distributed Computing, 5, 1998, pp. 271-290.

[6] Ignacio B, Pedro J, Manuel M, et al. Implementation In FPGAS Of Jacobi Method To Solve The Eigenvalue And Eigenvector Problem[C]. Field Programmable Logic and Applications. Madrid: IEEE, 2006, pp. 1-4.

[7] H. Zheng. Fast implementation of MUSIC algorithm on high-speed processor, University of Electronic Science and Technology of China, 2004, pp. 34-39.

[8] K Huang, J Sha, W Shi, Z Wang. An Efficient FPGA Implementation for 2-D MUSIC Algorithm, Circuits Systems & Signal Processing, 35(5), 2016, pp. 1795-1805.
[9] Lu Gan, W. Y. Zhao, H. S. Liao. Parallel computing method for estimating arrival direction through 2-D MUSIC algorithm, China. Patent 201710486807.4. (2017).