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## Key indicator system establishment for reliability evaluation of marine electrical system based on analytic hierarchy process

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# Key indicator system establishment for reliability evaluation of marine electrical system based on analytic hierarchy process

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**Abstract.** Reliability evaluation is widely used in electrical system which determines operation stability and environment safety of ships and vessels since it can discover potential safety hazard scientifically and accurately. Analytic hierarchy process (AHP) is an effective and useful method for reliability evaluation and indicator system establishment due to its simplicity and convenience. In this paper, a hierarchical structure of indicator system for ship's electrical system is established based on AHP, and three judgement matrixes of the reliability, basic reliability and mission reliability are constructed. Then a calculation method of the maximum eigenvalue is introduced to solve the ranking weight vector of the three matrixes which can determine the weight of the indicator system for reliability evaluation of marine electrical system. Consistency index and its evaluation method are given for checking the judgement matrixes and conclusions. Finally taking a ship's electrical system as an example, the above solutions were calculated and results show that AHP can guarantee the validity and accuracy of the establishment and application of the key indicator system. Lower hierarchies and indicators could be considered due to the growing demand of the mission and service of ship's electrical system.

## 1. Introduction

Electrical system plays a key role in stability and safety of ships and vessels, so electrical system reliability should be evaluated scientifically and accurately to ensure the operation and function of electrical system by discovering the operating condition variations and the potential safety hazard of the electrical system timely [1, 2].

As an important aspect of reliability evaluation for the electrical system, the establishment and application of indicator system are synthesis problems which have many objectives and properties. Thus, the indicator system not only contains various parameters and data, but also has complicated hierarchies and structures. The different indicators of the system should be set different weights due to their different functions and levels for the indicator system, so the indicator weights reflect the important degrees of



the all indicators in the evaluation processes immediately and measure the influence degrees of the different indicators for the evaluation objectives comprehensively.

There is a vast literature [3-10], most of which is concerned with many theories and methods of establishment and application for indicator systems, such as Delphi method [3], analytic hierarchy process [4], expert investigation method [5], scatter degree method [6], mean square difference method [7], variation coefficient method [8], maximizing difference deciding principle [9] and correlation function method [10]. Some indicator systems were established by evaluation criteria or expert experiences, but the establishment processes of the indicator systems were too simple to ensure the integrity of hierarchies or structures. Though some indicator systems established had a part of the integrity of hierarchies or structures, the processes and methods of the indicator systems were lack of related the theoretical support.

Accordingly, the aim of this paper is to develop a better method in the establishment and application of the key indicator system for reliability evaluation by using analytic hierarchy process, due to more safety and stable operating for marine electrical system.

## 2. Method and model

### 2.1. Hierarchical structure

As the most important part in the indicator system establishment for the reliability evaluation of the marine electrical system, the construction procedures of hierarchical structure based on AHP are as follow: First, the whole problem of the indicator weights can be decomposed into many smaller and smaller parts, and one part has one element. Second, those elements are classified as different teams depending on their properties. As analytical criterion, the elements of the same hierarchy dominate the elements of the sub-hierarchy and are dominated by the elements of the upper-hierarchy. Thus, the dominance relation of hierarchical structure of AHP is established from top to bottom. Last, the highest hierarchy, which has only 1 element, is the decision objective or satisfactory result generally.

In this study, it should be the reliability of marine electrical system. The intermediate hierarchies are the criteria and sub-criteria of the reliability evaluation, such as the parameters of the reliability of marine electrical system. The lowest hierarchy contains the evaluation methods and the data of reliability parameters. The dominance relations of the elements in the different hierarchies are not absolutely corresponding, so an element of the hierarchy could not dominate all the elements of the sub-hierarchy. The dominance relations of the elements depend on the technical features and service conditions of the marine electrical system.

### 2.2. judgment matrix

The dominance relations of the elements between different hierarchies are determined by the hierarchical structure of AHP. Supposing the  $i$ th element in the  $k$ th hierarchy  $A_i^{(k)}$  is an evaluation criterion and dominates the elements  $(A_1^{(k-1)}, A_2^{(k-1)}, \dots, A_n^{(k-1)})$  of the sub-hierarchy (the  $k-1$ th hierarchy). Then the weights of  $A_1^{(k-1)}, A_2^{(k-1)}, \dots, A_n^{(k-1)}$  can be determined by the relative importance between the elements of the  $k-1$ th hierarchy based on the criterion  $A_i^{(k)}$ .

When the comparison result between the  $i$ th element  $A_i^{(k-1)}$  and  $j$ th element  $A_j^{(k-1)}$  of the  $k-1$ th hierarchy is a judgement value  $a_{ij}^{(k)}$ , a judgment matrix based on the criterion  $A_i^{(k)}$  can be expressed as

$$A_i^{(k)} = \begin{pmatrix} a_{11}^{(k)} & a_{12}^{(k)} & \cdots & a_{1n}^{(k)} \\ a_{21}^{(k)} & a_{22}^{(k)} & \cdots & a_{2n}^{(k)} \\ \vdots & \vdots & & \vdots \\ a_{n1}^{(k)} & a_{n2}^{(k)} & \cdots & a_{nn}^{(k)} \end{pmatrix} \quad (1)$$

The judgement results of the degrees of importance using AHP can be classified as 9 degrees, which are called as proportion criteria that an effective method for quantifying human thinking and judgment.

All elements of the judgement matrix should be greater than 0. The judgement value of an element itself identically equals to 1 ( $a_{ii}^{(k)} = 1$ ). The two judgement values of intercomparison between two elements are reciprocals ( $a_{ij}^{(k)} a_{ji}^{(k)} = 1$ ), so the judgement matrix is a positive reciprocal matrix. Since it only needs to be judged its upper triangular elements for the  $n$  order judgement matrix, the total number of the judgement is  $n(n-1)/2$ . The judgement matrix is a consistent matrix for  $a_{ij}^{(k)} a_{jk}^{(k)} = a_{ik}^{(k)}$ . According to the matrix properties and proportion criteria, the judgement matrix can be expressed as

$$A_i^{(k)} = \begin{pmatrix} 1 & a_{12}^{(k)} & \cdots & a_{1n}^{(k)} \\ 1/a_{12}^{(k)} & 1 & \cdots & a_{2n}^{(k)} \\ \vdots & \vdots & & \vdots \\ 1/a_{1n}^{(k)} & 1/a_{2n}^{(k)} & \cdots & 1 \end{pmatrix} \tag{2}$$

Where

$$a_{ij}^{(k)} = 1, 2, \dots, 9 \quad \text{or} \quad 1, \frac{1}{2}, \dots, \frac{1}{9}$$

### 2.3. Indicator weight

The sequencing problem of the weights of the elements  $A_1^{(k-1)}, A_2^{(k-1)}, \dots, A_n^{(k-1)}$  can be solved for evaluating the reliability of the marine electrical system by developing the multiple comparison of  $A_1^{(k-1)}, A_2^{(k-1)}, \dots, A_n^{(k-1)}$  and solving the matrix eigenvalues. Supposing ranking weight vector is  $w_i^{(k)}$ , the function of the ranking weight vector can be expressed as

$$A_i^{(k)} w_i^{(k)} = \lambda_{i,\max}^{(k)} w_i^{(k)} \tag{3}$$

The judgement matrix  $A_i^{(k)}$  has the maximum eigenvalue which is unique. The ranking weight vector  $w_i^{(k)}$  of the judgement matrix  $A_i^{(k)}$  can be composed of the positive components which are unique also, so the ranking weight vector  $w_i^{(k)}$  can be expressed as

$$w_i^{(k)} = (w_{i1}^{(k)} \quad w_{i2}^{(k)} \quad \cdots \quad w_{in}^{(k)})^T \tag{4}$$

For solving the ranking weight vector  $w_i^{(k)}$ , the elements  $a_{ij}^{(k)}$  of the judgement matrix  $A_i^{(k)}$  should be normalized by column and then column vectors are expressed as

$$\begin{cases} b_1^{(k)} = \sum_{i=1}^n a_{i1}^{(k)} (a_{11}^{(k)} & a_{21}^{(k)} & \cdots & a_{n1}^{(k)})^T \\ b_2^{(k)} = \sum_{i=1}^n a_{i2}^{(k)} (a_{12}^{(k)} & a_{22}^{(k)} & \cdots & a_{n2}^{(k)})^T \\ \vdots \\ b_n^{(k)} = \sum_{i=1}^n a_{in}^{(k)} (a_{1n}^{(k)} & a_{2n}^{(k)} & \cdots & a_{nn}^{(k)})^T \end{cases} \tag{5}$$

Then a matrix  $\mathbf{B}_i^{(k)}$  can be expressed by the column vectors above.

$$\mathbf{B}_i^{(k)} = \begin{pmatrix} \mathbf{b}_1^{(k)} & \mathbf{b}_2^{(k)} & \cdots & \mathbf{b}_n^{(k)} \end{pmatrix} = \begin{pmatrix} b_{11}^{(k)} & \cdots & b_{1n}^{(k)} \\ \vdots & & \vdots \\ b_{n1}^{(k)} & \cdots & b_{nn}^{(k)} \end{pmatrix} \quad (6)$$

Adding the elements of the matrix  $\mathbf{B}_i^{(k)}$  by row, normalizing the results and transforming to column vector, the ranking weight vector  $\mathbf{w}_i^{(k)}$  can be derived and expressed as

$$\mathbf{w}_i^{(k)} = \frac{1}{\sum_{i=1}^n \sum_{j=1}^n b_{ij}^{(k)}} \begin{pmatrix} \sum_{i=1}^n b_{i1}^{(k)} & \sum_{i=1}^n b_{i2}^{(k)} & \cdots & \sum_{i=1}^n b_{in}^{(k)} \end{pmatrix}^T \quad (7)$$

The maximum eigenvalue  $\lambda_{i,\max}^{(k)}$  of the judgement matrix  $\mathbf{A}_i^{(k)}$  can be solved due to the ranking weight vector  $\mathbf{w}_i^{(k)}$ . A vector can be expressed as

$$\lambda_i^{(k)} = \mathbf{A}_i^{(k)} \mathbf{w}_i^{(k)} \quad (8)$$

Then the maximum eigenvalue  $\lambda_{i,\max}^{(k)}$  of the judgement matrix  $\mathbf{A}_i^{(k)}$  can be expressed as

$$\lambda_{i,\max}^{(k)} = \sum_{j=1}^n \frac{\lambda_{ij}^{(k)}}{n w_{ij}^{(k)}} \quad (9)$$

where  $n$  is the order of the matrix.

#### 2.4. consistency check

The key indicator system can be verified through the consistency check of the judgement matrix  $\mathbf{A}_i^{(k)}$ . There is no need to check the consistency when the judgement matrix  $\mathbf{A}_i^{(k)}$  is constructed. However, when the ranking weight vector  $\mathbf{w}_i^{(k)}$  is derived, the judgement matrix  $\mathbf{A}_i^{(k)}$  must has consistency to ensure the ranking weight vector  $\mathbf{w}_i^{(k)}$  completely reflects the degrees of importance between the elements and avoids the error of the indicator weights. Therefore, the consistency index of the judgement matrix  $\text{CI}_i^{(k)}$  can be expressed as

$$\text{CI}_i^{(k)} = \frac{\lambda_{i,\max}^{(k)} - n}{n - 1} \quad (10)$$

Supposing a random matrix, its rank is same as the judgement matrix  $\mathbf{A}_i^{(k)}$ . Comparing the consistency index of the judgement matrix  $\text{CI}_i^{(k)}$  and the consistency index of the random matrix  $\text{RI}$ , the consistency ratio  $\text{CR}_i^{(k)}$  can be expressed as

$$\text{CR}_i^{(k)} = \frac{\text{CI}_i^{(k)}}{\text{RI}} \quad (11)$$

For  $\text{CR}_i^{(k)} < 0.1$ , the judgement matrix  $\mathbf{A}_i^{(k)}$  is acceptable and can be solving the ranking weight vector  $\mathbf{w}_i^{(k)}$  to weight the key indicator for the reliability evaluation of marine electrical system.

**3. Case study**

The hierarchical structure of the evaluation indicator of a ship’s electrical system and the parameter symbols have been performed with the following data in Table 1. The reliability of the electrical system is the objective and its sub-hierarchy contains two criteria that are the basic reliability and mission reliability.

Three judgement matrixes of the reliability, basic reliability and mission reliability are be constructed by Equation 2 according to the evaluation experts. The orders of the judgment matrixes of the reliability, basic reliability and mission reliability are 2, 5, and 3, respectively. Therefore, the elements of the judgement matrixes of the reliability are shown in Table 2, and the judgement matrixes of the basic reliability and mission reliability and their elements are shown in Table 3 and Table 4. All of three matrixes are positive reciprocal matrixes and the values of their elements depends on the evaluation expert judgement. The accuracy and effectiveness would be increased by the growing of the number of the evaluation expert judgement.

**Table 1.** The hierarchical structure of the evaluation indicator of a ship’s electrical system.

Objective	Criterion	Indicator
Reliability (A)	Basic reliability (A <sub>1</sub> )	Mean time to failure (A <sub>11</sub> )
		Mean time between failures (A <sub>12</sub> )
		Mean time between preventive maintenance (A <sub>13</sub> )
		Mean time between repairs (A <sub>14</sub> )
		Failure rate (A <sub>15</sub> )
	Mission reliability (A <sub>2</sub> )	Mean time between critical failures (A <sub>21</sub> )
		Men time duration (A <sub>22</sub> )
		Mission completion success probability (A <sub>23</sub> )

**Table 2.** The judgement matrix of the reliability (A) from the expert evaluation.

Criterion	A <sub>1</sub>	A <sub>2</sub>
A <sub>1</sub>	1	1/3
A <sub>2</sub>	3	1

**Table 3.** The judgement matrix of the basic reliability (A<sub>1</sub>) from the expert evaluation.

Indicator	A <sub>11</sub>	A <sub>12</sub>	A <sub>13</sub>	A <sub>14</sub>	A <sub>15</sub>
A <sub>11</sub>	1	1	7	5	2
A <sub>12</sub>	1	1	7	5	2
A <sub>13</sub>	1/7	1/7	1	1/2	1/5
A <sub>14</sub>	1/5	1/5	2	1	1/3
A <sub>15</sub>	1/2	1/2	5	3	1

**Table 4.** The judgement matrix of the mission reliability (A<sub>2</sub>) from the expert evaluation.

Indicator	A <sub>21</sub>	A <sub>22</sub>	A <sub>23</sub>
A <sub>21</sub>	1	5	1/3
A <sub>22</sub>	1/5	1	1/8
A <sub>23</sub>	3	8	1

Based on Equation 2, Table 2, Table 3 and Table 4 can be rewritten as follow. The superscripts of the judgement matrixes I and II indicate the number of the hierarchical structure. The subscripts of the judgement matrixes 1 and 2 illustrate the number of the criterion.

$$A^{(1)} = \begin{pmatrix} 1 & 1/3 \\ 3 & 1 \end{pmatrix}, A_1^{(ii)} = \begin{pmatrix} 1 & 1 & 7 & 5 & 2 \\ 1 & 1 & 7 & 5 & 2 \\ 1/7 & 1/7 & 1 & 1/2 & 1/5 \\ 1/5 & 1/5 & 2 & 1 & 1/3 \\ 1/2 & 1/2 & 5 & 3 & 1 \end{pmatrix}, A_2^{(ii)} = \begin{pmatrix} 1 & 5 & 1/3 \\ 1/5 & 1 & 1/8 \\ 3 & 8 & 1 \end{pmatrix}$$

Then three matrixes are given as follow in order to solve the ranking weight vectors by Equation 5 and Equation 6.

$$B^{(1)} = \begin{pmatrix} 0.25 & 0.25 \\ 0.75 & 0.75 \end{pmatrix}, B_1^{(ii)} = \begin{pmatrix} 0.35 & 0.35 & 0.32 & 0.35 & 0.36 \\ 0.35 & 0.35 & 0.32 & 0.35 & 0.36 \\ 0.05 & 0.05 & 0.05 & 0.04 & 0.04 \\ 0.07 & 0.07 & 0.09 & 0.07 & 0.06 \\ 0.18 & 0.18 & 0.23 & 0.21 & 0.18 \end{pmatrix}, B_2^{(ii)} = \begin{pmatrix} 0.24 & 0.36 & 0.23 \\ 0.05 & 0.07 & 0.09 \\ 0.71 & 0.57 & 0.69 \end{pmatrix}$$

Combing the three matrixes and Equation 7, the ranking weight vectors of the judgement matrixes of the reliability, basic reliability and mission reliability can be derived as follow.

$$w^{(1)} = \begin{pmatrix} 0.25 \\ 0.75 \end{pmatrix}, w_1^{(ii)} = \begin{pmatrix} 0.34 \\ 0.34 \\ 0.04 \\ 0.09 \\ 0.19 \end{pmatrix}, w_2^{(ii)} = \begin{pmatrix} 0.28 \\ 0.07 \\ 0.65 \end{pmatrix}$$

Then three eigenvectors can be given by using Equation 8.

$$\lambda^{(ii)} = \begin{pmatrix} 0.5 \\ 1.5 \end{pmatrix}, \lambda_1^{(iii)} = \begin{pmatrix} 1.79 \\ 1.79 \\ 0.22 \\ 0.37 \\ 1 \end{pmatrix}, \lambda_2^{(iii)} = \begin{pmatrix} 0.85 \\ 0.21 \\ 2.05 \end{pmatrix}$$

The results of the consistency check for the judgement matrix of the basic reliability and mission reliability are shown in Table 5. The maximum eigenvalues of the judgement matrix of the basic reliability and mission reliability are 5.08 and 3.07 by using Equation 9, respectively. Thus, the consistency ratios of them can be given as 0.02 and 0.06 by Equation 10 and Equation 11. Since all the consistency ratio is less than 0.1, the judgement matrix of the basic reliability and mission reliability are acceptable. The key indicator system for reliability evaluation of marine electrical system has been established and their relative weights and absolute weights are shown in Table 6.

**Table 5.** The results of the consistency check for the judgement matrix.

Matrix	Maximum eigenvalue	CI	RI	CR	Consistency
A <sub>1</sub>	5.08	0.02	1.12	0.02	Acceptable
A <sub>2</sub>	3.07	0.03	0.58	0.06	Acceptable

#### 4. Conclusion

The key indicator system for reliability evaluation of marine electrical system can be established by analytic hierarchy process. Developing the hierarchical structure and constructing the judgement matrix

play important roles in AHP, while solving the eigenvalue and check the consistency can grantee the validity and accuracy of the establishment and application of the key indicator system. AHP is an effective and useful method for discovering the operating condition variations and the potential safety hazard of the electrical system timely.

Because the database of design and operation of the ship's electrical system while be extended and updated furthermore by the growing demand of the mission and service, the lower hierarchies and indicators could be considered. New method for the weights of those additional indicators can be studied for improving the feasibility and advancement of the reliability evaluation.

**Table 6.** The key indicator system for reliability evaluation of marine electrical system.

Objective	Criterion	Indicator	Relative weight	Absolute weight
A	(0.25)	A <sub>11</sub>	0.34	0.09
		A <sub>12</sub>	0.34	0.09
		A <sub>13</sub>	0.04	0.01
		A <sub>14</sub>	0.09	0.02
		A <sub>15</sub>	0.19	0.05
	(0.75)	A <sub>21</sub>	0.28	0.21
		A <sub>22</sub>	0.07	0.04
		A <sub>23</sub>	0.65	0.49

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